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STABILITY AND CONTROL

Air Force Flight Test Center
Edwards Air Force Base, California

September 1968

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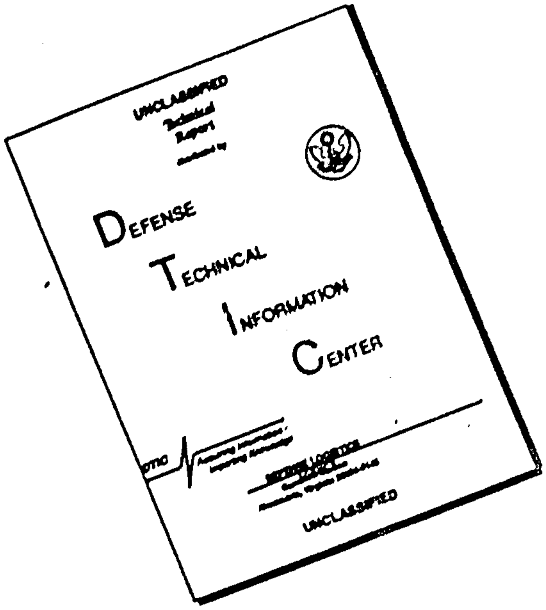

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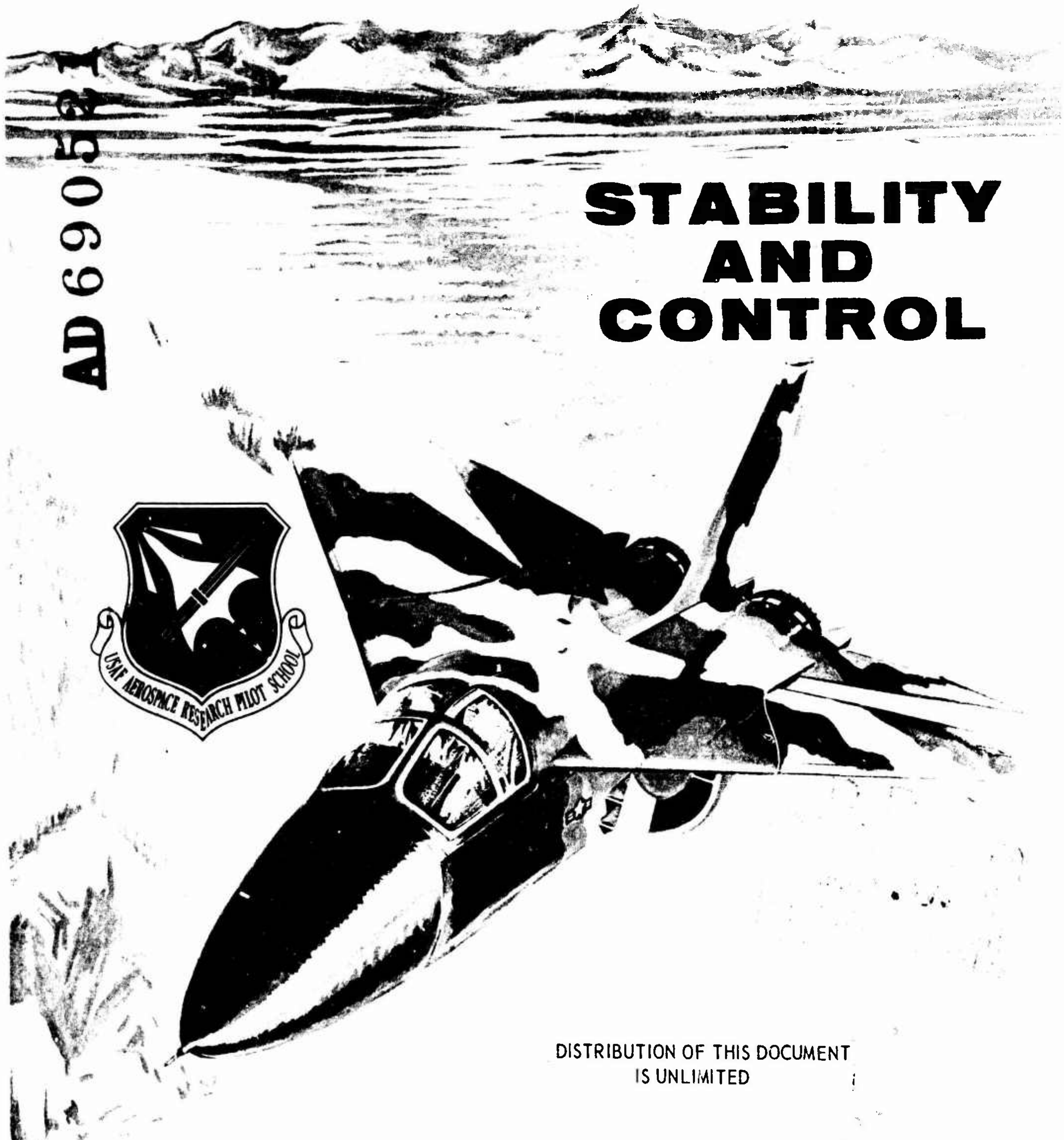

STABILITY AND CONTROL

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13. ABSTRACT

The stability and control handbook was compiled by the instructors of the USAF Aerospace Research Pilot School at Edwards AFB. The flight test techniques and data reduction methods presented have been developed at Edwards AFB. Although the theory portion of this handbook is oriented toward the academic portion of the training of test pilots, the handbook may be useful to others.

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14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
aircraft flight testing stability and control test pilot training stability and control theory stability and control test techniques test data reduction methods						

ERRATA SHEET

STABILITY AND CONTROL

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VOLUME I, CHAPTER 2

Page	Location	Correction
2.2	Fig 2.2 (Lower left Figure)	Delete: or slat
2.3	Column 1 Fig 2.3	Should read: α = Geometric angle of attack; α_o = Effective angle of attack; α_i = Induced angle of attack; ϵ = Downwash angle far behind the wing
2.5	Column 1 Bottom of page	Should read: $q_{stall} = \frac{\rho V^2_{stall}}{2} = \frac{nW}{C_{L_{max}} S}$
2.6	Column 1 First paragraph 2nd line	Delete parenthesis in the: W cos ([illegible]) Should read: W cos [illegible]
2.8	Column 2 Table 2.1	Delete. Change to:

TRIM POINTS

ALTITUDE (ft)	AIRSPEED (KIAS)	REMARKS
20,000	200	IP Demo
20,000	250	Student Practice
STALLS		
CRUISE CONFIGURATION		
20,000		V_{TRIM} 130 KIAS
n = 1.0		n = 2.0
IP Demonstrate Each, Student Practice Each		
α_{OL} Determination Trim Point at 20,000 Ft, 300 KIAS		
POWER APPROACH		
20,000		V_{TRIM} 140 KIAS
n = 1		n = 1.5
STUDENT PRACTICE		
α_{OL} Determination Trim Point at 20,000, 15[illegible] & 16[illegible] KIAS		

VOLUME I, CHAPTER 2 - ERRATA SHEET - STABILITY AND CONTROL

Page	Location	Correction
2.10	Column 2 2.8 DATA	Delete through f.

Change to:

2.8 DATA

Data to be Recorded

A continuous photo panel recording will be taken from before stall warning until after recovery on all stalls on which quantitative data are collected. The photo panel will be used to record the trim points. Alpha is one parameter of primary interest from both the trim points and the accelerated stalls. It will be used to develop a C_L vs α curve so that α_{0L} can be determined. α_s and α_{0L} will be used in the determination of MIL-F-8785A compliance for accelerated stall warning.

The following data should be hand recorded for each stall:

a. Indicated airspeed at stall warning (also actuate the event marker).

b. Indicated airspeed at stall (also actuate the event marker).

c. Altitude lost in the recovery.

d. Qualitative comments on:

1. Type and adequacy of stall warning.
2. Stall characteristics such as pitch and roll.
3. Control characteristics during all the three phases of the stall.
4. Type and effectiveness of recovery techniques.

e. Fuel counter readings.

f. Correlation number.

VOLUME I, CHAPTER 2 - ERRATA SHEET - STABILITY AND CONTROL

Page	Location	Correction
2.11	Column 1 Under "Data Presentation" 3rd line	Change: will to may. Delete: in the report. Delete: All the parameters for the time history may be obtained from the oscillograph record.
2.11	Column 1 2nd paragraph 1st line	Change: report to results
3.11	Column 1 2nd paragraph last line	Delete: paragraph 3.4.1.) Add: the military specification.)
2.11	Column 2 Figure 2.14	On Graph titled "Normal Acceleration" Change q to g

ERRATA SHEET

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VOLUME I, CHAPTER 3

Page	Col	Para	Line	Correction
3.3	1	-	6	Delete sentence beginning "It is used ---"
3.6	Figure 3.3			Ordinate is $\frac{d\delta_e}{dC_L}$
3.11	2	1	-	Delete entire paragraph and Fig's 3.17 and 3.18.

Insert "Para 3.6 Flight Path Stability.

Large aircraft or aircraft whose final approach speed is on the backside of the power curve are prone to have poor flight path stability characteristics. The requirements for acceptable flight path stability are found in MIL-F-8785 para 3.2.1.2. If an aircraft does not meet these requirements then it may be necessary to increase $V_{o_{min}}$, add direct lift control or an auto throttle.

To test for flight path stability, the aircraft is placed in the proper configuration for the power approach flight phase at low altitude. The airspeed is set at the proper $V_{o_{min}}$ for that gross weight and the power is reduced to acquire an approximate 3° glide path. This may be best determined by establishing a rate of descent calculated to give a 3° glide path. The steeper the glide path, the less power required to maintain $V_{o_{min}}$ and thus less help from power effects

in meeting the requirements. Smooth air and careful attitude flying are required to get good data. The altitude band must be narrow (2,000 feet or less) or the increase in power with decreasing altitude will invalidate the data. Since it is necessary to get a minimum of 4 points ($V_{o_{min}}$; $V_{o_{min}}$ -5, -10, +5), it may require more than one pass through the altitude band.

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VOLUME I, CHAPTER 4

Page	Col	Para	Line	Correction
4.5	2	last	-	ADD: "Sport" provides television (video tape playback capability) in lieu of the 16 mm film. "Sport" should be notified ASAP after the data mission if video tape replay is desired.

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ERRATA SHEET

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VOLUME I, CHAPTER 5

Page	Col	Para	Line	Correction
5.1	2	-	26	Delete "lb/g".
5.8	Figure 5.9			Put parentheses around "lb/g" and "g/RAD".

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ERRATA SHEET

STABILITY AND CONTROL

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VOLUME I, CHAPTER 7

Page	Col	Para	Line	Correction
7.3	Figure 7.4			Reverse sign of β on right and left abcissa.
7.5	2	2	8	CHANGE: 3.3.6.3a and 3.3.6.3b TO 3.3.6.3.1 and 3.3.6.3.2. ADD: 3.3.5.1, 3.3.5.2, 3.3.7.1 and 3.3.8.
7.7	2	2	3	Change VI to IX
			7	Change VI to IX
			12	Change 3.3.4.1a, 1b, 1c, 1d TO 3.3.4.1.1, .2, .3, .4.
			17	Change VI to IX
7.8	1	3	7.8	Change 3.3.4.1a, 3.3.4.1b, 3.3.4.1c and 3.3.4.1d TO 3.3.4.1.1, 3.3.4.1.2, 3.3.4.1.3 and 3.3.4.1.4.
			12, 13, 14	Delete 3.3.5, 3.3.5.1, 3.3.5.1a and 3.3.5.2. ADD: 3.3.2, 3.3.2.2, 3.3.2.2.1, 3.3.2.4.1, 3.3.2.5 and 3.3.4.2.
7.8	2	2	4	Change 10,000 feet AGL to 10,000 feet MSL (over flat terrain).
7.11	2	10	-	Add "This input is designed to get an aircraft response to a step input that will include at least one period of the Dutch Roll."

//

ERRATA SHEET

STABILITY AND CONTROL

FTC-TIH-68-10C2

VOLUME I, CHAPTER 9

Page	Col	Para	Line	Correction
9.3	1	3	6	Change 3.2.2.1a, 3.2.2.1b and Figure 1 to 3.2.2.1.1, 3.2.2.1.2, Table IV and Figure 1, 2, and 3.
9.11	1	5	7	Change Figure 1 to Figure 1, 2, and 3.
9.12	1	2	2	Delete "in Figure 2" ADD: "Applicable paragraphs and tables are 3.3.1.1 and Table VI.
9.14	2	3	2	Change V to VIII.
9.15	1	2	4	Change V to VIII.

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ERRATA SHEET

STABILITY AND CONTROL

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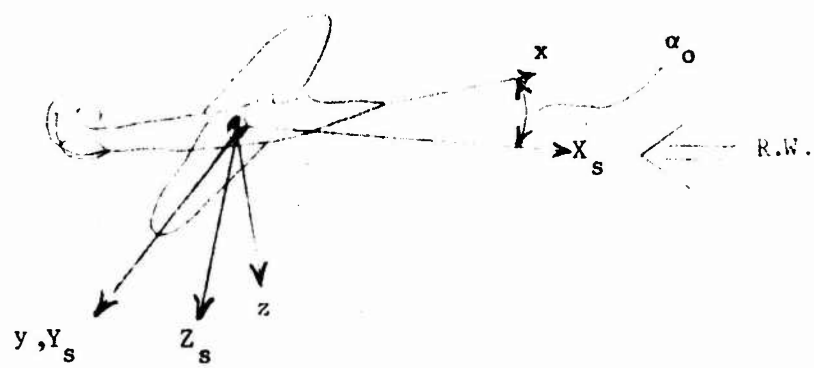
VOLUME II, CHAPTER 1

Page	Location	Correction
1.5		Correct Figure to look like this:



Page	Location	Correction
1.7	Fig. 1.6	Change Z to z, and place v in line out y axis.
1.7	Fig. 1.6	Should read: Caution - Other definitions of β are possible.
1.7	Fig. 1.7	Change α in Figure to γ.
1.8	Par 1.3.4.2	Change α to γ.
1.11	4th line from bottom	Should read: $P = -\dot{\psi} \sin \theta$
1.13	Fig. 1.10	Delete Describes.
1.15	Under(1.19)	Should read:

$$= \underbrace{\frac{d}{dt}\left|\vec{A}\right|_a + \vec{\varphi} \times \vec{A}}_{\text{By Equation (1.18) this is } \left.\frac{d\vec{A}}{dt}\right|_{xyz}} + \vec{\omega} \times \vec{A}$$

VOLUME II, CHAPTER 1 - ERRATA SHEET - STABILITY AND CONTROL

Page	Location	Correction
1.16	above(1.22)	Should read: From Eqn(1.16) this is By inspection this is $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ p & q & r \\ x & y & z \end{vmatrix} = \vec{\omega} \times \vec{A}$
1.19	(1.27)	Should read: $H = \sum_{o}^{i} m_i \vec{\rho}_i \times \vec{\mu}_i$
1.21	Fig. 1.18	Change Z to z
1.23	4th line from bottom	Delete the
1.29	Fig. 1.21	Change α^o to α_o

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STABILITY AND CONTROL

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<u>VOLUME II, CHAPTER 2</u>

Page	Col	Para	Line	Correction
2.2	2	-	6	Change L_w to N_w
2.9	2	-	5	Change α_w to a_w
2.11	Fig. 2.12			Change Proller to Propeller
2.14	(2.48)			Change h_h to h_n
2.18	Fig. 2.17a			Bottom intercept should be $\delta_{e_{C_L}} = 0$
2.20	(2.64)			Change $\frac{\delta C_h}{\delta \delta_e}$ to $\frac{\partial C_h}{\partial \delta_e}$
	(2.65)			Change $\frac{\delta C_h}{\delta \alpha_e}$ to $\frac{\partial C_h}{\partial \alpha_e}$

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VOLUME II, CHAPTER 3

Page	Col	Para	Line	Correction
3.2	Fig. 3.1			Ordinate should be $C_{L_{\delta_e}} \Delta\delta_e$
3.3	(3.15)			Place parentheses around remainder of line after $\frac{n_o W}{q\ S}$
	(3.16)			Place parentheses around remainder of line after $\frac{nW}{qS}$
3.5	(3.35)			Change + to = after $\frac{\Delta\delta_e}{n - n_o}$
3.7	Fig. 3.7			Horizontal vector should be labeled V
	2	-	16	Change $\frac{dC_m}{dq}$ to $\frac{\partial C_m}{\partial q}$
3.9	1	5	2	Change left to lift
3.10	2	last	4	Change low to high
3.17	Fig. 3.10			Move all decimal points one place to left
	2	2	-	Delete first sentence beginning "The stick-free ...."

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VOLUME II, CHAPTER 5

Page	Col	Para	Line	Correction
5.9	1	2	1	Change 5.13a to 5.14a
5.24	2	1	6	Change f(x) to f(t)
5.32	2	last	2	Line should read: "function of t. To emphasize this we rewrite (5.91) as:"

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VOLUME II, CHAPTER 6

Page	Col	Para	Line	Correction
6.1	2	last	title	Should read: "6.2 Equations of Motion"
6.2	1	-	-	Change (613) to (6.3)
6.2	2	-	-	Next to last equation - Should Read: $\bar{\beta} = \arcsin \frac{\bar{v}}{V_T} = \frac{\bar{v}}{u_o}$
6.3	1	-	note	Should read: " * D_o, L_o, etc., includes the normalizing factors $1/m$, $1/I_x$, i/I_y and $1/I_z$ as applicable"
6.4	1	-	-	Place a minus(-) sign in front of X in all equations.
6.5	1	eqn (6.24)		Change right hand side of equation to include $- \left[D_o + D_\alpha \bar{\alpha} + D_{\dot{\alpha}} \dot{\alpha} + D_{o_{w_t}} + D_u \bar{u} + D_q \bar{q} + D_{\delta e} \delta e + D_\theta \bar{\theta} \right] + \text{-------}$
6.5	1	-	-	Should read: "$D_o + \frac{To}{m} \cos \varepsilon + D_{o_{w_t}} = 0$ (steady state)
6.5	2	Eqn (6.27)		Add $\frac{L_{o_{w_t}}}{V_T}$

VOLUME II, CHAPTER 6, ERRATA SHEET, STABILITY AND CONTROL

Page	Col	Para	Line	Correction
6.5	2	-	-	Add $\frac{L_{\alpha_{w_t}}}{V_T}$ to L_o + Thrust Terms = 0
6.7	1	2	2	Should read: "essentially a constant airspeed, varying angle of attack motion (Figure 6.3).
6.7	1	Eqn (6.38)		2nd line, 1st column, numerator should read: "$s^2 - sM_q$"
6.8	1	Fig. 6.4		Change δ to ζ
6.15	1	Eqn (6.55)		Should read: "$\frac{F_y}{mV_T} = \frac{\dot{v} + ru - pw}{V_T} = Y_o + Y_\beta \bar{\beta} + Y_{\dot{\beta}} \dot{\beta} + Y_{\alpha_{w_t}} + Y_p \bar{p} + Y_r \bar{r} + Y_{\delta a} \delta a + Y_{\delta r} \delta r + Y_\varphi \bar{\varphi}$
6.15	2	Eqn (6.56)		Should read: "$\frac{G_x}{I_x} = \dot{p} - \dot{r} \frac{I_{xz}}{I_x}$ + --------"
6.17	2	last	6	Change X_{xz} to I_{xz}

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VOLUME II, CHAPTER 7

Page	Col	Para	Line	Correction
7.6	2	Eqn (7.19)		Should Read: "$\dot{q} = \frac{V^2 C_m}{2\mu K_y^2} + \left(\frac{I_z - I_x}{I_y}\right) pr + \frac{M_{gyro}}{I_y} + \frac{M_{other}}{I_y}$"
7.11	2	Eqn (7.21)		Add minus (-) sign in front of $pr\ (I_z - I_x)$
7.12	2	7.8.4		Change (7.18) to (7.21)
7.13	1	last	3	Change or perturbations to of perturbations.
7.15	2	-	-	Top of page; Change $\vec{H}_E \times \bar{\omega}$ to $\bar{\omega} \times \vec{H}_E$
7.15	2	last	2	Change last $\dot{q}_o$ to q_o

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VOLUME II, CHAPTER 8

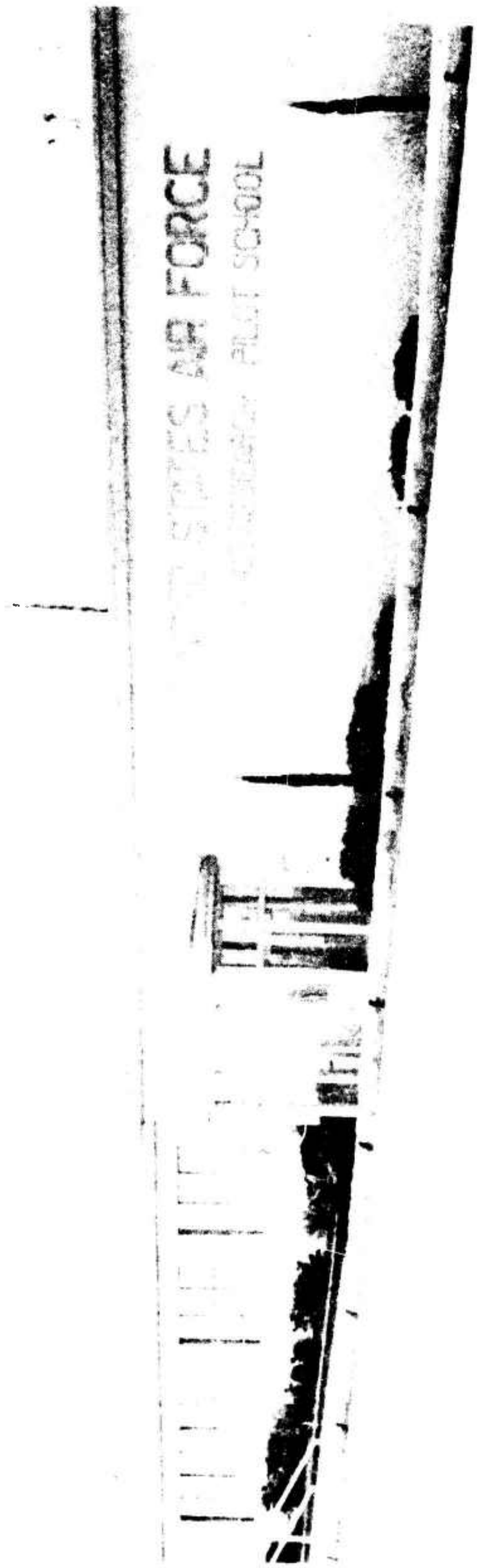
Page	Col	Para	Line	Correction
8.4	1	Eqn (8.7)		Delete "T" subscript to V
		Eqn (8.8)		Delete "T" subscript to V
8.5	-	Fig. 8.5		Add M_2 by weight in tail and M_1 by weight under canopy.
8.7	-	Fig. 8.7		Title should read: "Kinematic Coupling, Stable Rolling of an Aircraft with Infinitely Large Inertia or Negligible Stability in Pitch and Yaw.
8.7	-	Fig. (8.8)		Delete "Kinematic Coupling" in title
8.9	1	6	3	Change N_p to N_r

foreword........

Stability and Control is that branch of the aeronautical sciences that is concerned with giving the pilot an aircraft with good handling qualities. As aircraft have been designed to meet greater performance specifications, new problems in Stability and Control have been encountered. The solving of these problems has advanced the science of Stability and Control to the point it is today.

This handbook has been compiled by the instructors of the Aerospace Research Pilot School for use in the Stability and Control course. Most of the material in Volume II of this handbook has been extracted from several reference books and is oriented towards the test pilot. The flight test techniques and data reduction methods in Volume I have been developed at the Air Force Flight Test Center, Edwards Air Force Base, California. I received my first copy of this handbook in 1952 and have managed to acquire each subsequent revision to keep up with the state-of-the-art in Stability and Control. This handbook is primarily intended to be used as an academic text in our School, but if it can be helpful to anyone in the conduct of Stability and Control testing, be our guest.

HAROLD W. CHRISTIAN, JR., Colonel, USAF
Commandant
USAF Aerospace Research Pilot School


STATES AIR FORCE
PILOT SCHOOL

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DEFINITIONS, ABBREVIATIONS, AND SYMBOLS

● GENERAL

a	speed of sound, ft sec^{-2}, linear acceleration, ft sec^{-2}
a	slope of the lift curve, $\partial C_L/\partial a$, deg^{-1} or $radian^{-1}$
a.c.	aerodynamic center
A/C	aircraft
AR	aspect ratio, b^2/S
b	span, ft
b_o	lumped damping, $2\zeta\,\omega_n$, sec^{-1}
c	chord, ft
$\bar{c}$	mean aerodynamic chord, ft
cg	center of gravity
c.p.	center of pressure
C	chordwise force, x component of resultant aerodynamic force, lb
$C_{1/2}$, $_{1/10}$	cycles to damp to 1/2 amplitude, 1/10 amplitude
CO	combat configuration
C.N.	counter number
CR	cruise configuration
D	drag, lb or dive configuration
e	wing efficiency factor
f	frequency, cycles sec^{-1}
F	force, lb
$F_{x,y,z}$	force along the x, y, or z body axis, lb
F/C	fuel counter
FLT	flight
FWD	forward
g	acceleration of gravity, ft sec^{-2}
G	gearing, $\delta_e/l_s\,\delta_s$, ft^{-1}, or glide configuration
h	altitude, ft or cg position
h_m	maneuver point
h_n	neutral point
H_e	hinge moment, ft lb., e elevator, r rudder, ai aileron
HP	horsepower, 550 ft lb sec^{-1}
$I_{x,y,z}$	moment of inertia about x, y, or z axis, slug ft^2
I_{xz}	product of inertia, slug ft^2
$k_{x,y,z}$	radius of gyration for particular axis noted, ft
k_o	lumped spring constant, ω_n^2, sec^{-2}
K	constant with various definitions
l	length, ft
ln	natural logarithum
L	lift, lb., rolling moment, ft-lb., or landing configuration
Lim	limit
Long	longitudinal
m	mass
MAC	mean aerodynamic chord, ft
M	pitching moment, ft lb., or Mach number
MAN	maneuvering
MAX	maximum
MIL	military
MRP	military rated power, maximum power (not including augmentation) at which engine can be operated for a specified period
n	normal acceleration or load factor in g units
N	normal force, z component of resultant aerodynamic force, lb., yawing moment, ft-lb.
NRP	normal rated power, maximum power for continuous engine operation
p	roll rate, (x component of body-axis angular velocity), deg sec^{-1} or rad sec^{-1}, static pressure
p_a	ambient static pressure, lb ft^{-2}
P	power on configuration
PA	power approach configuration
PLF	power for level flight
q	dynamic pressure, $1/2\,\rho_o V_e^2$, $0.7\,p_a M^2$, lb ft^{-2}
r	yaw rate (z component of body-axis angular velocity) deg sec^{-1} or rad sec^{-1}
R	resultant force, lb
R/C	rate of climb
RSF	rudder side force, lb
RT	remote trigger
RW	relative wind
s	linear distance, ft
S	wing area, ft^2
SF	single frame
t	time, sec
T	period, sec, trigger, temperature, deg., thrust, lb
T.E.	trailing edge
TO	takeoff configuration

TOT	total
u, v, w	incremental velocity along x, y, and z body axis, respectively, ft sec^{-1}
VT	total true axial velocity, ft sec^{-1}
$V_H V_V$	tail volume, $l_t S_t / c_w S_w$, horizontal, vertical
VSSF	vertical stabilizer side force, lb
W	weight, lb
x	distance in x body axis direction, ft
y	distance in y body axis direction, ft
Y	side force, lb
z	distance in z body axis direction, ft
$\dot{\theta}, \dot{\alpha}$ etc	time rate of change d/dt
<	less than
>	greater than

● GREEK SYMBOLS

α or α	angle of attack, deg or radian
β	angle of sideslip, deg or radian
γ	flight path angle with respect to horizontal, deg or radian
δ	control deflection, deg or pressure ratio P_a/P_o
Δ	incremental change in particular parameter referred to
ϵ	downwash angle, deg
ζ	damping ratio
η	efficiency with varied meanings
θ	pitch angle, deg or radian
μ	relative aircraft density, $m/\rho Sc$ or $m/\rho Sb$
π	pi, 3.1416
ρ	density, slug ft^{-3}
σ	density ratio, ρ_a/ρ_o
τ	aerodynamic time, $m/\rho SV$, sec or elevator effectiveness parameter α_T/e
Φ	bank angle, deg or radian
ψ	yaw angle, deg or radian
ω	oscillating frequency, rad sec^{-1}
ω_n	undamped natural frequency, rad sec^{-1}
Ω	total angular velocity, rad sec^{-1}

● SUBSCRIPTS

a	aileron, ambient
B	body
c	calibrated
D	damped
e	elevator or equivalent
eff	effective
f or fus	fuselage
h	horizontal
i	indicated or induced
ic	instrument corrected
L	left, landing, or limit
L.E.	leading edge
max	maximum
n	natural undamped
o	sea level condition, trim value or zero lift condition
O.L.	zero lift
p	pressure, propeller, or parasite
pc	position corrected
r	rudder
R	right
S	stabilizer, stall, stick
t	tail, tab, or true
T	true
v	vertical
w	wing or warning

● AERODYNAMIC COEFFICIENTS AND NON-DIMENSIONAL STABILITY DERIVATIVES

C_c	chordwise coefficient, C/qS_w
C_D	drag coefficient, D/qS_w
C_h	hinge moment coefficient, HM/qS_eC_e
C_L	lift coefficient, L/qS_w
C_l	rolling moment coefficient, L/qS_wb_w
C_m	pitching moment coefficient, M/qS_wc_w
C_n	yawing moment coefficient, N/qS_wb_w
C_N	normal force coefficient, N/qS_w
C_T	thrust coefficient, T/qS_w
C_y	side force coefficient, Y/qS_w
$C_{h\alpha}$	$\partial C_h/\partial \alpha$ floating tendency, deg^{-1} or rad^{-1}
$C_{h\delta}$	$\partial C_h/\partial \delta$ restoring tendency, deg^{-1} or rad^{-1}
$C_{L\alpha}$	$\partial C_L/\partial \alpha$ slope of the lift curve, deg^{-1} or rad^{-1}
$C_{m\alpha}$	$\partial C_m/\partial \alpha$ static longitudinal stability derivative, deg^{-1} or rad^{-1}
$C_{m\alpha}$	$\partial C_m/\partial(\alpha c/2U_o)$, rad^{-1}
$C_{m\theta}$	$\partial C_m/\partial(\theta c/2U_o)$, rad^{-1}
$C_{m\delta}$	$\partial C_m/\partial \delta$ longitudinal control power, deg^{-1} or rad^{-1}
$C_{n\beta}$	$\partial C_n/\partial \beta$ static directional stability derivative, deg^{-1} or rad^{-1}
C_{n_r}	$\partial C_n/\partial(rb/2V_T)$ damping in yaw, deg^{-1} or rad^{-1}
C_{n_p}	$\partial C_n/\partial(pb/2V_T)$ deg^{-1}
$C_{n\delta_a}$	$\partial C_n/\partial \delta a$, deg^{-1}
$C_{n\delta_r}$	$\partial C_n/\partial \delta r$, rudder control power, deg^{-1}
$C_{l\beta}$	$\partial C_l/\partial \beta$, dihedral effect, deg^{-1}
C_{l_r}	$\partial C_l/\partial(rb/2V_T)$, deg^{-1}
C_{l_p}	$\partial C_l/\partial(pb/2V_T)$, damping in roll, deg^{-1}
$C_{l\delta a}$	$\partial C_l/\partial \delta a$, aileron power, deg^{-1}
$C_{l\delta_r}$	$\partial C_l/\partial \delta r$, deg^{-1}
$C_{Y\beta}$	$\partial C_Y/\partial \beta$, deg^{-1}
$C_{Y\delta_r}$	$\partial C_Y/\partial \delta r$, deg^{-1}
C_{Y_r}	$\partial C_Y/\partial(rb/2V_T)$, deg^{-1}

● GREEK ALPHABET

Caps	Lower Case	Greek Name	English Sound
Α	α or α	Alpha	A
Β	β	Beta	B
Γ	γ	Gamma	G
Δ	δ	Delta	D
Ε	ε	Epsilon	E short
Ζ	ζ	Zeta	Z
Η	η	Eta	E long
Θ	θ	Theta	Th
Ι	ι	Iota	I
Κ	κ	Kappa	K
Λ	λ	Lambda	L
Μ	μ	Mu	M
Ν	ν	Nu	N
Ξ	ξ	Xi	X
Ο	ο	Omicron	O short
Π	π	Pi	P
Ρ	ρ	Rho	R
Σ	σ	Sigma	S
Τ	τ	Tau	T
Υ	υ	Upsilon	U
Φ	φ	Phi	F
Χ	χ	Chi	Ch
Ψ	ψ	Psi	Ps
Ω	ω	Omega	O long

• CONVERSION FACTORS AND USEFUL NUMBERS

CONVERSION FACTORS

MULTIPLY NO. OF	BY	TO OBTAIN
DEGREES	1.745×10^{2}	RADIANS
RADIANS	57.3	DEGREES
KNOTS	1.689	FEET PER SECOND
FEET PER SECOND	592p	KNOTS

USEFUL NUMBERS

$\pi = 3.1416$

$g = 32.2 \text{ ft sec}^{-2}$

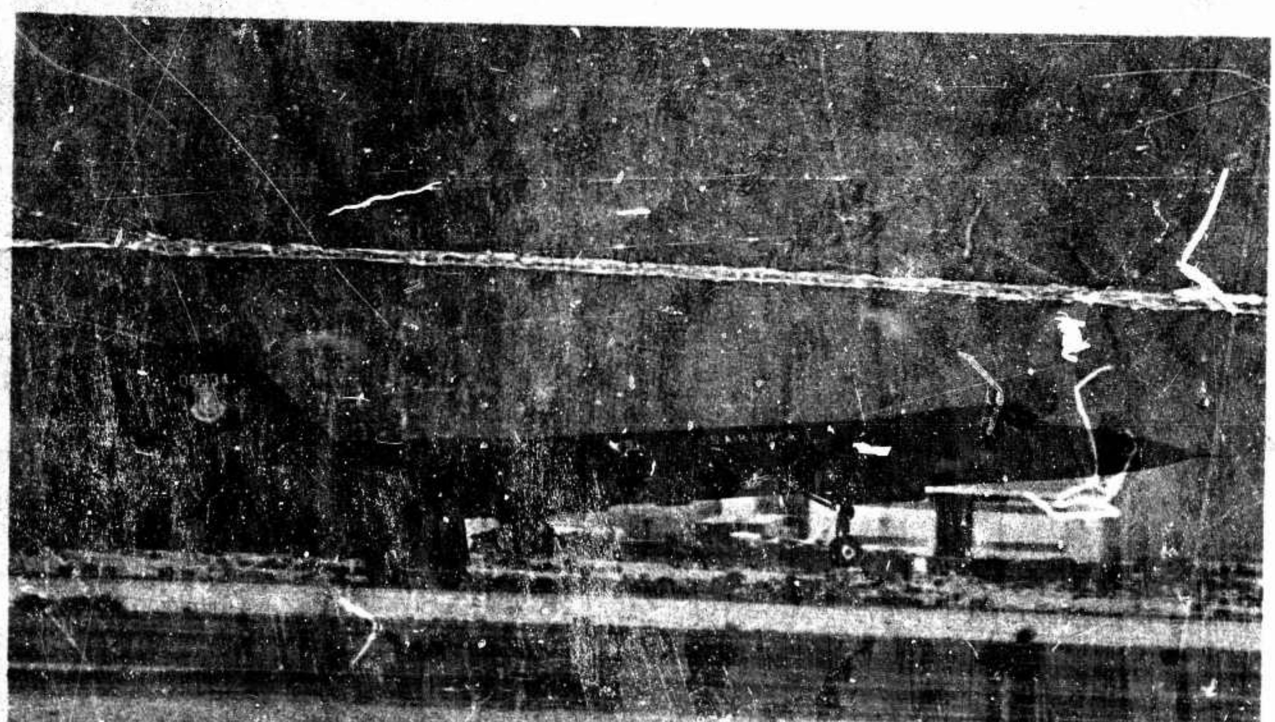
$\rho_o = 0.002378 \text{ slug ft}^{-3}$

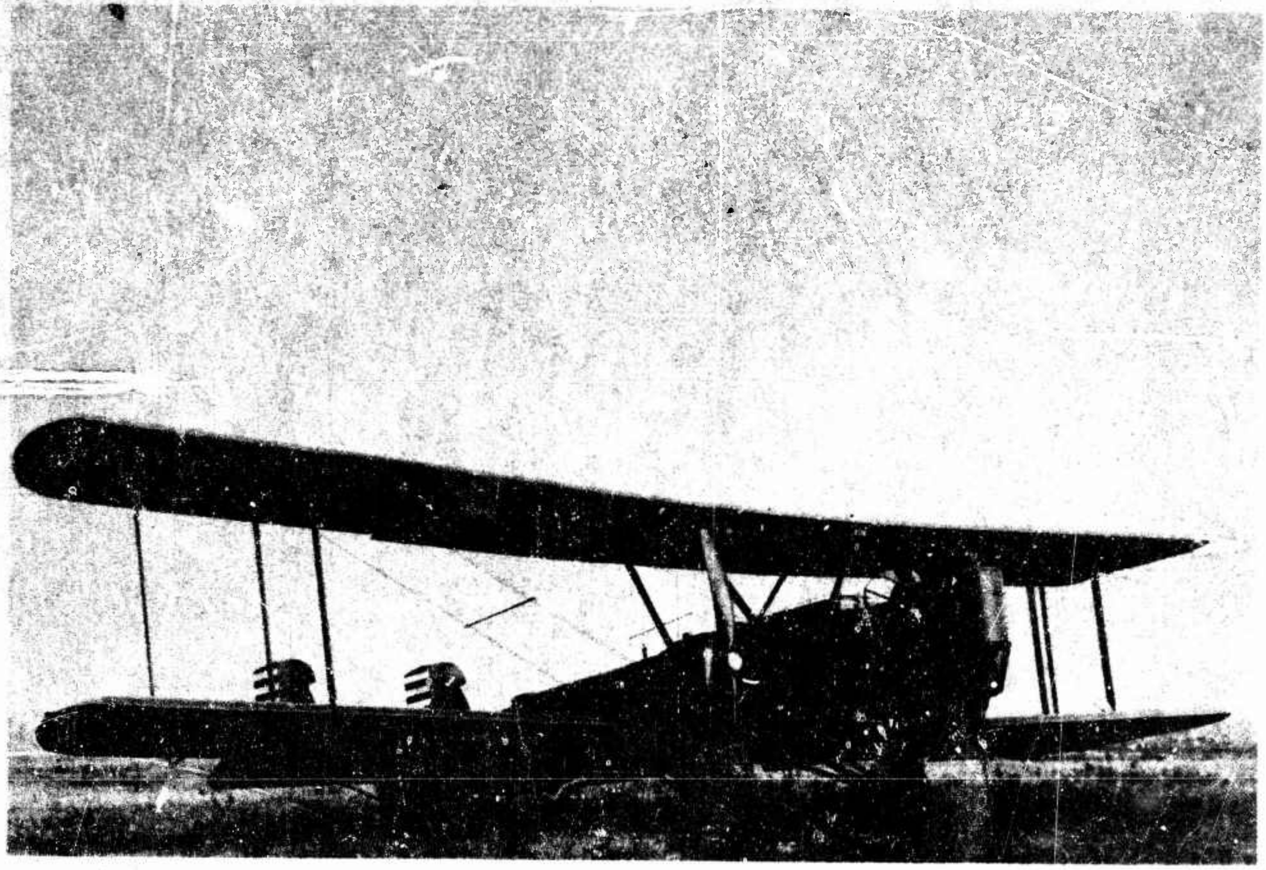
$P_o = 2116 \text{ lb ft}^{2}$

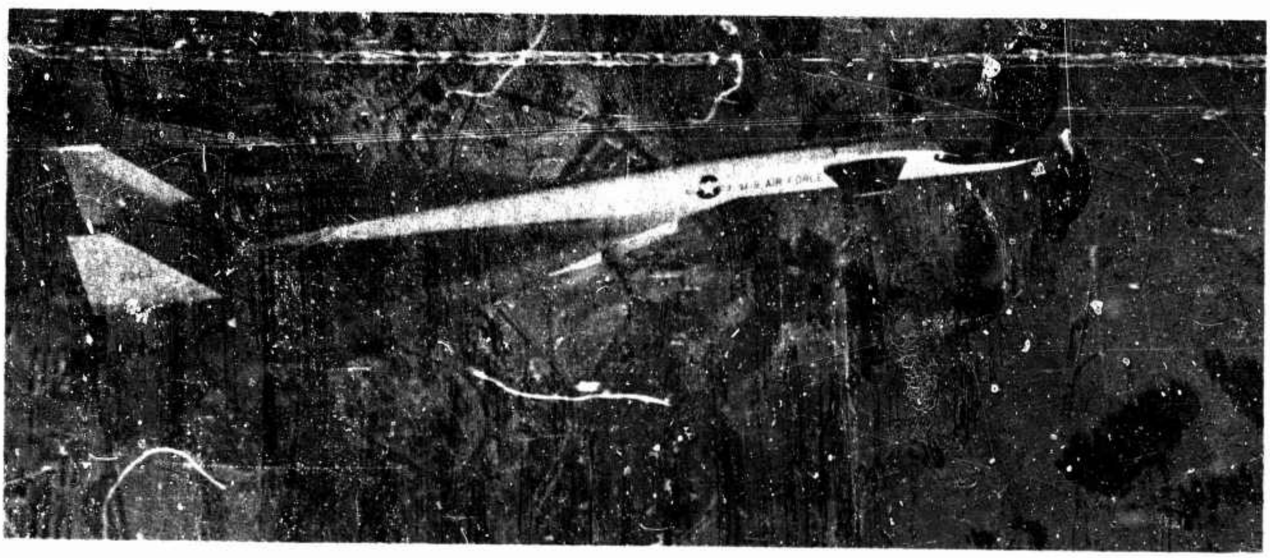
		F-104	T-38	T-33	B-57
	S ft^2	196.1	170	234.8	960
	b ft	21.94	25.25	37.54	64.0
	C ft			6.72	16.208
I_x (FULL FUEL) SLUG-Ft^2				47687	153442
I_x (50% FUEL) SLUG-Ft^2		362.0	1438	34910	98425
I_x (EMPTY) SLUG-Ft^2				13480	97393
I_y (FULL FUEL) SLUG Ft^2				21761	122057
I_y (50% FUEL) SLUG Ft^2		70881	25874		115000
I_y (EMPTY) SLUG Ft^2				20552	108048
I_z (FULL FUEL) SLUG Ft^2				67967	
I_z (50% FUEL) SLUG Ft^2		59598	26779		
I_z (EMPTY) SLUG Ft^2				32589	

USAF AEROSPACE RESEARCH PILOT SCHOOL

VOLUME I
FLIGHT TEST TECHNIQUES






U.S. AIR FORCE

CHAPTER 1

INTRODUCTION TO STABILITY FLIGHT TEST TECHNIQUES

1.1 ATTITUDE FLYING

In stability flight testing, attitude flying is absolutely essential. Under a given set of conditions (altitude, power setting, center of gravity location, etc.) the aircraft's speed is entirely dependent upon the attitude. This being the case, the pilot's ability to fly the aircraft accurately depends upon his ability to see and interpret small attitude changes. This can best be done by reference to the outside horizon. Any change in aircraft attitude will be noticed by reference to the distant horizon long before the aircraft instruments (airspeed, etc.) show a change. Thus, it is often possible to change the attitude of the aircraft from a disturbed position back to the required position before the airspeed has a chance to change. The outside horizon is also very useful as a rate instrument. If a stabilized point is required, hold zero rate of change of pitch; i.e., hold aircraft attitude fixed in relation to some outside reference which calls for one particular speed. If, as in acceleration run, the airspeed is continuously increasing or decreasing, one should look for a steady, smooth, and extremely slow rate of change of the aircraft's attitude.

It is suggested that the method of lining up a particular spot on the aircraft with some outside reference can be useful at times but is often wasteful of time. A general impression is often all that is necessary; i.e., it is possible to see that the aircraft rate of pitch is zero by use of the pilot's peripheral vision while also glancing at the airspeed indicator or some other cockpit instrument. As soon as the pilot notes a rate of change of pitch, he can make proper control movements to correct the attitude of the aircraft. The pilot should always be aware of the outside view even while reading the instruments.

In flight tests involving turning flight, this overall view of the horizon is of utmost importance in order to be able to hold constant velocity or Mach number. If the airspeed is high the nose should be raised and then stabilized at the new position required, using the horizon as a displacement and a rate instrument. If a change of aircraft attitude is necessary, this change should be made relative to the present picture until the rate of change of aircraft attitude again goes to zero at the new stabilized condition.

If it is necessary to stabilize on an airspeed several knots from the existing airspeed, time can be saved by overshooting the required pitch attitude and using the rate of change of airspeed as an indication as to when one should raise or lower the nose to the required position. A little practice will allow the pilot to stabilize on a new airspeed with a minimum amount of airspeed overshoot in the least time.

01.2 TRIM SHOTS

Prior to each stability flight test requiring photopanel or oscillograph data, a trim shot will be taken near the test pressure altitude (±100 feet). The trim shot will be made using the remote camera and oscillograph switch (not the stick trigger) so that no control forces will be inadvertently fed to the system. The trim shot is used primarily to make any necessary corrections to the force-measuring equipment readings. For example if the stick force gage reads +0.1 with no force applied, it is apparent that 0.1 must be subtracted from all stick force readings for this particular test.

The means of obtaining the different force information will be covered in detail in class; however, it should be kept in mind that it is possible to get erroneous rudder force information if the foot is placed improperly on the rudder bar. The strain gages are located on the lower rectangular pad and the foot should be placed centrally on this pad. Care should be taken not to apply any force to the toe pads since force applied here will not register on the force-measuring equipment. When making force measurements using the instrumented stick grip, it is very important that no extraneous force inputs are made by torquing the stick grip. The force measurements should be taken by using straight fore and aft or left and right force inputs on the center of the stick grip.

The importance of proper trim in stability flight testing cannot be overemphasized. Most of the tests involve force information and therefore it is essential that the aircraft be properly trimmed at the desired speed since one is interested in forces necessary to fly in conditions differing from the trim condition.

In order to stabilize and trim at a particular speed or Mach number at a constant altitude, the speed should first be established by placing the aircraft in the required attitude to give this speed. While obtaining this approximate attitude by reference to the outside horizon, the throttle setting should be changed to give zero rate of climb at the proper test altitude. Minute changes in attitude may be necessary in order to hold the exact airspeed as the power is changed. Once the proper attitude and power setting have been established, the force should be trimmed to zero while holding the required control position to give the required attitude. Release the stick and check for a change in pitch attitude. If the nose starts up or down put the nose back at the trim position with the stick and retrim. Then repeat the procedure. The lateral and directional controls (aileron and rudder) should be used in the proper manner to hold the wings level, maintain a constant heading, and keep the ball in the center of the turn and bank indicator. The necessary forces should be held in order to accomplish this and then the forces should be relieved by proper trim actuation. As in all flying, the pilot who can get the aircraft trimmed most accurately and quickly is the pilot who can do the most things simultaneously. For example, the pilot who can make the required attitude correction while adjusting the power will become trimmed before the pilot who flies strictly by the numbers. The often-used method of moving the trim device and allowing the aircraft to seek a new speed "hands off" is very time-consuming and inaccurate. Hold the aircraft attitude fixed and then relieve the existing control forces. If the aircraft gains or loses a little altitude during the trimming process the parameters of interest in stability testing will not have changed significantly. Therefore if the altitude is within

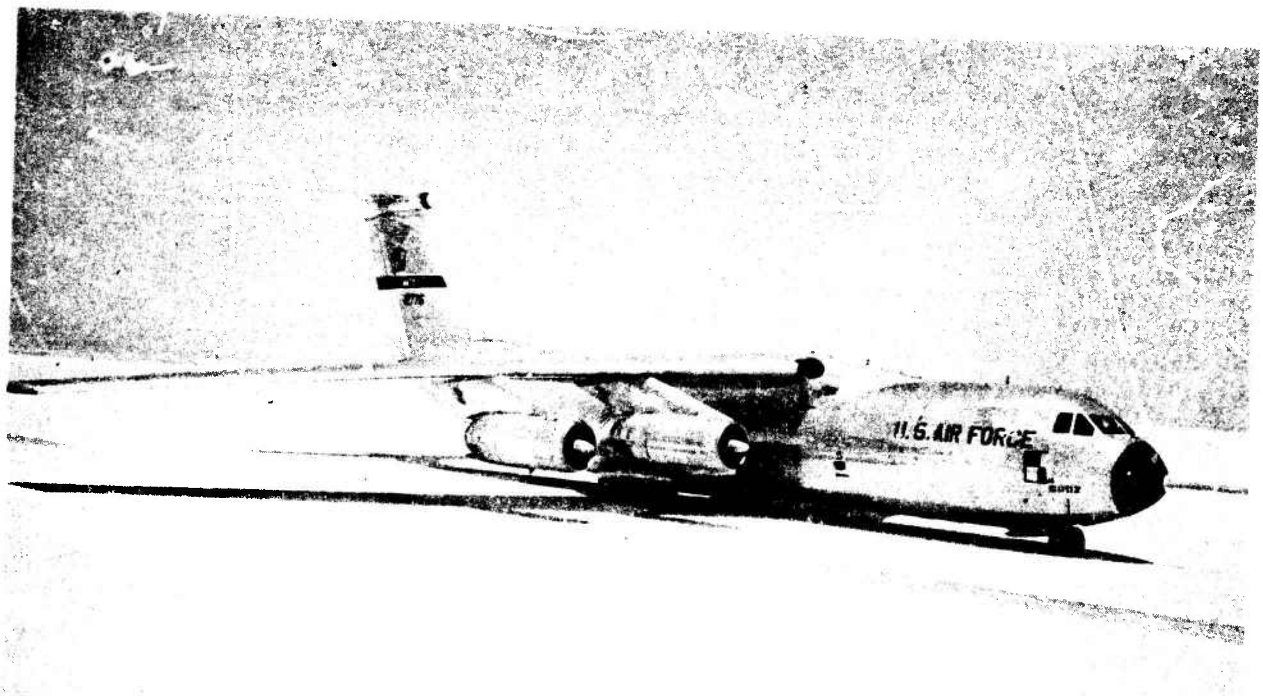
100 feet of the test altitude, the pilot should then take his trim shot.

1.3 TIMING

Proper utilization of time is always of paramount importance in conducting a test program. As a consequence, proper preflight preparation is absolutely essential. The complete test should be well in mind prior to takeoff. The pilot should always be thinking ahead and planning what he is to do next, keeping himself properly positioned in respect to the airfield. A flight that is well planned prior to flight has a very good possibility of working out well.

1.4 PRIMARY OBJECTIVES

All stability flight tests will be flown with the prime objective of giving the prospective user the most information possible about the particular aircraft. This will be done by noting the aircraft's degree of compliance with the latest specification of flying qualities along with any other information that the test pilot feels essential for safe, effective use of the machine under all conditions.



1.4

CHAPTER 2

STALLS

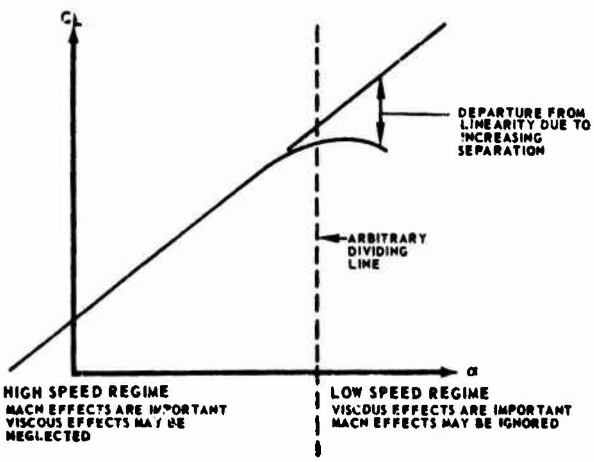
●2.1 INTRODUCTION

Stall speed is the minimum steady speed attainable, or usable, in flight. A sudden loss of lift occurring at a speed just below that for maximum lift is considered the "conventional" stall, although it has become increasingly common for the minimum speed to be defined by some other characteristic, such as a high sink rate, an undesirable attitude, loss of control about any axis, or a deterioration of handling qualities.

For rather obvious safety and operational reasons, determination of stall characteristics is a first-order-of-business item in flight testing a new aircraft. Stall speeds are also required early in the test program for the determination of various test speeds.

●2.2 SEPARATION

Figure 2.1



Separation, a condition wherein the streamlines fail to follow the body contours, produces a large disturbed wake behind the body and results in a pressure distribution greatly different from the that of attached flow. On an aircraft, these changes in turn produce:

a. A loss of lift

b. An increase in drag

c. Control problems due to:

   1. Control surfaces operating in the disturbed wake

   2. Changes in the aerodynamic pitching moment due to a shift in the center of pressure and an altered downwash angle.

Separation occurs at a point where the boundary layer kinetic energy has been reduced to zero, therefore the position and amount of separation is a function of the transport of energy into and out of the boundary layer and of dissipation of energy within the boundary layer.

Some factors which contribute to energy transport are:

a. <u>Turbulent (non-laminar) flow:</u> Higher energy air from upper stream tubes is mixed into lower stream tubes. This type flow, characterized by a full velocity profile, occurs at high values of Reynolds number (R_e) and involves <u>microscopic</u> turbulence.

b. Vortex generators: These devices produce macroscopic turbulence to circulate high energy air down to lower levels.

c. Slats and slots: These devices inject high energy air from the underside of the leading edge into the upper surface boundary layer.

d. Boundary Layer Control: The blowing type of boundary layer control (BLC) injects high energy air into the boundary layer; while the suction type removes low energy air.

Two examples of energy dissipation functions are:

a. Viscous friction: Energy loss varies with surface roughness and distance traveled.

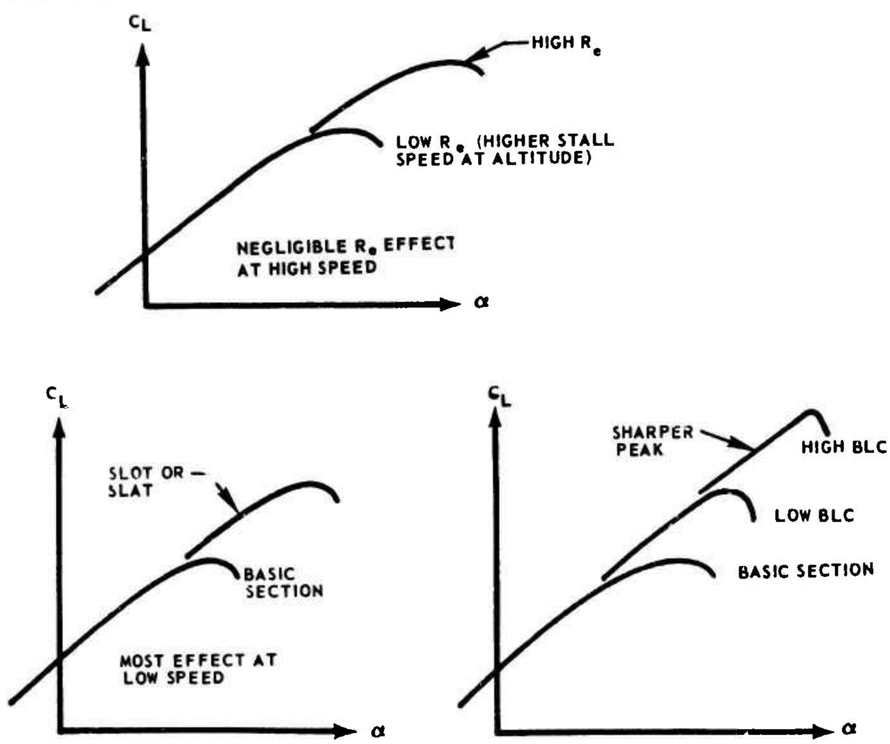
b. Adverse pressure gradient: Boundary layer energy is dissipated as the air moves against the adverse pressure gradient above a cambered airfoil section. The rate of energy loss is a function of:

1. Body contours - such as camber, thickness distribution, and sharp leading edges.

2. Angle of attack - Increased angle of attack steepens the adverse pressure gradient.

Some typical coefficient of lift versus angle of attack (C_L versus α) curves illustrating these effects are shown in figure 2.2.

Figure 2.2



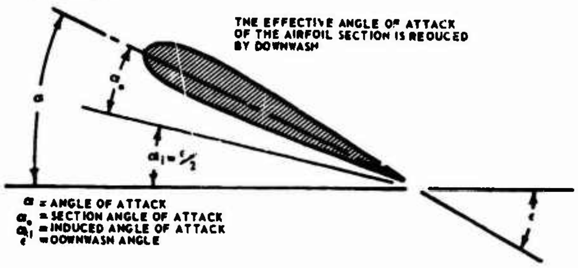
2.3 THREE-DIMENSIONAL EFFECTS

2.3 A three-dimensional wing exhibits aerodynamic properties considerably different from those of the two-dimensional airfoil sections of which it is formed. These differences are related to the planform and the aspect ratio of the wing.

Planform:

Downwash, a natural consequence of lift production by a real wing of less than infinite span, reduces the angle of attack at which the individual wing sections are operating.

Figure 2.3



An elliptical wing has a constant value of downwash angle along its entire span. Other planforms, however, have downwash angles that vary with position along the span. As a result, the lift coefficient for a particular wing section may be more or less than that of nearby sections, or that of the overall wing. Airfoil sections in areas of light downwash will be operating at high angles of attack, and will reach stall first. Stall patterns therefore depend on the downwash distribution, and vary predictably with planform as shown in figure 2.4.

Sweptback and delta planforms suffer from an inherent spanwise flow. This is caused by the outboard sections being located to the rear, placing low pressure areas adjacent to relatively high pressure areas.

Figure 2.4

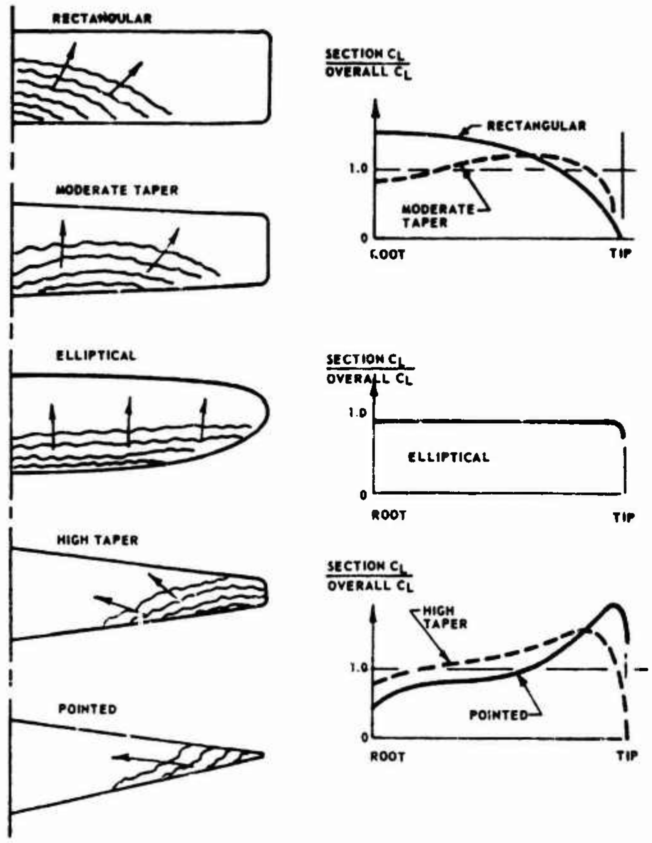
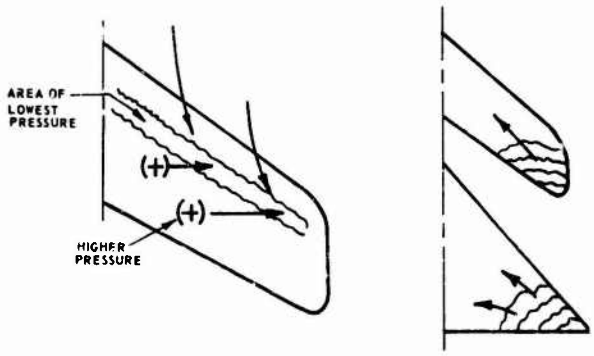


Figure 2.5



This spanwise flow transports low energy air from the wake of the forward sections outboard toward the tips, inviting early separation. Both the sweptback and delta planforms display tip-first stall patterns.

Pointed or low chord wing tips are unable to hold the tip vortex, which moves further inboard with increasing angle of attack.

Figure 2.8

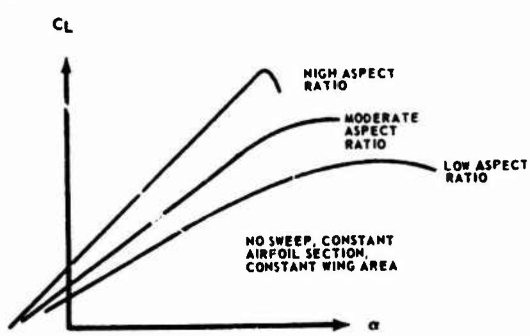
The extreme tips operate in upwash and in the absence of aerodynamic fixes such as twist or droop are completely stalled at most angles of attack.

Aspect Ratio:

Aspect ratio may be considered an inverse measure of how much of the wing is operating near the tips. Wings of low aspect ratio (much of the wing near the tip) require higher angles of attack to produce a given lift.

The curves of figure 2.7 illustrate several generalities important to stall characteristics. High aspect ratio wings have relatively steep lift curve slopes with well defined peaks at $C_{L_{max}}$. These wings have a relatively low angle of attack (and hence pitch angle) at the stall, and are usually characterized by a rather sudden stall break.

Figure 2.7



Low aspect ratio wings display the reverse characteristics; high angle of attack (high pitch angles) at slow speeds and poorly defined stalls. They can frequently be flown in a high sink rate condition to the right of $C_{L_{max}}$ where drag increases rapidly.

Aerodynamic Pitching Moment:

On almost all planforms the center of pressure moves forward as the stall pattern develops, producing a noseup pitching moment about the aircraft center of gravity (cg).

This moment is not great on most straight wing planforms and the characteristic root stall of these wings adds a compensating nosedown moment such that a natural pitchdown tendency exists at high angles of attack. This occurs because the stalled center section produces much less downwash in the vicinity of the horizontal tail, decreasing its download. If the tail actually enters the turbulent wake the nosedown moment may be further intensified due to a decrease in elevator effectiveness. This latter case usually provides a natural stall warning in the form of airframe and control buffet.

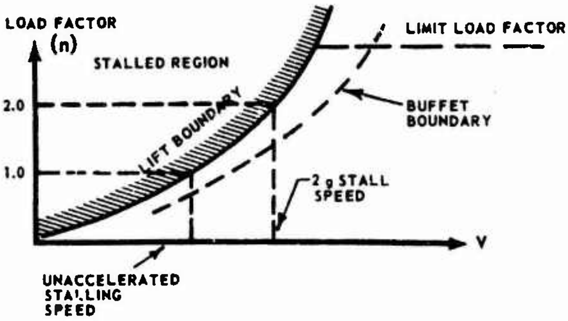
On swept-wing and delta planforms the moment produced by the center of pressure (c.p.) shift is usually more pronounced and the moment contributed by the change in downwash at the tail in this case is noseup. This occurs because the wing root section remains unstalled, producing greater lift and greater downwash as the angle of attack increases. The inboard movement of the tip vortex system also increases the downwash behind the center of the wing. Horizontal tails even in the vicinity of this increased downwash will produce more download. If the tail is mounted such that it actually enters the downwash area at high angles of attack, such as on the F-101, an uncontrollable pitchup may occur.

Many fixes and gimmicks have been used to alter lift distribution and stall patterns. Tip leading edge extensions, tip slots and slats, tip washout and droop, fences and root spoilers are but a few. Horizontal tail position is also subject to much adjustment such as has been necessary on the F-4C.

• 2.4 LOAD FACTOR CONSIDERATIONS

The relationship between load factor (n) and velocity may be seen on a V-n diagram.

Figure 2.8



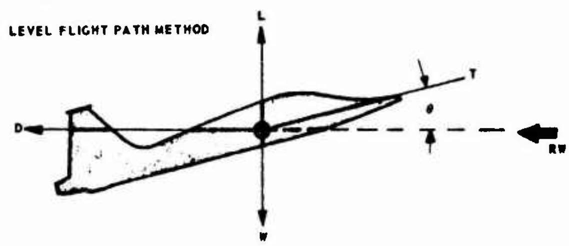
Every point along the lift boundary curve, the position of which is a function of gross weight, altitude, and aircraft configuration, represents a condition of $C_{L_{max}}$ (neglecting cases of insufficient elevator power). It is important to note that for each configuration, $C_{L_{max}}$ occurs at a particular α_{max}, independent of load factor, i.e., an aircraft stalls at the same angle of attack and C_L in accelerated flight, with n = 2.0, as it does in unaccelerated flight, with n = 1.0. The total lift (L) at stall for a given gross weight (W) does however vary with load factor since L = nW. The increased lift at the accelerated stall must be obtained by a higher dynamic pressure (q).

$$q_{stall} = 1/2\rho V^2_{stall} = \frac{nW}{C_{L_{max}} S}$$

Thus stall speed is proportional to n, making accurate control of normal acceleration of primary importance during stall tests.

Two flight test methods are described below. The first, involving a level flight path, is an older method that is valid only for unaccelerated stalls. It has several disadvantages that limit its application, but in certain cases such as VSTOL testing or initial envelope extension it might prove useful. It has been largely replaced by the second method that involves a curved flight path and is valid for both accelerated and unaccelerated stalls.

Figure 2.9



L + T sin θ = W and the flight path is straight. In order to slow the aircraft to stall speed, however, an acceleration (a_D) in the drag direction must be obtained by adjustment of thrust or drag such that D is greater than T cos θ. This represents a disadvantage of the method, since a particular trim power or drag configuration cannot be maintained to the stall.

Examination of figure 2.10 shows that the load factor will be at the desired value of 1.0 only if a_D is large enough. The size of a_D will be indicated by the rate of change of airspeed, termed the <u>bleed rate</u>. Experience has shown that undesirable dynamic effects are encountered if bleed rates much in excess of 1 or 2 knots per second are used. In practice, this usually restricts a_D to a value insufficient to close the acceleration diagram to the desired

n = 1.0, another disadvantage of this method.

Figure 2.10

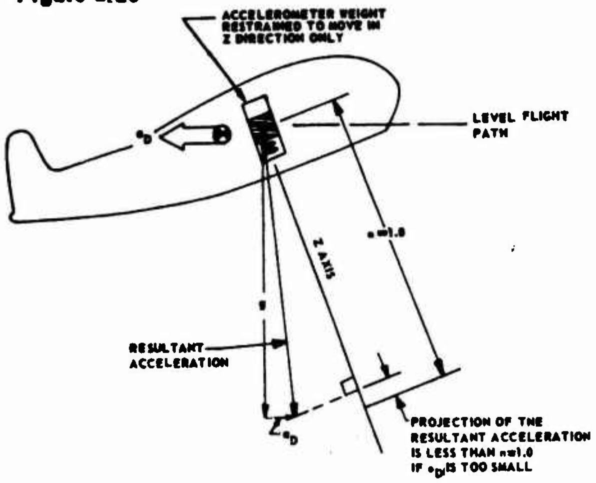
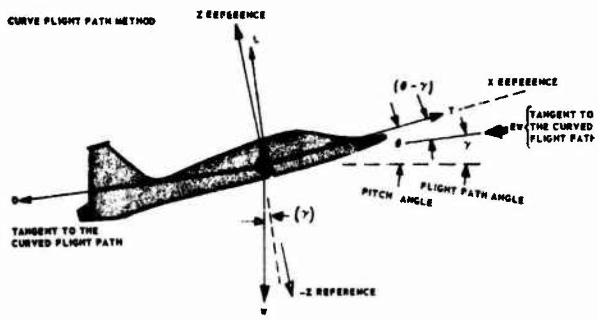


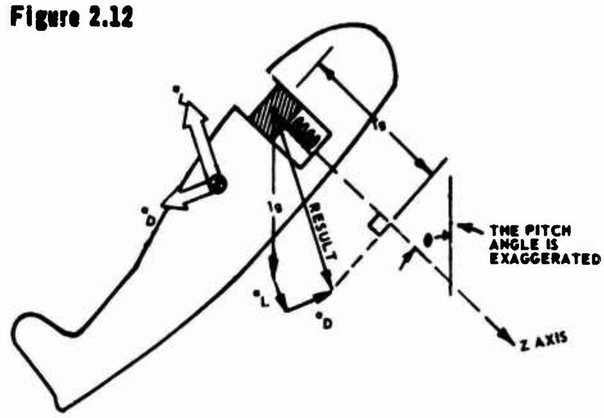
Figure 2.11



Here L + T sin $(\theta - \gamma)$ is greater than W cos(γ) and the aircraft is accelerating (a_L) toward the center of curvature. If trim power was set in level flight, D + W sin γ will be greater than T cos $(\theta - \gamma)$ and there will [illegible] additional acceleration [illegible] drag direction (a_D).

Note that a_L is closely aligned with the vertical reference and that adjustment of the radius of curvature may be used to close the acceleration diagram to the desired value of n. Thus load factor may be easily controlled with the elevator while maintaining trim power and configuration.

Figure 2.12



It is important to realize that the proper value of load factor may be maintained even with large bleed rates (a_D) simply by changing the radius of curvature (a_L), which will of course require a different pitch rate. The steady state diagrams above do not illustrate the need for a small bleed rate. They in fact indicate that the desired load factor may be obtained within a wide range of bleed rates. No rigorous limit for bleed rate can be calculated. In order to remove the possibility of dynamic effects, however, maximums of 1 knot/second for unaccelerated stalls, and 2 knots/second for accelerated stalls have been arbitrarily set on the basis of experience.

2.5 STALL FLIGHT TESTING

General:

Stalls, a familiar maneuver mastered by every pilot when he first learned to fly, must not be taken for granted in a test program. There is a rather large collection of examples from flight test history to document the need for caution. Designs that combine an inherent pitchup tendency with miserable spin characteristics have contributed much to these examples. Stalls are

usually first demonstrated by a contractor pilot, but it is possible for a military test pilot to find himself doing the first stalls in a particular configuration, especially on test bed research programs where frequent modifications and changes are made after the vehicle has been delivered by the contractor.

The cautious approach starts with good preplanning. Discuss with the appropriate engineering talent the predicted stall characteristics. Develop with them the most promising recovery technique for each stage of the stall, to include possible post-stall gyrations. In marginal cases, a suggestion for further wind tunnel testing or other alternative investigations might be warranted. Determine the most favorable loading and configuration to be used in the initial stages. Stall and spin practice in trainer aircraft will enhance pilot performance during any out-of-control situations that might develop.

If pitchup or other control problems seem remotely possible, the first runs should terminate early in the approach to the stall and the data carefully examined (on the ground) for trends such as lightening or reversal of control, excessive attitudes or sink rates. Advance this data systematically on subsequent flights - avoid the mistake of suddenly deciding in flight, because things are going well, to take a bigger step than planned.

A stall test point will in general involve three phases; the approach, the stall, and the recovery.

Approach to the Stall:

As will be described later, the aircraft must be flown through this phase in a manner to insure that the stall occurs at the desired altitude and load factor.

Stall warning, if any, will occur during this phase. This requires a subjective judgement by the pilot - only he can tell when he has been warned. This judgement should be extrapolated to the conditions under which the aircraft will be used in service, when distractions such as combat maneuvering may be present. A warning barely discernable during the test program would be of little use under these conditions. Excessive warning is also not desirable; MIL-F-8785 specifies definite upper and lower airspeed limits within which warning should occur. Control shake or airframe buffet is desired although artificial warning devices such as stick and rudder shakers are becoming increasingly common.

The Stall:

Stall has been defined as the minimum steady speed attainable, or usable, in flight. This minimum may be set by a variety of factors, for example:

a. Reaching $C_{L_{max}}$ - the conventional stall.

b. Insufficient longitudinal control to further decrease speed - lack of elevator power.

c. Onset of control problems. (Loss of control about any axis.)

   1. Pitchup

   2. Insufficient lateral-directional control to maintain attitude

   3. Poor dynamic characteristics

d. Back-side problems.

   1. High sink rate

   2. Insufficient wave-off capability

   3. Excessive pitch attitude

Aircraft with lift curves having sharp peaks may be prone to wing drop near the stall if local gusts or control motion cause a high angle of attack to occur on one wing before the other. MIL-F-8785 prescribes definite limits on pitch and bank angle at the stall. The test pilot should describe any other undesirable characteristics that may be evident.

The Recovery:

The recovery is started when the stall or minimum steady speed has been attained. For a conventional stall this is indicated by the inability to maintain the desired load factor - usually a sudden break is apparent on the cockpit accelerometer.

The goal of the recovery must be specified. For example, it might be to keep the altitude lost to a minimum or to obtain the fastest acceleration to a maneuver speed. In a test program all promising recovery procedures consistent with the objectives should be tried. It is important to have the recovery specified in detail before each stall - do not wait until the stall breaks to decide what procedure is to be used. There are no iron-clad rules for recovery - a "standard procedure" such as full military power could be disastrous in certain vehicles. Keep the instrumentation running throughout the recovery until the goal has been attained. In the case of minimum altitude loss this would be when rate of descent is zero and the aircraft is under control (the altimeter is the first indication of R/C = 0).

2.6 DEMONSTRATION MISSION

The student will fly an Attitude and Stall Demonstration Mission from the rear cockpit of the B-57.

Attitude and Stall Demonstration:

It has been found advantageous to devote a portion of the first Stability and Control flight to an exercise in attitude flying and aircraft trim techniques. The purpose of the exercise is to demonstrate and practice the proper techniques for rapidly getting an aircraft on altitude and airspeed in order to obtain an accurate trim shot.

The visual attitude technique for stabilized points will be flown to a zero/zero airspeed and altitude tolerance. Front side and back side trim techniques will be employed; the latter receiving more stress in its application to Stability and Control flight testing.

The technique for stalls will also be demonstrated and practiced. No data will be recorded. The student should however be thoroughly familiar with the B-57 instrumentation operation by the completion of this flight.

TABLE 2.1

DEMONSTRATION FLIGHT DATA

TRIM POINTS		
ALTITUDE (ft)	AIRSPEED (KIAS)	REMARKS
20,000	200	IP Demonstration
24,000	150	Student Practice
20,000	300	Student Practice
STALLS		
CRUISE CONFIGURATION		
20,000		V_{TRIM} 300 KIAS
n=1.0		n=2.0
IP Demonstrates each student practices each.		
POWER APPROACH CONFIGURATION		
20,000		V_{TRIM} 140 KIAS
n 1.0 Student Practice		n 1.5

• 2.7 STALL TEST TECHNIQUES

As on all missions, the student will be expected to keep abreast of the overall progress of the flight, including such considerations as airspace boundaries, restricted areas, turbulence conditions, flight time remaining and proximity to the base. Avoid getting "tunnel vision" on the immediate details of the task to the exclusion of all else.

Trim Point:

a. Set the configuration

b. Check for symmetric engine power and good lateral-directional trim

c. Record a stabilized trim point (±1 knot ±100 feet)

d. Record trim power

Entry Conditions:

Decide on an entry airspeed and altitude (based on previous experience) and do not settle for other values. A smooth well established entry is essential to good results.

Unaccelerated Entries.

a. Get established straight and level on entry airspeed and altitude with trim power reset.

b. Make a slight initial pitch rotation to start the bleed rate, using the outside visual attitude for reference, not the airspeed indicator.

Accelerated Entries.

a. Establish a roughly level turn at the entry airspeed, altitude and load factor. Reset trim power.

b. Using the visual attitude and the accelerometer as reference, substitute an increment of pitch for an increment of bank angle while maintaining the aim load factor. The nose should begin to follow a chandellelike helix above the horizon. The angle of this helix with the horizon determines the bleed rate.

Approach to the Stall:

Pitch control is used primarily in this phase to maintain the aim load factor, although some adjustment of the bleed rate may be made.

Unaccelerated Stall Approach.

a. Check the bleed rate and correct with pitch if necessary.

b. Start the instrumentation at some predetermined speed.

c. Call out the airspeed and actuate the event marker at stall warning. Mentally note the type and adequacy of the warning.

d. Use pitch control to keep $n = 1.0$. Do not attempt bleed rate corrections after stall warning. Keep wings level. Pitch rotation must increase as the stall is approached to keep the aim load factor, and the accelerometer must be closely monitored to catch the stall break.

Accelerated Stall Approach.

a. Bleed rate is largely determined by the initial helix angle. A bank angle increase will slow the bleed rate and a bank angle decrease will speed it up, provided the load factor is maintained. Concentrate on the aim load factor; once it is deviated

from it is difficult to salvage a run.

b. Start the instrumentation at a speed well in advance of stall warning.

c. Check the bleed rate again only to evaluate the stall. Corrections late in the approach are useless.

d. Call out airspeed and actuate the event marker at stall warning. Qualitatively evaluate the warning for the type and adequacy.

e. Maintain the load factor until the stall breaks.

The Stall:

The accelerometer must be closely cross checked to catch the break. A good positive pitch rotation will make the stall easy to identify. If the stick is relaxed near the stall (a natural tendency) a pseudo-break will confuse the issue. At the break call out the airspeed and altitude. Initiate recovery controls and configuration change, if any. Qualitatively evaluate the aircraft stall characteristics.

The Recovery:

Keep the instrumentation running and follow the predetermined procedure. Qualitatively evaluate the recovery characteristics. Call out the final recovery altitude.

Clean-Up Phase.

a. Stop the instrumentation.

b. Check the general situation and start the aircraft toward the next point.

c. Hand record warning speed, stall speed, altitude lost, and qualitative comments. The verbal call-outs aid in retaining the numbers until they can be written down.

d. Decide if a correction to the entry conditions is required.

2.8 DATA

Data to be Recorded:

A continuous oscillograph recording will be taken from before stall warning until after recovery for each of the stalls on which data are collected. The oscillograph will also be used to record trim points. The following data should be hand recorded for each stall:

a. Indicated speed at stall warning (actuate event marker).

b. Indicated stall speed.

c. Altitude lost during recovery.

d. Qualitative comments on:

   1. Type and adequacy of stall warning.

   2. Stall characteristics such as pitch and roll.

   3. Control characteristics during the three phases of the stall.

   4. Recovery technique and effectiveness.

e. Fuel counter readings.

f. Oscillograph run number.

The format of the flight data cards is not specified. However, the stall mission is a very busy one and it will tax the pilot's concentration and agility. Extra time should be spent to devise data cards that will aid in keeping track of the details. A space for every type of comment desired should be provided beforehand; then the pertinent information may be rapidly entered during the clean-up phase. Do not crowd the cards. Prepare an outline type flight card

for the IP with space for remarks but not necessarily a duplicate of the data cards.

Data Presentation:

A table and a time history similar to figures 2.13 and 2.14 will be presented in the report. The time history should be of a particularly well flown stall, and/or of one during which some unusual characteristic was observed. All the parameters for the time history may be obtained from the oscillograph record.

The report should include a discussion of the qualitative findings and an evaluation of the aircraft in comparison to the requirements of MIL-F-8785. (The School flight profile was chosen to demonstrate as much as possible in a reasonable time - it does not necessarily fulfill all the requirements of paragraph 3.4.1.)

Figure 2.14

TYPICAL STALL TIME HISTORY

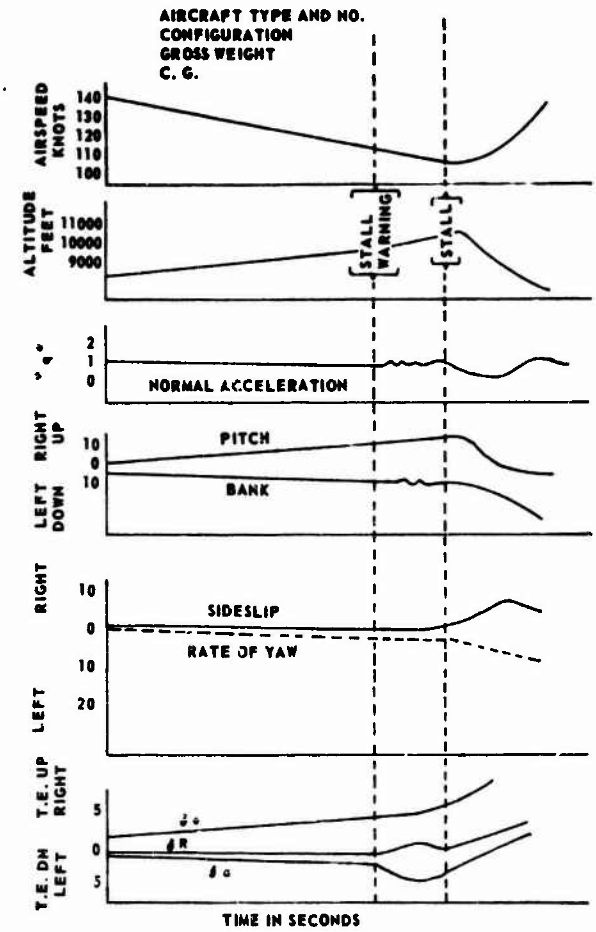


Figure 2.13

Conf	n	Weight	CG	TRIM		ALTITUDE	IAS		$C_{L_{Max}}$	V_W/V_S	Remarks
				M	IAS		Warn	Stall			
CR											
etc.											

2.9 DETERMINATION OF THE LIFT BOUNDARY

The purpose of this test is to determine the limiting normal acceleration or g's that can be pulled at various speeds and Mach numbers. This may be determined by buffet and/or pitchup. From this data it is possible to determine the best maneuvering Mach number.

Data Recording:

The data for this test will be hand recorded. Normally this test would be flown at several center of gravity positions. However, at the School, to conserve time, the test will be flown while the center of gravity is being moved from the forward to the aft position.

Test Techniques:

Trim the aircraft at 250 KIAS at 20,000 feet. Since the center of gravity is shifting, it will be impossible to maintain trim for an extended period of time. Furthermore, the data obtained on this test is not a function of stabilizer position, therefore, do not take a trim shot. Place the aircraft in a steady level turn and increase power in an attempt to hold constant altitude, trim velocity and Mach number. Continually increase the load factor until initial buffet is reached and note the load factor at this time. Continue to increase the bank and load factor until moderate buffet is reached and again note this load factor.

The same technique will be used at airspeeds of 280, 300, 320, and 350 KIAS. At the higher airspeeds, when it may be impossible to hold constant altitude at full power, plan the entry from a higher altitude so that the aircraft will reach buffet at the required speed and altitude. A tolerance of plus or minus 500 feet will be allowed.

Care should be taken not to increase load factor more than one half g per second in order to minimize dynamic effects.

If any intolerable condition of flight is experienced prior to initial or heavy buffet the run will be discontinued and appropriate mention made of this fact.

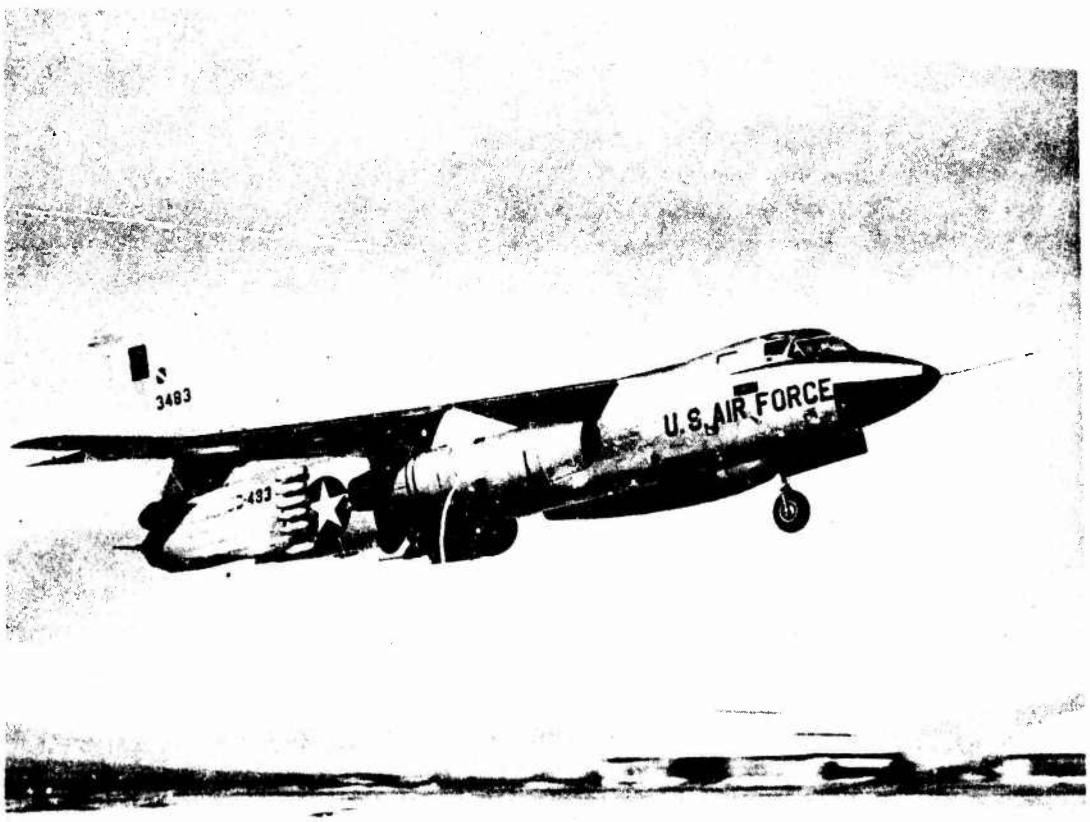
Data Reduction Outline:

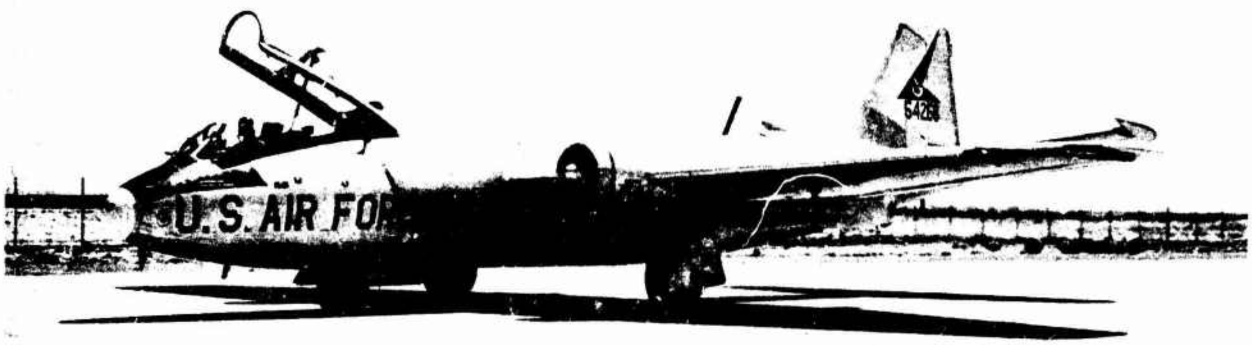
The data reduction is as follows:

	Parameter	Source	How Obtained
1	V_i	Instrument Panel	Data Card
2	ΔV_{ic}	Calibration Sheets	
3	V_{ic}		1 + 2
4	ΔV_{pc}	Position error	
5	V_c		3 + 4
6	H_i	Instrument Panel	
7	ΔH_{ic}	Calibration Sheets	
8	H_{ic}		6 + 7

	Parameter	Source	How Obtained
9	ΔH_{pc}	Position error	
10	H_c	Calibration Altitude	8 + 9
11	M_c	Appropriate charts at 10	
12	n (Initial buffet)	Instrument Panel	
13	n (Moderate buffet)	Instrument panel	
14	W_t	Gross Weight	
15	δ	Pressure ratio	Appropriate charts at 10
16	W/δ		14 ÷ 15
17	nW/δ (Initial)		12 x 16
18	nW/δ (Moderate)		13 x 16

Plot 17 and 18 versus 11 showing lines of initial buffet and moderate buffet.





2.14

CHAPTER 3

LONGITUDINAL STABILITY

The purpose of this flight test is to determine the longitudinal static stability characteristics of an aircraft. More specifically, it determines the gust and speed stability level of the aircraft, and the limiting aft cg location based on the static margin derived from an investigation of the neutral point.

An aircraft is said to be statically stable longitudinally (positive gust stability) if the moments created when the aircraft is disturbed from trimmed flight tend to return the aircraft to the condition from which it was disturbed. Longitudinal stability theory shows the flight test relationships relating to the stick-fixed and stick-free gust stability, dC_m/dC_L, to be:

stick-fixed

$$\frac{d\delta_e}{dC_L} = - \frac{dC_m/dC_L}{C_{m_{\delta_e}}} \text{ Fixed} \quad (3.0)$$

stick-free:

$$\frac{d}{dC_L}(Fs/q) = - A\frac{C_{h_{\delta_e}}}{C_{m_{\delta_e}}} \frac{dC_m}{dC_{L_{Free}}} \quad (3.1)$$

Therefore, during the flight test, control stick forces (F_s) and elevator deflections (δ_e) must be measured. These test parameters are a direct indication of handling qualities and show the variation of out-of-trim forces with airspeed. The measurement of these flight test parameters also provides a means of extracting a neutral point and, consequently, a static margin for any cg position.

The neutral point extracted from steady level flight indicates the cg position for neutral stability. The neutral point concept of static restoring tendencies has questionable utility if the neutral point variation with angle of attack, Mach number, or aeroelasticity is large. Generally flight at high angles of attack or in the transonic region results in large neutral point variations. For this reason, neutral point locations are often of secondary or strictly academic interest. The variation of stick forces and elevator deflection with Mach number for a given range of cg gives the stability engineer all the desired information.

Speed stability, which indicates how the control stick forces and elevator deflections vary with Mach number, is obtained directly from camera or oscillograph data. Neutral speed stability is indicated at points of zero slope on the Mach versus stick force or elevator position plots. In addition, the simultaneous measurement of forces and deflections permits both stick-fixed and stick-free static stability determinations.

Flight path stability is a requirement to be satisfied on the "backside" of the drag curve for the landing approach flight phase. This means that when the pilot changes airspeed by use of the elevator alone, increases in airspeed may be accompanied by decreases in the flight path climb angle. De-

creases in airspeed would likewise be accompanied by increases in the flight path climb angle. MIL-F-8785 (para 3.2.1.3) prescribes the allowable gradients over the airspeed range to be considered. The data required to test for compliance with this specification are vertical velocity and indicated airspeed both hand-recorded at stabilized airspeed points.

3.1 MILITARY SPECIFICATION REQUIREMENTS

General:

The 1954 version of MIL-F-8785 established longitudinal stability requirements in terms of the neutral point. While the neutral point criteria is still valid for testing certain types of aircraft, this criteria was not optimum for aircraft operating in flight regimes where other factors were more important in determining longitudinal stability. The 1968 version of MIL-F-8785 does not even mention neutral points, instead, section 3.2.1 of MIL-F-8785 specifies longitudinal stability with respect to speed and flight path. The requirements of this section are relaxed in the transonic speed range except for those aircraft which are designed for prolonged transonic operation.

3.2 NEUTRAL POINT THEORY

The neutral point has sometimes been defined as the cg location at which the derivative $dC_m/dC_L = 0$. Under the condition of a gliding, rigid airplane at low Mach number, C_L is a unique function of α, and

$$dC_m/dC_L = \frac{\partial C_m/\partial\alpha}{\partial C_L/\partial\alpha} \quad . \text{ Then if}$$

dC_m/dC_L is zero, then $\partial C_m/\partial\alpha$ must also be zero.

Mathematically speaking, the derivative dC_m/dC_L does not exist unless M, T_c and $1/2\rho\ V^2$ are definite functions of C_L. When that is the case, then

$$\frac{dC_m}{dC_L} = \frac{\partial C_m}{\partial\alpha}\frac{\partial\alpha}{\partial C_L} + \frac{\partial C_m}{\partial M}\frac{\partial M}{\partial C_L} + \frac{\partial C_m}{\partial T_c}\frac{\partial T_c}{\partial C_L} + \frac{\partial C_m}{\partial(1/2\rho\ V^2)}\frac{\partial(1/2\rho\ V^2)}{\partial C_L}$$

This equation has meaning only during straight, one-g flight. When a condition of this kind is imposed, then M, T_c, and the dynamic pressure are definite functions of C_L. The derivative dC_m/dC_L exists, and the neutral point determined from it is not an index of stability with respect to gust disturbances. In this case, dC_m/dC_L relates to the trim curves of the airplane.

A plot of the elevator angle required to trim versus speed will have a zero slope when dC_m/dC_L is zero, and a negative slope when the cg lies aft of the neutral point. The reversal of slope indicates a tendency toward instability with respect to speed, but only a dynamic analysis can show whether or not the airplane is stable in this condition. A neutral point thus defined does not lead to any definite and clear conclusion about either the gust stability or about the general static stability involving both speed and gusts. It does have a definite bearing on the handling qualities and nature of the instabilities that can occur when the cg lies forward and aft of this neutral point.

3.3 EXAMPLE TEST METHODS

Stabilized Method:

This method is used for those aircraft with a small airspeed

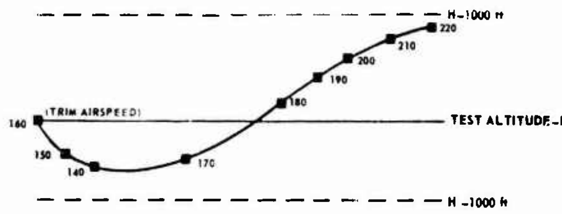
range and virtually all aircraft in the power approach configuration or at low subsonic speeds at low altitudes. It is the less difficult method but requires more flying time. It is used to test for both speed and flight path stability.

The aircraft is trimmed carefully at the desired airspeed ($V_{o_{min}}$) and altitude. Once the trim is established and a trim shot recorded, the trim should not be changed for the remainder of the test. The starting altitude for the test depends on the type aircraft. The altitude band for the test is the trim altitude plus or minus a 1,000 feet. For a low aspect ratio aircraft like the T-38, the test may be initiated at the trim altitude. In any event, when the aircraft flies out of the altitude band, the test should be terminated and the aircraft flown back into the test band for completion of the test.

Without changing power or trim, increase aircraft attitude to slow the airspeeds to below trim airspeeds (use approximately 10 kt increments). When stabilized, (attitude fixed, airspeed fixed, and no control stick movement) record the data. The pilot should be careful not to induce additional control stick forces or movement when taking the data. The aircraft may be climbing or descending when stabilized. The aircraft's attitude is then increased to further reduce the airspeed. The aircraft is again stabilized, and the data recorded. For airspeeds above trim, the aircraft's attitude is lowered to increase the airspeed (use approximately 10 kt increments). If small flight path pitch changes are made and aim airspeeds are lead by timely pitch corrections, airspeed overshoot may be prevented. In the power approach configuration, the test is terminated at the gear down limiting airspeed. The test is then repeated for different altitudes and cg positions.

A typical T-38 flight path variation during the stabilized method is shown in figure 3.0.

FIGURE 3.0
STABILIZED METHOD



<u>Acceleration Method:</u>

This method is most commonly used for an aircraft with a large airspeed/altitude range capability and for aircraft in the cruise and/or combat configuration. This method is less time consuming but more difficult than the Stabilized Method.

As in the Stabilized Method, the aircraft is carefully trimmed hands-off at the designated trim airspeed at the desired altitude. The trim shot is recorded. The power is then reduced to idle and the aircraft is decelerated to the required low speed point. During deceleration the pilot should increase the aircraft attitude smoothly to maintain a nearly zero rate of climb (a slight constant climb or descent is acceptable). Aft stick forces should increase during deceleration. At no time should the stick forces be relaxed or reversed during the deceleration. Data should be recorded at intervals of about 10 KIAS.

When below the specified minimum speed for data, the power should be increased to stabilize the airspeed prior to initiating the acceleration portion of the test. Power is then increased smoothly to full power. Positive and expeditious throttle changes

to full power or idle will minimize power and acceleration contributions to the trim curves. As airspeed increases towards trim airspeed, the aircraft's attitude will decrease as aft stick forces are relaxed. Before trim airspeed is reached the relaxation of aft stick forces will transition to a push force which will increase as the airspeed increases above trim airspeed. Preventing stick force relaxation or reversals is of primary importance. Smooth attitude changes are made to control rate of climb. At approximately 0.90 Mach, the aircraft's attitude should be fixed and stick movements and forces used to maintain this attitude during the transonic portion of the run. Data should again be recorded every 10 KIAS.

The aircraft is then decelerated back to the trim airspeed using the same stick and attitude techniques.

The aircraft is then trimmed at another desired flight condition and the maneuver is repeated.

During this test the aircraft's attitude will gradually change and a rapid crosscheck between the outside horizon and the rate of climb indicator will assist in establishing the proper rate of aircraft rotation.

3.4 DATA RECORDING

The data card should list pertinent test information to help in identifying the particular test flight. The card may be used as an outline to maintain the continuity of the flight. A place for camera number and oscillograph correlation, if applicable, is necessary in order to identify the film or oscillograph trace recording.

The sample data card shown is the responsibility of the flight test engineer or the pilot.

PILOT DATA CARD — AFFTC FORM 202

ITEM	C.N.	Hi	Vi	Hi	FUEL	
GND SHOT			0			
TRIM		15000				
START						
FINISH						
TRIM		30000				
START						
FINISH						
TRIM		45000				
START						
FINISH						

GROUND TEMPERATURE		GROUND ALTIMETER [illegible] Hg	
TAKEOFF	LANDING	TAKEOFF	LANDING

TIME			PLANE
TAKEOFF	LANDING	TOTAL	T-38A 596

DATE	FLIGHT NO.	PILOT	
20 JULY 64	3	MORRISON	
ENGINE NO.	FUEL FULL INTERNAL	MAIN CONTROL NO.	EMERG CON NO.
CONFIGURATION	CR AFT C.G.		

3.5 DATA REDUCTION

3.5 Table 3.1 is a data reduction outline that can be used to present the results of the stabilized and/or acceleration method for the T-33, B-57, and T-38 aircraft. Columns 21-24 are used only for flight path stability determination.

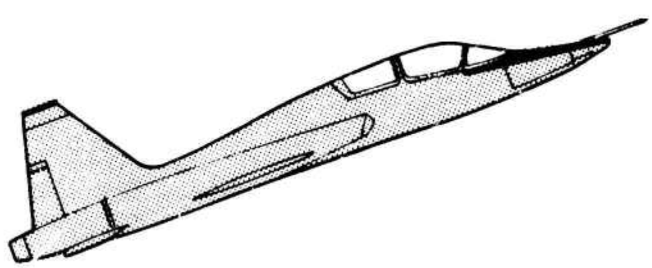


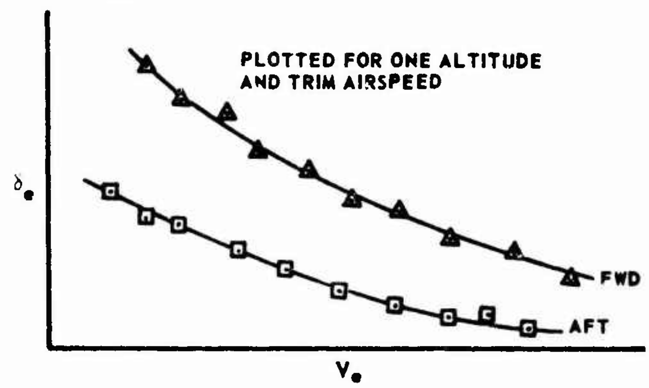
Table 3.1

DATA REDUCTION OUTLINE

(For T-33, B-57, and T-38 Aircraft)

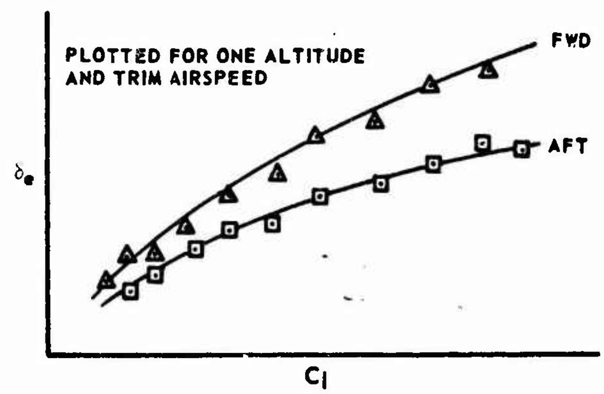
Column	Symbol	Units	Source
1	C.N.		Counter number
2	V_i	kt	Indicated airspeed
3	ΔV_{ic}	kt	Instrument corrections
4	ΔV_{pc}	kt	Position error
5	ΔV_c	kt	Airspeed compressibility correction
6	V_e	kt	Equivalent airspeed $V_i + \Delta V_{ic} + \Delta V_{pc} + \Delta V_c$
7	V_e^2	kt^2	Equivalent airspeed squared
8	q	lb/ft^2	Dynamic pressure $1/2\rho\ V_e^2$ equals $V_e^2 \times 0.003392$
9	qS	lb	Dynamic pressure wing area T-33 $S = 234.8\ ft^2$ T-38 $S = 170\ ft^2$ F-104 $S = 196.1\ ft^2$ B-57 $S = 960\ ft^2$
10	F/C	gal	Fuel counters
11	W	lb	Aircraft gross weight plus weight of fuel remaining (plot weight vs fuel counter reading and take W from that)
12	C_L		Lift coefficient = W/qS
13	δ_{e_i}		Indicated elevator position
14	δ_e		Degrees elevator position from calibration sheet
15	$d\delta_e/dC_L$	deg	The trim curve slope for each C_L and cg
16	F_{s_i}	lb	Indicated stick force
17	F_{s_c}	lb	Corrected stick force. Indicated stick force minus the trim force recording
18	F_s	lb	Stick force from calibration sheets
19	F_s/q	ft^2	Stick force divided by dynamic pressure
20	$\frac{dF_s/q}{dC_L}$	ft^2	Trim curve slope for various C_L's at each cg
21	VV	fps	Cockpit R/C ÷ 60
22	V	fps	TAS from No. 6
23	γ	deg	Climb angle $\sin^{-1} \left(\frac{\text{Vert Vel}}{\text{True Airspeed}}\right)$
24	$\frac{d\gamma}{dV}$	$\frac{\text{deg-sec}}{\text{ft}}$	Plot of γ vs. V

FIGURE 3.1



Equivalent airspeed is selected as a plotting variable because the coefficient of lift is normally obtained from it at incompressible speeds. From this plot, longitudinal control requirements and speed stability evaluation at various speeds can be determined.

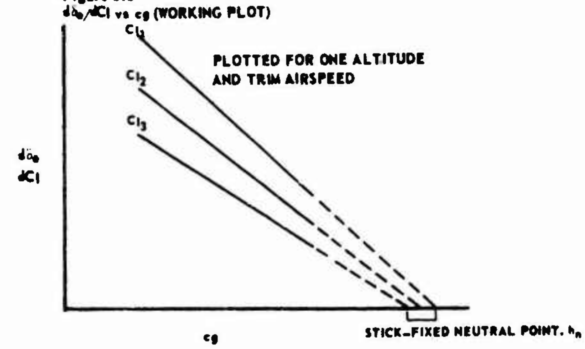
FIGURE 3.2



If elevator position plots linearly with lift coefficient, only one stick-fixed neutral point exists. Otherwise, the neutral point varies with angle of attack (or lift coefficient). The derivative $d\delta_e/dV_e$ (at one V_e and cg) does not change with weight unless the neutral point varies with angle of attack. The derivative $d\delta_e/dC_L$ serves better than $d\delta_e/dV_e$ as a plotting variable in locating neutral points because nonlinear weight effects are included.

The rate of change of elevator deflection with respect to lift force coefficient is measured from the plot of δ_e versus C_L. The slope is taken at three or more C_L's over the airspeed range for all center of gravity loadings.

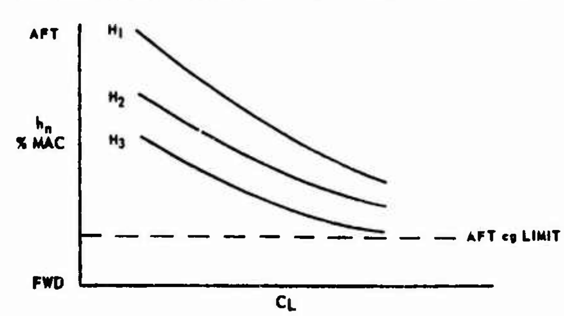
Figure 3.3
$d\delta_e/dCl$ vs cg (WORKING PLOT)



The point where $d\delta_e/dC_L = 0$, is the stick-fixed neutral point for that particular C_L. These neutral points for this one altitude are plotted versus C_L as the curve H_1 in figure 3.4. Additional altitude data would plot as curves H_2 and H_3.

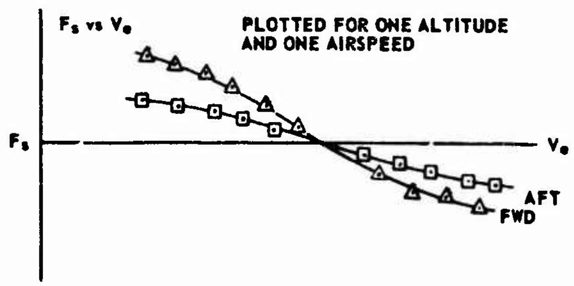
Figure 3.4 indicates the change in stick-fixed stability as the aircraft traverses the speed range at three representative altitudes.

FIGURE 3.4
STICK-FIXED NEUTRAL POINT VERSUS LIFT COEFICIENT



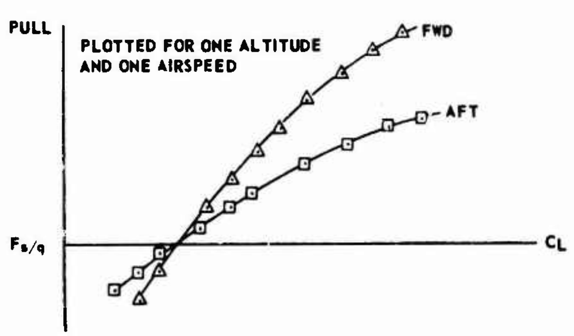
Stick-free neutral points are determined from data in the following manner:

FIGURE 3.5



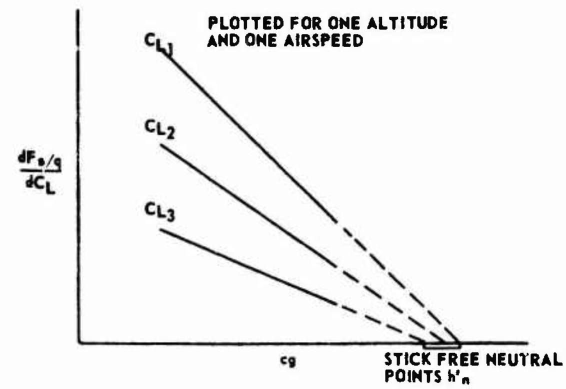
This plot indicates the apparent stability to the pilot. It is used to evaluate handling qualities and compliance with military specifications. The derivative dF_s/dV_e is a function of aircraft trim as well as stability. This reduces the value of an extracted neutral point. When the stick force is divided by dynamic pressure, the derivative of this quantity, $\frac{dF_s/q}{dC_L}$, is a function of stability only and produces a more valid stick-free neutral point.

FIGURE 3.6



From the plot of F_s/q vs C_L, the rate of change of stick force with respect to lift coefficient is measured and the slope determined at three or more C_L's over the airspeed range for both center of gravity loadings.

FIGURE 3.7



The point where $\frac{dF_s/q}{dC_L} = 0$

is the stick free neutral point, h'_n at that particular C_L. The neutral point movement with C_L for one altitude is curve H_1 in figure 3.8. Additional altitudes would plot as H_2 and H_3.

FIGURE 3.8

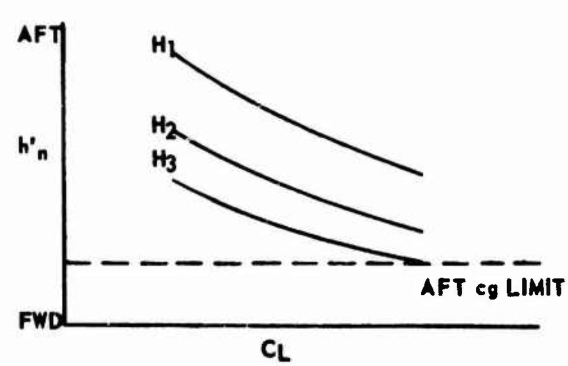


Table 3.2 is a data reduction outline that can be used for the T-38 or other transonic/supersonic aircraft.

Table 3.2

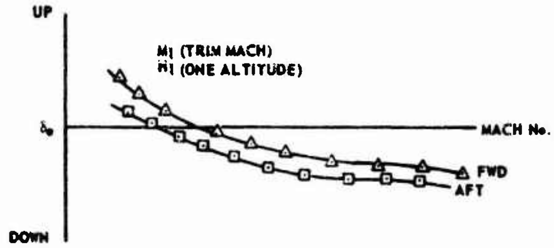
DATA REDUCTION OUTLINE

(T-38 Aircraft)

Column	Symbol	Units	Source
1	C.N.		Counter number
2	V_i	kt	Indicated airspeed
3	ΔV_{ic}	kt	Instrument corrections
4	ΔV_{pc}	kt	Position error
5	V_c	kt	Calibrated airspeed $V_i + \Delta V_{ic} + \Delta V_{pc}$
6	H_i	ft	Indicated altitude
7	ΔH_{ic}	ft	Instrument corrections
8	ΔH_{pc}	ft	Position error
9	H_c	ft	Calibrated altitude
10	M	ft	Mach number from V_c and H_e
11	$\frac{d\delta_e}{dM}$	deg	Trim curve slope at various Mach numbers throughout the speed range
12	F_{s_i}		Indicated stick force
13	F_{s_c}		Corrected stick force. Indicated stick force minus the trim force reading
14	F_s		Stick force from calibration sheets
15	dF_s/dM		Trim curve slope for each cg position at a trim speed

3.8

FIGURE 3.9



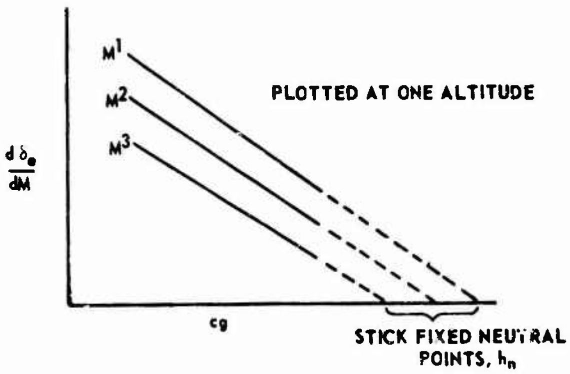
This plot indicates the change in apparent speed stability of the aircraft and is used in evaluating handling qualities.

The neutral point thus defined does not necessarily occur at the same cg position where $dF_s/dV = 0$. Therefore, it is permissible to shift the cg behind the point where $\frac{dF_s/q}{dC_L} = 0$ if a pull force is still required to fly slower than the trim airspeed.

It would be desirable to flight test for the separate effects of Mach number, angle of attack, and elasticity on neutral point location, but this is not normally possible. A practical evaluation would consist of combining the angle of attack and elastic effects for a constant Mach number.

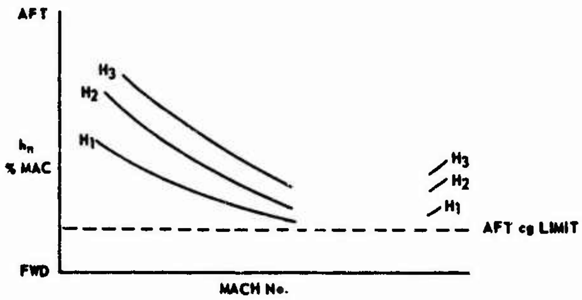
From a plot of δ_e vs M, the curve slope at constant Mach number is measured.

FIGURE 3.10



The stick fixed neutral points are then plotted versus Mach number for all test altitudes as in figure 3.11.

FIGURE 3.11
STICK-FIXED NEUTRAL POINTS vs MACH NUMBER



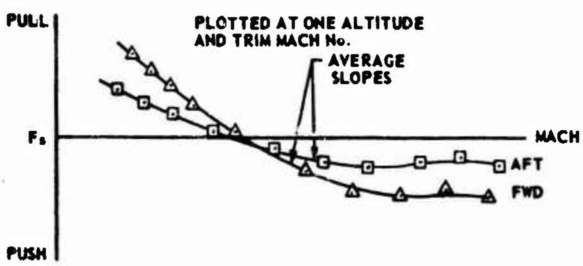
Neutral points vary with configuration, angle of attack, Mach number, and static elastic airframe distortion. The above analysis and plots are for constant weight. Tests should be conducted at high and low weight at each altitude, Mach number, and cg position. However, testing for relative neutral point location from selected flight conditions is recommended in view of the extensive flight test time required to conduct a complete analysis.

No extensive attempts should be made to locate transonic stick-fixed or stick-free neutral points because the cg would never be shifted to correct for transonic speed instability in any case. Also the dynamic stability is highly nonlinear with Mach number in this region and the neutral point concept has little utility other than to qualitatively specify an instability.

An aircraft of relatively low speed capability which displays essentially linear aerodynamic variations with angle of attack and which is structurally rigid will have one neutral point per configuration. On the other hand Mach number effect may dominate, in

which case neutral point versus Mach number becomes one curve. If the aircraft is elastic, has supersonic speed capability, and is nonlinear with angle of attack as are many modern types, the results will appear similar to figures 3.12 through 3.16.

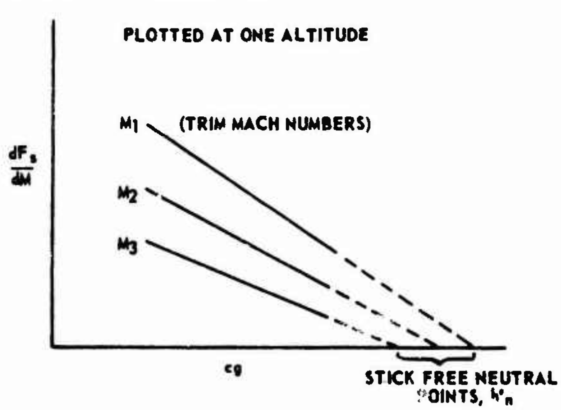
FIGURE 3.12



The derivative dF_s/dM may be used as a plotting variable for determing a neutral point at a trim Mach number for a particular altitude and configuration. When dealing with an irreversible control system, stick force versus Mach number trim curves should be evaluated. Neutral points thus have meaning only at the trim Mach number.

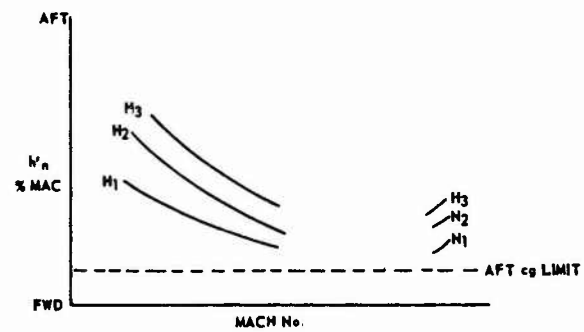
In reducing force data for neutral point determination, the slopes should be averaged on either side of trim speed for each cg position.

FIGURE 3.13
TRIM CURVE SLOPES vs cg POSITION AT A TRIM MACH NUMBER (WORKING PLOT)



Variation of Neutral Point with trim Mach number is shown in figure 3.14.

FIGURE 3.14
NEUTRAL POINT vs TRIM MACH NUMBER



For an irreversible control system, force trim curves should be obtained for various trim Mach numbers to evaluate the trim curve variation with Mach number. Force data at Mach numbers differing appreciably from trim Mach number are obtained in order to evaluate the forces required to maintain one-g flight if the trim mechanism fails. A single neutral point (for a particular Mach number, altitude, and configuration) also is valuable in showing the apparent stick-free stability at the particular test conditions.

In the transonic speed regime, the trim curve (stabilator deflection versus speed curves) is usually characterized by a hump or a reversal in deflections. This does not indicate aircraft instability. If the aircraft were displaced in the transonic regime the pitching moment would still tend to restore the aircraft to equilibrium flight. The aircraft is still stable and has a negative slope of C_m versus α. At subsonic speeds the curve is insensitive to speed; while at transonic speeds, the curve is affected by Mach number. In traversing the transonic speed range, the aircraft is traversing many stable C_{m_α} curves. The aircraft

is statically stable throughout the transonic speed, but has apparent speed instability.

By accelerating and decelerating through the same speed range, any noticeable power effects will reflect in the trim curve plot as shown in figure 3.15. If the power effects are large, the acceleration and deceleration curves should be shown separately. If the effects are small, an average curve may be drawn (figure 3.16).

FIGURE 3.15
LARGE POWER EFFECTS

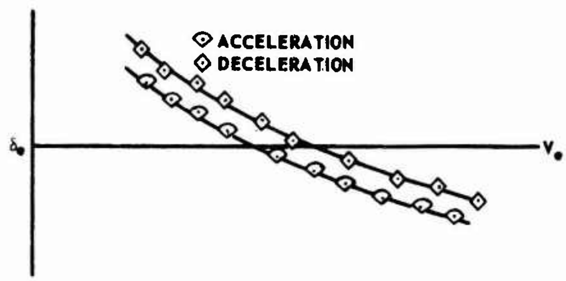
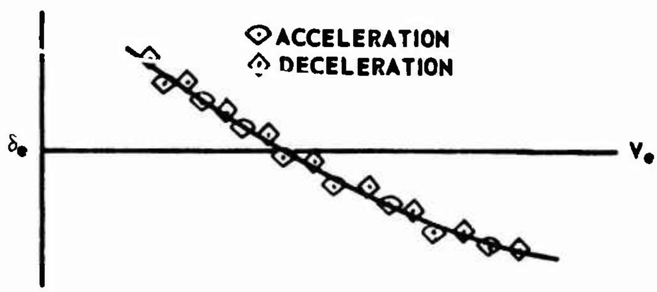


FIGURE 3.16
SMALL POWER EFFECTS



Climb angle plotted versus true airspeed is shown in figure 3.17. This figure is typical of the T-38 in the PA configuration, trimmed at $V_{o_{min}}$. The criteria for flight stability is contained in figure 3.18 which is derived from figure 3.17.

Figure 3.17
T-38 P.A.

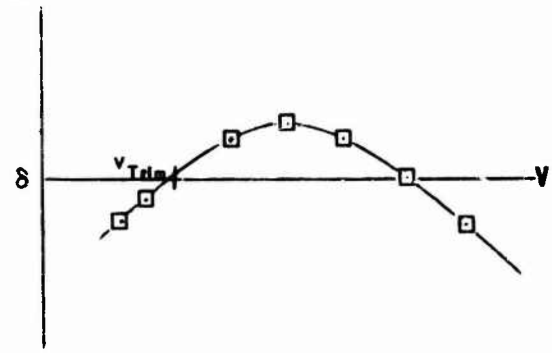
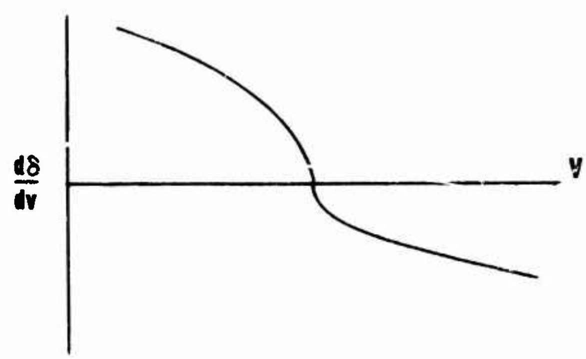
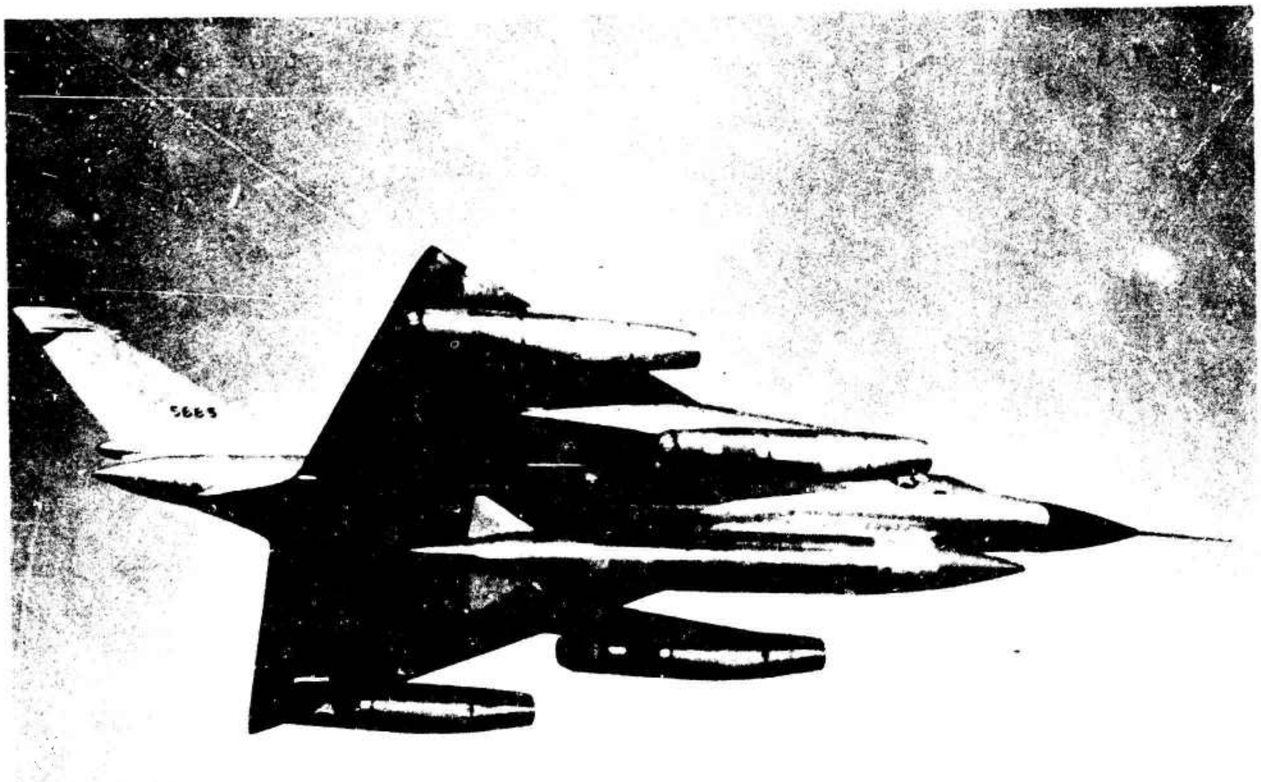


Figure 3.18
Flight Path Stability





3.12

CHAPTER 4

SPIN FLIGHT TEST TECHNIQUES

• 4.1 PREPARATION

The test pilot who intends to spin an aircraft should be able to study its design and obtain a good idea of the spinning characteristics. By comparing the test aircraft's design parameters with the equations of motion, he should also be able to tell what the best recovery procedure is to be. In the event the test pilot is advised to recover in a manner contradictory to general theory, he should be able to intelligently couple the aircraft's particular design parameters with the theory and understand why the recommended technique will produce anti-spin accelerations.

The inherent dangers of spin testing are well recognized; however, experience has shown that high performance aircraft can be safely spin tested if the tests are properly planned and executed.

Spin Project Checklist:

The following preparations should be accomplished prior to actual spin test missions. The AFFTC Hazardous Test Checklist further expands upon these areas.

a. Theoretical Analysis (by pilot and project engineer)

1. Literature research of previous results and studies. Spin reports of similar aircraft should be thoroughly reviewed.
2. Equations of motion should be examined by computer and/or hand analysis.
3. Model studies in a wind tunnel and/or free flight.
4. Results of previous stall missions flown in the test aircraft should be thoroughly studied.

b. Predictions

1. Spin characteristics of each phase.
2. The effect of various entries and control positions during the spin.
3. Optimum recovery techniques.
4. Optimum configuration and loading for the initial spins.
5. Plan for the worst possible conditions. (Control motions may produce more than one effect.)

c. Modifications to the Test Aircraft

1. A last-ditch recovery device must be provided if predictions leave the slightest doubt about recovery. Rockets and spin chutes are most frequently utilized.
2. Modify and/or backup each aircraft system as necessary (hydraulic, electrical, fuel, oil, etc.).
3. Provide a dependable escape system and adequate pilot restraints.

4. The control system must be checked and/or modified to insure sufficient control power. The effects of interconnects and limiters must be determined.

Test Instrumentation:

A determination of the quantitative data required must be made. On-board recording and telemetry systems will normally be required in all spin test programs. Regardless of the amount of quantitative instrumentation required, it is imperative that a redundant recording capability be provided for pilot comments. A quick-look capability should be provided so that qualified engineering personnel on the ground can monitor the aircraft motion and other critical items during the spin. Recent spin tests in high performance aircraft have pointed out the need for cockpit lights or other indicators that give the pilot positive and unmistakable indications of spin direction and rates.

Pilot Spin Proficiency, Experience, and Practice:

The spin test pilot should be experienced in testing high performance aircraft and have an adequate engineering background, especially in spin theory. Since the aircraft will have to be stalled in order to enter the spin, the test pilot should thoroughly investigate normal one-g and accelerated stall characteristics. The same holds for possible post-stall gyrations. In addition, the pilot should spin similar aircraft that are cleared for intentional spins. This allows the pilot to become completely familiar with and at ease in the general spin environment. Experience gained in centrifuge rides would also help the test pilot to become familiar with the forces expected during the actual spin tests.

Other Factors:

Highly qualified chase pilots should accompany the test aircraft on all spin test flights. The chase pilots should also participate in the preparation phase in order to become familiar with the expected spin aircraft characteristics.

As will all flight tests, a thorough knowledge of the aircraft is essential. A study of MIL-S-25015, "Spinning Requirements for Airplanes" should also be made.

4.2 EXAMPLE TEST METHODS

The aircraft used to perform the spin test at the Aerospace Research Pilot School will be the T-33A. The preparation recommended in the preceding paragraphs will not be accomplished for obvious reasons. However, the student will have two flights in which he can evaluate the spin characteristics of the T-33A; one will be with an instructor and the other with a student.

Preparation for Spin Testing:

a. Prepare flight cards to help you remember exactly what you want to do, see, and record.

b. Use these cards during the last stages of each climb to review the next spin - it is extremely easy to overlook important items.

c. Preplan (and modify in flight, as necessary) approach points to provide spin entries at the altitude and load factor desired. Use good stall test techniques during the approach.

d. Remove or stow securely all books, pins, etc., in the cockpit. Keep seat belt and shoulder harness tight and secure. Zip pockets.

e. Engine power control is optional through approach and entry. Power is idle for all demo spins, optional for data spins. Quickly trim elevator forces to zero as airspeed bleeds through 170 KIAS for all spins except hands off recoveries, which are trimmed for 140 KIAS.

f. Apply entry controls briskly and properly. Hold extra force to keep controls on the stops as speed and buffet buildup. If necessary, check aileron position visually to insure $\delta_a = 0$.

g. When aileron-with spins are to be accomplished, aileron will be placed full with the spin during turns 2, 3, and 4. Aileron will be placed with on the turn count of "one" and placed neutral on the turn count of "four".

h. Pick (at the moment of entry, if necessary) a spot for turn count. Watch for this spot but do not lead or lag the call. Avoid extra words, call out "THREE!" instead of "TURN NUMBER THREE." If airspeeds and altitudes are to be called out, read them both as you call the turn, then spit out the words as quickly as possible afterwards. Again devise a code and a sequence to eliminate extra words. Turns will be counted in all spins.

i. Devise some technique for giving a rapid oral report of altitude and airspeed immediately after each turn count. EXAMPLE - Four digits: the first two state altitude to nearest 100 feet (since you know altitude within 10,000 feet, the first digit is thousands and the second digit is nearest hundred); the last two digits represent tens and units of airspeed (again, you already know hundreds). Always read altitude first since it changes faster than airspeed.

j. Briskly apply recovery controls precisely at the proper turn call. Attention must be split to get this airspeed and altitude, but watching the motion is of primary importance at this point - you must see the rotation stop and neutralize the rudder. Reasonable and proper use of the controls is required thereafter. Power must be at idle for all recoveries.

k. Pull-out technique and load factor can be varied from light to heavy buffet to investigate altitude loss. Check during the pull-out how many turns were used in the recovery; delay will lead to erroneous answers.

l. Preflight inspection of aircraft will include a careful inspection of known stress areas. These are as follows:

1. All control surfaces and their hinges (check for bulges, wrinkles, and cracks).
2. Particular attention should be paid to the horn balances on the rudder and elevator.
3. Tail pipe tracks - if one of these tracks is broken, the tailpipe will be loose and/or not centered in the aircraft.

Demonstration Mission:

Table 4.1 presents the demonstration mission outline. The spin to be performed, the entry, recovery as well as the qualitative data to be acquired are all outlined. However, it is recommended that each student design a better card for the actual demo mission. Cards need not be prepared for the IP.

Data Mission:

Each student will fly one spin data mission in the front seat, one in the rear seat, and will chase at least one spin test from either the front or rear seat.

The student in the front seat of the spin aircraft is the flight commander and will conduct a formal preflight briefing of both crews.

The class will be divided into four equal groups. Each pilot will perform the four spins assigned his group as well as two to four optional spins. Optional spins may be right or left. The type recovery from any spin is optional; however, the recoveries should be divided so that representative data is gathered on each type. Likewise, a sufficient number of the different type entries should be performed so a correct evaluation of the effects of entry load factor can be made. Baseline spins should be performed at the beginning and end of the flight to evaluate weight effects. The four groups will perform the spins outlined in table 4.3.

Review Flight Manual section on spins, especially the discussion on out-of-control recoveries.

Each student reduces his own spin data and places it in a common file. Report writers and oral reporters may make use of any of the data as desired.

Cautions.

a. Visually check to see that the stick is properly positioned. It is particularly easy to not have ailerons neutral.

b. Hold controls firmly against the desired stops when required: full aileron is difficult to maintain.

c. Hands off recovery - move controls smoothly to a guarded neutral position. Don't just let go of the controls!!

d. NASA Standard Recovery - EASE stick forward to neutral (not beyond).

Common Mistakes.

a. Forget to reduce power to idle

b. Forget to count turns.

c. Leave recovery controls in too long on first spin.

d. Take recovery controls out too soon on second spin.

e. Ease in pro-spin controls.

f. Ease in recovery controls.

g. Recovery rudder not full against.

h. Not keeping aileron in desired position.

i. Moving stick forward of neutral.

j. Trying to use aileron to stop roll before spin is broken during hands off recovery.

k. Not neutralizing rudder completely when spin rotation stops.

l. Letting airspeed build excessively before pull out during recovery. This is particularly prevalent during hands off recovery. In this case, the spin breaks and a steep dive with a moderate roll rate follows. During initial hands off recoveries, pilots tend to mistake the roll for continued spin rotation and are therefore late in stopping the roll and pulling out of the dive.

m. Excessive altitude loss during recovery due to delay and letting airspeed buildup.

Spin Chase Mission:

This mission when properly performed represents an exercise of pilot judgment and flying technique. The T-33 spin aircraft is chased by a T-38 or a T-33. Chasing with the T-38 presents some special problems to the chase pilot. The difference in performance of the two aircraft when combined with the requirements of the chase mission make this an interesting and enjoyable flight. Proper preflight planning and correct airborne maneuvering are mandatory if the chase plane is to have sufficient fuel for the last spin or two. It is necessary for the spin pilot to keep the chase pilot informed of airspeed, maneuvers, etc. if proper spacing is to be maintained.

The primary reason for the chase mission is safety. After each spin, the test aircraft must be carefully checked by the chase pilot. It is therefore necessary that the chase pilot get in fairly close, but not too close; USE EXTREME CAUTION. In the event of even the slightest abnormality, abort the mission and get the aircraft on the ground.

The second reason for the chase mission is to provide further observation of the spin. An "outside-in" observation can be quite helpful in qualitatively describing the spin. This point of view also helps give the chase pilot a better "big picture" of spins because he has normally seen spins from the "inside-out" vantage point only.

There will be no discussion or instructions on the specific techniques to use during the chase mission. However, the general objective is to keep the test aircraft under close observation from entry through recovery, while staying out of the way. Some pilots like to imagine themselves with a movie camera mounted on their helmets and attempt to maneuver the chase aircraft so good quality film coverage can be taken continuously from entry through recovery. In either case, the objectives are the same.

The spin pilot will conduct a formal briefing for the chase pilot prior to spin data mission.

Space Positioning and Communications:

Although spin areas one and four and the dive corridor are normally used for the spin demo mission, only spin area one will be used for the data mission. Space Positioning (call sign "Sport") will provide photo and radar coverage during spins. "Sport" should be contacted on taxi-out so they can track the test aircraft from the ground. Approach control should also be contacted while taxiing or immediately after becoming airborne to advise them of your intentions. However, "Sport" will normally be worked during the climb to altitude. The test aircraft will be tracked and vectored to spin area one, advised when in the area by "Sport's" transmission, and "cleared to spin." It may take another thirty seconds before the spin pilot is ready to begin the spin. In order for "Sport" to get complete coverage on film and trackers, the spin pilot should give a 10- to 15-second warning. This warning also assists the chase pilot in his planning. After the spin recovery is complete, the spin pilot should call "DATA OFF" so "Sport" can turn off their cameras.

Color 16mm movie film as well as ground tracks for each data mission will be produced by "Sport" and sent to the test pilot involved. FPS-16 "Digital Data" will also be available for some of the data flights.

Table 4.1

DEMONSTRATION MISSION OUTLINE

Spin Number	Entry	Spin Direction	Spin Controls	Recovery	Primary Qualitative Data	Remarks
DEMO	30M n = 1	5 L	Normal	MOD	A/S, ALT	
2	30M n = 1	5 L	Normal	MOD	Spin Motion	Call Alt & A/S at initiation of recovery & level
3	30M n = 1	5 L	δ_a with during turns 2, 3 & 4	STD	Spin Motion	Same as above
4	30M n = 1	5 R	Normal	HANDS OFF	A/S, ALT	
5	30M n = 2 left turn	5 R	Normal	Optional	A/S, ALT	
6	30M n = 2 left turn	5 L	Normal	Optional	Optional	
7 DEMO	34M n = 0	8 L	Normal	Optional	Optional	Watch for gyroscopic tendencies at top

Table 4.2

RECOVERY TECHNIQUES

NASA STANDARD	NASA MODIFIED
(If ailerons were held during spin, neutralize)	Same
A. Full opposite rudder	A. Full opposite rudder and at the same time ease stick forward to neutral
B. Stick full aft	B. Neutralize rudder when rotation stops
C. When rotation stops - neutralize rudder (immediately)	
D. EASE stick forward to approximately neutral position	

Get sufficient airspeed and fly out of dive to level flight

Normal recovery above 20,000 feet

At 15,000 feet and recovery hasn't started - jettison canopy

Eject at 12,000 feet if recovery not completed or well underway (pilot judgment)

HANDS OFF RECOVERY

A. Move all controls (smoothly) to their neutral position.

B. With hands off the stick, monitor this position and prevent any large excursions from neutral; in other words, this is a hands off, stick guarded to neutral recovery.

C. When spin is broken (normally under one turn for T-33) stop the ensuing roll and pull out of the dive.

D. Airspeed jump from the steady state value is the key recognition point.

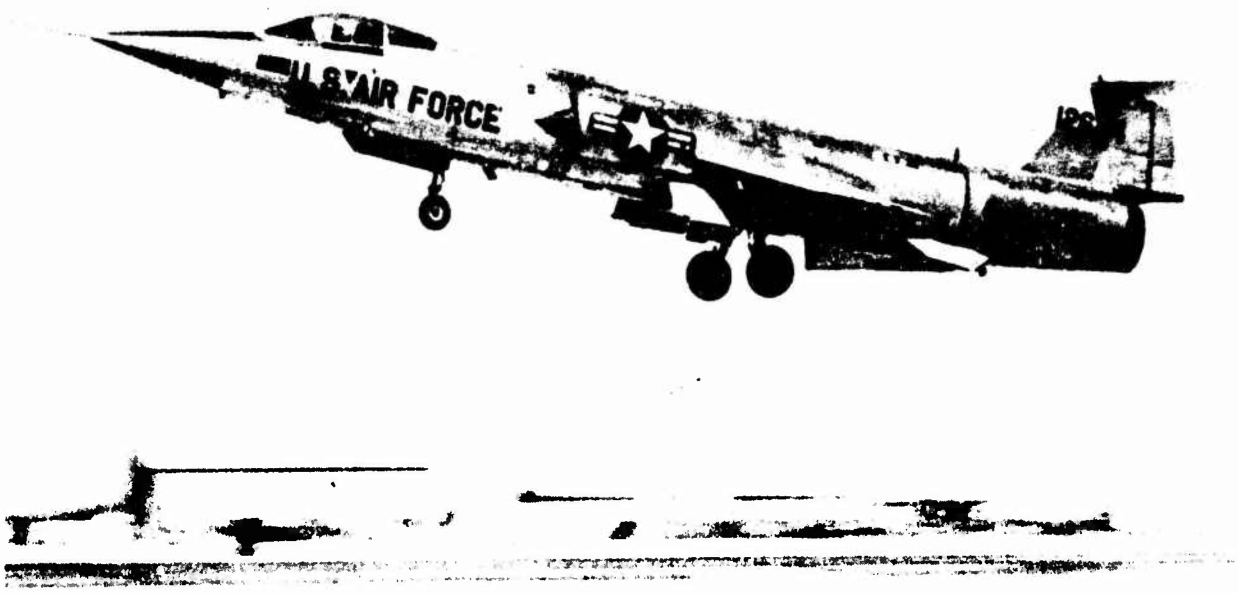
Table 4.3

GROUP SPIN ASSIGNMENTS

	Left Spins		Right Spins
Group A	2 Baseline	Group B	2 Baseline
	2 δ_a With		2 δ_a With
Group C	2 Baseline	Group D	2 Baseline
	2 Power On		2 Power On

The two to four optional spins will be checked by an IP and will comply with the following restrictions:

No δ_a against the spin (at anytime) } Could cause tumble
No δ_a with during recovery
No δ_e (stick) forward of neutral
No incipient phase recoveries (intentionally)
No vertical entries
No fuel in tip tanks
No intentional inverted spins
Cruise configuration spins only.



4.8

CHAPTER 5
MANEUVERABILITY

5.1 INTRODUCTION

The purpose of maneuvering flight is to determine the stick force versus load factor gradients and the forward and aft center of gravity limits for an aircraft in accelerated flight conditions.

To maneuver an aircraft longitudinally from its equilibrium condition, the pilot must apply a force, F_s, on the stick to deflect the elevator an increment, $\Delta\delta_e$. The requirements that must be met during longitudinal maneuvering are covered in MIL-F-8785, section 3.2.2.

5.2 MIL-F-8785

MIL-F-8785 specifies the allowable stick/wheel force per "g" gradient during maneuvering flight. It also specifies that the stick/wheel force gradients be approximately linear with pull forces on the stick/wheel required to maintain or increase normal acceleration. The pilot must also have sufficient aircraft response without excessive cockpit control movement. These requirements and associated requirements of lesser importance provide the legitimate background for good aircraft handling qualities in maneuvering flight.

The backbone of any discussion of maneuvering flight is stick/wheel force per "g". The amount of stick/wheel force that the pilot must apply to maneuver his aircraft is an important parameter. If the force per "g" is very light, a pilot could overstress or overcontrol his aircraft with very little resistance from the aircraft. The T-38, for instance, has a 5 lb/g gradient at 25,000 feet, Mach 0.9, and 20 percent MAC cg position. With this condition, a ham-fisted pilot could pull 10 g's with only 50 lbs of force and bend or destroy the aircraft. The designer could prevent this possibility by making the pilot exert 100 lb/g to maneuver. This would be highly unsatisfactory for a fighter type aircraft, but perhaps about right for a cargo type aircraft. The mission and type of aircraft must therefore be considered in deciding upon acceptable stick/wheel force per "g". Furthermore, the gradient of stick/wheel force per "g" at any normal load factor must be within 50% of the average gradient over the limit load factor. If it took 10 lb/g to achieve a 4 g turn, it would be unacceptable for the pilot to reach the limit load factor of 7.33 g's with only a little additional force.

The position of the aircraft's cg is a critical factor in stick/wheel force per "g" consideration. The fore and aft limits of cg position may therefore be established by maneuvering requirements.

5.3 EXAMPLE TEST METHODS

5.3 Generally speaking, there are four flight test methods for determining maneuvering flight

characteristics such as stick force gradients, maneuver points, and permissible cg locations. The names given to these different methods may vary among test organizations. Therefore, care should be exercised when discussing a particular test method to make certain that everyone involved is speaking the same language.

Stabilized g Method:

This method requires holding a constant airspeed and varying the load factor. The aircraft is trimmed at the test altitude for hands-off flight, and a trim shot is taken. The power setting is then noted, and the aircraft is climbed to the upper limit of the altitude band (±2,000 feet). The power is then reset to trim power and the aircraft is slowly rolled into a 15-degree bank while the nose is lowered slowly. Data is recorded when the aircraft has been stabilized on an airspeed and bank angle with no stick movements. The attitude indicator should be used to establish the bank angle. The bank angle is then increased to 30 degrees and data is again recorded when the aircraft has been stabilized. Stabilized data points are also obtained at bank angles of 45 and 60 degrees.

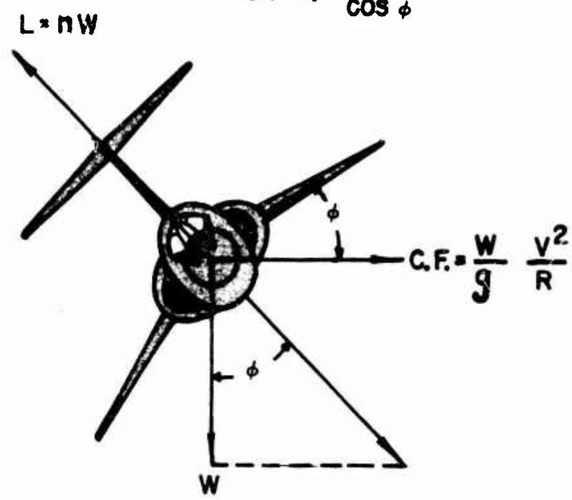
After taking data at the 60-degree bank angle, the bank angle should be increased so as to obtain 0.5 g-increments in load factor. At each 0.5 g-increment, the aircraft is stabilized and data recorded. The test should be terminated when heavy buffet or the limit load factor is reached. Above a load factor of 2.0, only slight increases in bank angle are needed to obtain 0.5-g increments. An idea of the approximate bank angle required can be reached by exploiting the relationship between load factor and bank angle for a constant altitude (figure 5.1).

Figure 5.1 LOAD FACTOR vs BANK ANGLE RELATIONSHIP

W = GROSS WEIGHT
n = LOAD FACTOR
ϕ = BANK ANGLE

$$\cos\phi = \frac{W}{nW} = \frac{1}{n}$$

$$\text{OR} \quad n = \frac{1}{\cos\phi}$$



Little altitude is lost at the lower bank angles up to approximately 60 degrees, and thus more time may be spent stabilizing the aircraft. At 60 degrees of bank angle and beyond, altitude is being lost rapidly; therefore every effort should be made to be on speed and well stabilized as rapidly as possible in order to stay within the allowable altitude block (test altitude ±2,000 feet). If the lower altitude band is approached before reaching limit load factor, the aircraft should be climbed to the upper limit and the test continued. No attempt should be made to obtain data at exact values of g since a good spread is all that is necessary.

The method of holding airspeed constant within a specified altitude band is recommended where Mach number is not of great importance. In regions where Mach number may be a primary consideration, every effort should be made to hold Mach number and airspeed constant.

If power has only a minor effect on the maneuvering stability and trim, altitude loss and the resulting Mach number change may be minimized by adding power as load factor is increased. At times, constant Mach number is held at the sacrifice of varying airspeed and altitude. For constant Mach number tests, a sensitive Mach meter is required or a programmed airspeed/altitude schedule is flown. The stabilized g method is usually used for testing bomber and cargo aircraft and fighters in the power approach configuration.

Slowly Varying g Method:

The aircraft is trimmed as before at the desired altitude. The power is noted and the aircraft is climbed to the upper limit of the altitude band (±2,000 feet). Power is reset at the trimmed value. The data recording switch is activated and the aircraft is slowly banked into a turn. With the airspeed held constant, load factor is slowly increased by increasing bank angle and descending. The slow increase of bank angle and the resulting load factor increase continue until heavy buffet or limit load factor is reached. The rate of g onset should be approximately 0.1 g per second. Again airspeed is of primary importance and should be held to within ±1 knot of aim airspeed. Care should also be taken not to reverse stick forces during the maneuver.

If the airspeed varies excessively, or if the lower limit of the altitude band is approached, the data recorder should be turned off. The aircraft should then be restabilized at the upper altitude limit at a lower g loading. The maneuver should then be continued until heavy buffet onset or limit load factor is reached.

The greatest error made in this method is overbanking beyond 60 degrees of bank. Overbanking, causes the aircraft to traverse the g increments too quickly to be able to accurately hold airspeed. Good bank control is important to obtain the proper g rate of 0.1 g per second.

The slowly varying g method is more applicable to fighter aircraft. Often a combination of the two methods is used in which the stabilized g method is followed until a 60-degree bank angle is reached. The slowly varying g method is then followed from the 60-degree bank angle until heavy buffet or limit load factor is reached.

Constant g Method (Wind-Up Turn):

The aircraft is stabilized and roughly trimmed at the desired altitude and at maximum airspeed for the test. The aircraft is then placed in a constant g turn. Data recording is started and the aircraft is climbed or descended to obtain a 2 to 5 knot per second airspeed bleed rate at the desired constant load factor. Normally the aircraft is climbed to obtain a bleed rate at low load factors and descended to obtain a bleed rate at high load factors. For high thrust-to-weight ratio aircraft at low altitudes, the maneuver may have to be initiated at reduced power to avoid a too rapid traverse of the altitude band. Maintaining the aim load factor is the primary requirement while establishing the bleed rate is secondary. During the maneuver the aircraft should be kept within the altitude band of ±2,000 feet. The airspeed should be noted as the aircraft flies out of the altitude band. When the aircraft is returned to the altitude band, the maneuver is started at an airspeed above the just previously noted airspeed so

that continuity of g and airspeed can be maintained for data purposes. Airspeed is again noted at buffet onset and the g break (when aim load factor can no longer be maintained). The buffet and stall flight envelope is determined or verified by this test method. The maneuver is then repeated at 0.5 g increments at high altitudes and 1-g increments at low altitudes.

Symmetrical Pull-Up Method:

The aircraft is trimmed at the desired test altitude and airspeed. The aircraft is then climbed to an altitude above the test altitude using power as required. Trim power is reset and the aircraft is pushed over into a dive. The dive angle is a function of the load factor to be applied (steeper angle for higher g values).

The aircraft is then maneuvered to reach a point, above the test altitude at a lead airspeed below the test airspeed, such that a "g pull" can be established that will place the aircraft at a given constant load factor while passing through the test altitude at the test airspeed. The lead airspeed is determined by the desired load factor - higher load factors and their resultant steeper dive angles require greater leads (lower lead airspeeds). The aircraft should pass through level flight (±15 degrees from horizontal) just as the airspeed reaches the trim airspeed with aim g loading and steady stick forces. Achieving the trim airspeed through level flight, ±15 degrees, and holding steady stick forces to give a steady pitch rate are of primary importance. The variation in altitude (±1,000 feet) at the pull-up is less important. The g loading need not be exact, but should be steady. Data is recorded as the aircraft passes through level flight ±15 degrees. The aircraft is then climbed to an altitude above the test altitude and the maneuver is repeated at another load factor at the same trim airspeed.

5.4 DATA REDUCTION METHODS

The maneuvering flight test is conducted at high, medium, and low altitudes at three different airspeeds or Mach numbers throughout the flight envelope. The aircraft is flown with a forward and an aft cg. Oscillograph recordings give a readout of stick force, elevator deflection, angle of attack, and load factor obtained with the constant airspeed, varying g, flight test method. Data will normally be recorded by use of the camera in both the T-38 and B-57 aircraft. The oscillograph will be used in the T-38. A sample data card is shown.

STUDENT FLIGHT RECORD		DATE 1 APR 68	
NAME OF STUDENT DENT		NAME OF INSTRUCTOR	
AIRCRAFT NUMBER T-38 596		TEST MAN FLT	
CONFIGURATION CRUISE - COMBAT AFT CG			
PRESS ALTITUDE	RUNWAY TEMP	TAKEOFF ROLL	TAKEOFF V_1

GROUND BLOCK AT TAKEOFF	ALTIMETER (at 29.92)	TEMP	TOD	CAM NR	OSC NR

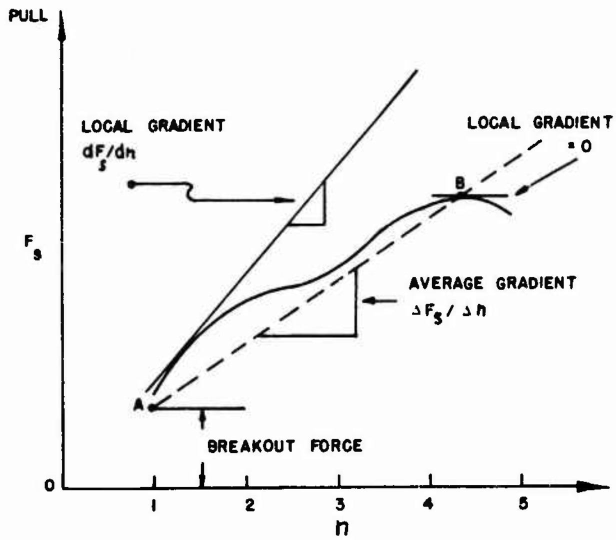
U_i	H_i	TRIM C/N	START C/N	END C/N	LEFT FUEL	RIGHT FUEL
.45	15M					
.60	"					
.75	"					
.60	"					
.75	25M					
.85	"					
.75	35M					
.85	35M					
1.1	"					

GROUND BLOCK AT LANDING	ALTIMETER (at 29.92)	TEMP	TOD	CAM NR	OSC NR

AFFTC FORM JAN 65 0-112   PREVIOUS EDITION OF THIS FORM WILL BE USED UNTIL STOCK IS EXHAUSTED.

The first step in data reduction is to plot stick force, F_s, against load factor for each test point. A sample plot is shown in figure 5.2. In this figure, the breakout force is determined and labeled Point A. Point B is located where the stick force curve becomes erratic. This point (approximately 85 percent of the limit load factor) may be defined by heavy buffet or change of sign of stick force gradient. The line connecting points A and B is the average gradient. The local gradient is the slope at any point along the curve.

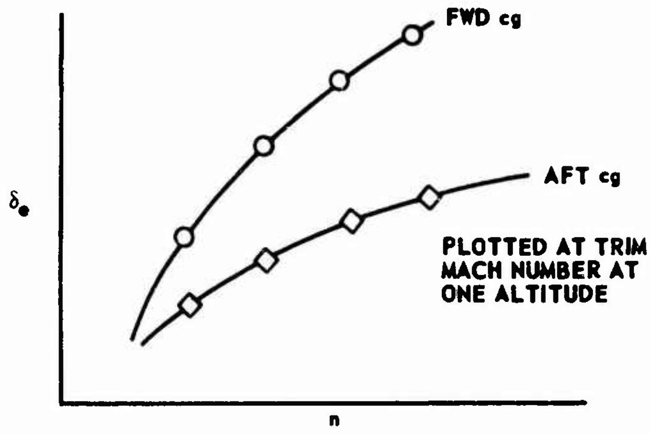
Figure 5.2 DETERMINATION OF AVERAGE GRADIENT FOR IRREGULAR CURVE OF F_s vs n



<u>Stick-Fixed Maneuvering Flight:</u>

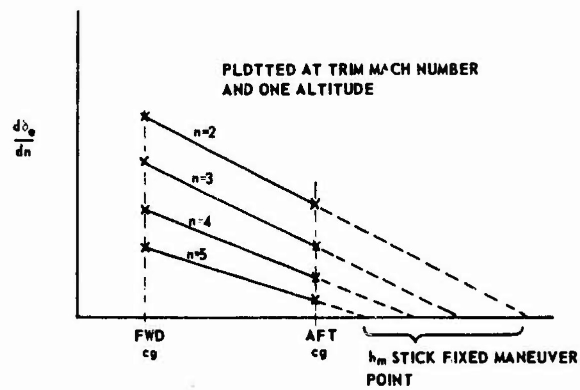
Elevator deflections, δ_e, are plotted versus load factor, n, for each trim airspeed at a particular altitude.

FIGURE 5.3 ELEVATOR DEFLECTION vs LOAD FACTOR



The slope, $d\delta_e/dn$, is determined for several load factors at the two cg positions and is plotted as shown in figure 5.4.

FIGURE 5.4 SLOPE OF ELEVATOR DEFLECTION PER LOAD FACTOR vs cg POSITION (WORKING PLOT)



The cg position where the slope $d\delta_e/dn$ is zero, is the maneuver point location for that load factor at the designated trim Mach and airspeed. Nine plots of δ_e vs n and nine plots of $d\delta_e/dn$ vs cg yield the following summary plot.

<u>Stick-Free Maneuvering Flight:</u>

Stick force, F_s, is plotted versus load factor, n, for each of nine trim Mach numbers (three at

FIGURE 5.5
STICK-FIXED MANEUVER POINT
VARIATION WITH LOAD FACTOR

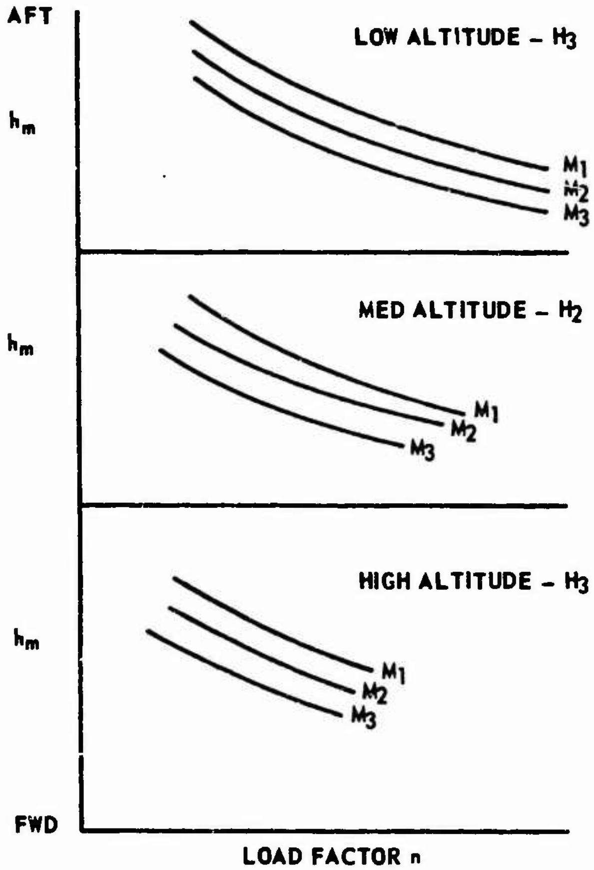
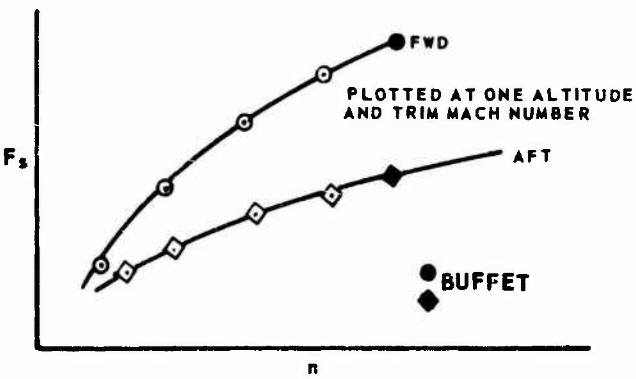


FIGURE 5.6
STICK-FORCE VERSUS LOAD FACTOR



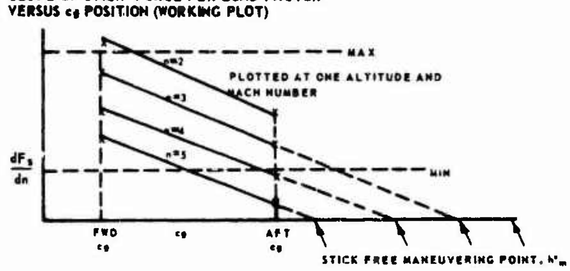
each of three altitudes). One plot for a single trim Mach, M_1, at a particular altitude, H_1, is shown in figure 5.6. The load factor at buffet onset should also be indicated.

It should be noted that the above curves do not necessarily go through zero stick force at one g. This is because there will usually be some breakout force required to allow movement of the longitudinal control.

The average stick force gradient should be found and the local gradient examined to determine whether or not the local gradient is within 50 percent of the average gradient.

The plot of dF_s/dn versus cg position is derived from the stick force versus load factor curves to determine the stick-free maneuver points.

FIGURE 5.7
SLOPE OF STICK-FORCE PER LOAD FACTOR
VERSUS cg POSITION (WORKING PLOT)

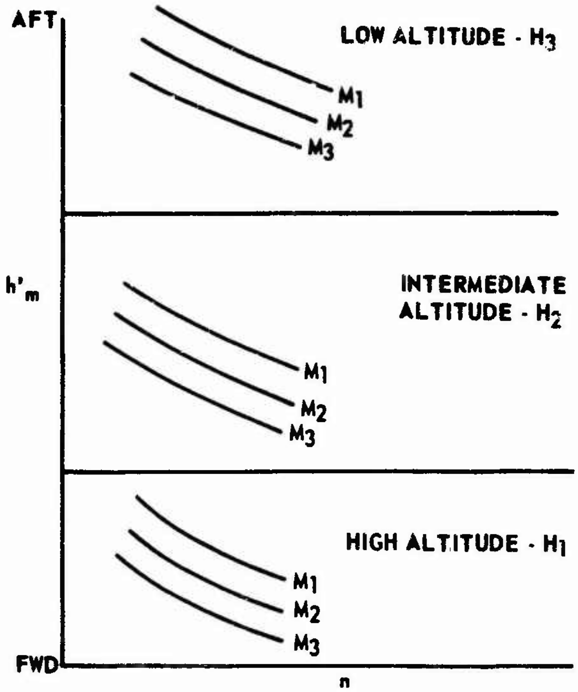


The cg location where dF_s/dn equals zero is the stick-free maneuver point for that particular load factor at the given trim Mach and altitude.

Nine plots for the three altitudes and three trim Mach numbers yield the following summary plots of stick free maneuver point variation with load factor.

A cross plot of figure 5.8 can also be made to show how stick-free maneuver points vary with Mach number.

FIGURE 5.8 STICK FREE MANEUVER POINTS vs LOAD FACTOR



Stick Force Gradients:

It is not sufficient to say that local stick force per "g" gradients do not differ from the average gradient by more than 50%. MIL-F-8785 specifies minimum and maximum values of the local gradient which must be met. These gradients are specified in absolute numbers, $x/n_L - 1$, or in $x/n_Z/\alpha$. Requirements for local gradients specified in terms of $x/n_L - 1$ or in absolute numbers can be determined from the data used to plot figure 5.2. To satisfy the requirements in terms of n_Z/α, another type of plot is required. First form the ratio of n_Z/α in lb/radian (where $\alpha = \alpha_{point} - \alpha_{trim}$). From figure 5.2 determine the F_S/g gradient at the corresponding test point. Finally plot F_S/g vs n_Z/α on a log-log plot as seen below to see if it lies within the MIL-F-8785 requirements.

5.5 DEMONSTRATION MISSION

A demonstration mission will be flown in the T-38 to demonstrate the methods and techniques used to determine stick force gradients and forward and aft center of gravity limits for an aircraft in accelerated flight. No data reduction or plots are required. The procedures to be followed are outlined below.

Procedures:

1. The student will perform an afterburner takeoff and climb to 30,000 feet.
2. He will then stabilize at 300 KIAS and 30,000 feet.
3. The instructor will demonstrate the stabilized g method at 300 KIAS and 30,000 feet.
4. The student pilot will then perform using the stabilized g method under the same trim, altitude, and airspeed conditions.
5. The instructor will demonstrate the slowly varying g method at 300 KIAS and 30.000 feet.
6. The student pilot will next perform the slowly varying g method at 300 KIAS and 30,000 feet.
7. The student will then stabilize at 350 KIAS and 30,000 feet.
8. The student will perform the stabilized g method at 350 KIAS and 30,000 feet.
9. Next, he will perform the slowly varying g method at 350 KIAS and 30,000 feet.

10. Then the student will perform the constant g, slowly varying airspeed method - pulling 3 g's at 20,000 feet.

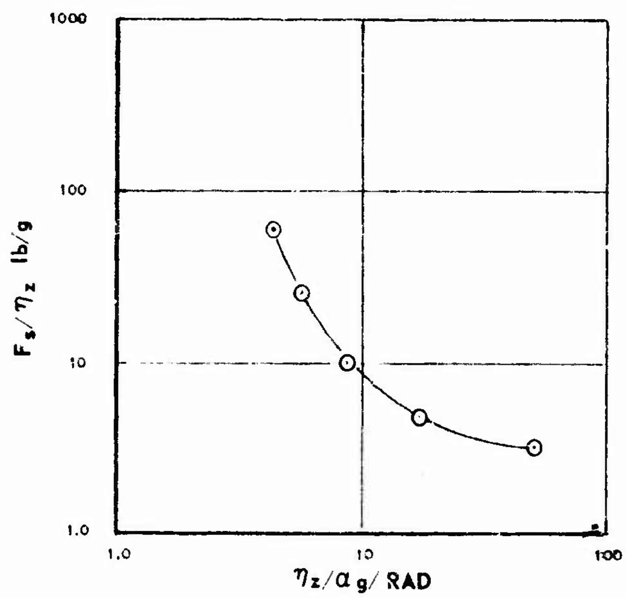
11. Finally, the student will perform two pull-up maneuvers: one of 2 g's at 300 KIAS and 20,000 feet; and the other of 3 g's at 300 KIAS and 20,000 feet.

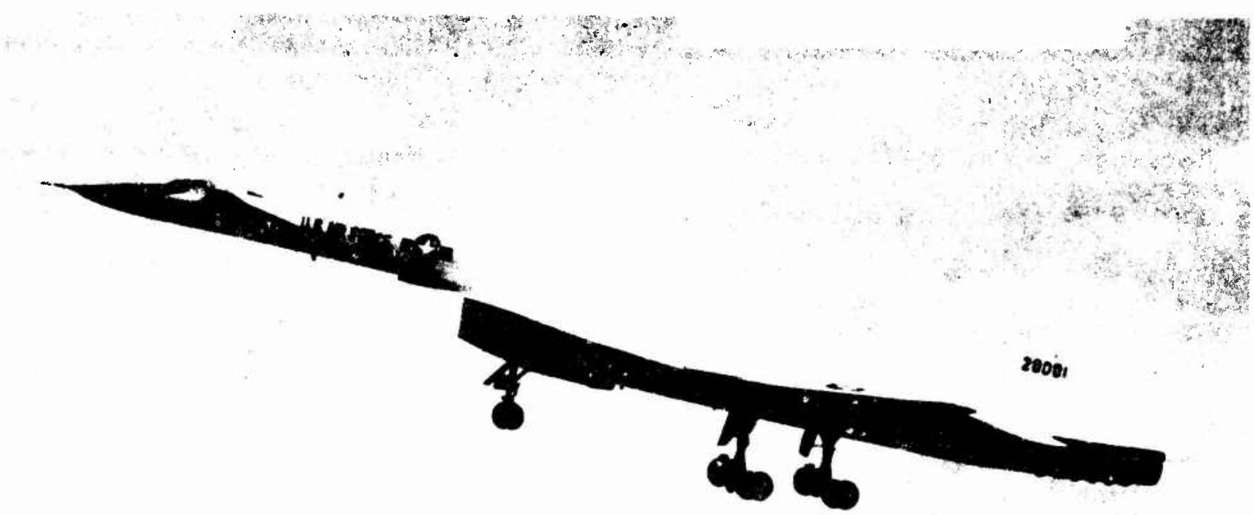
<u>Instrumentation:</u>

1. None.

2. Simulate picture taking at stabilized g points by calling out "picture."

Figure 5.9 STICK FORCE PER "g" vs n_z/α





CHAPTER 6

TRIM CHANGES

6.1 PURPOSE

The purpose of this test is to ascertain the magnitude of control force changes associated with normal configuration changes, trim system failure, or transfer to alternate control systems in relation to specified limits. It must also be determined that no undesirable flight characteristics accompany these configuration changes.

6.2 TEST CONDITIONS

Pitching moments on aircraft are normally associated with changes in the condition of any of the following: landing gear, flaps, speed brakes, power, bomb bay doors, rocket and missile doors, or any jettisonable device. The magnitude of the change in control forces resulting from these pitching moments is limited by Military Specification F-8785, and it is the responsibility of the testing organization to determine if the actual forces are within the specified limits.

The pitching moment resulting from a given configuration change will normally vary with airspeed, altitude, cg loading, and initial configuration of the aircraft. The control forces resulting from the pitching moment will further depend on the aircraft parameter being held constant during the configuration change. These factors should be kept in mind when determining the conditions under which the given configuration change should be tested. Even though the specification lists the altitude, airspeed, initial conditions, and parameter to be held constant for most normal configuration changes, some variations may be necessary on a specific aircraft to provide information on the most adverse conditions encountered in operational use of the aircraft. The altitude and airspeed should be selected as indicated in the specifications or for the most adverse conditions. In general, the trim change will be greatest at the highest airspeed and the lowest altitude. The effect of cg location is not so readily apparent and usually has a different effect for each configuration change. A forward loading may cause the greatest trim change for one configuration change, and an aft loading may be most severe for another. For this reason, a mid cg loading is normally selected since rapid movement of the cg in flight will probably not be possible. If a specific trim change appears critical at this loading, it may be necessary to test it at other cg loadings to ascertain its acceptability.

Selection of the initial aircraft configurations will be dependent on the anticipated normal operational use of the aircraft. The conditions given in the specifications will normally be sufficient and can always be used as a guide, but again variations may be necessary for specific aircraft. The same holds true for selection of the aircraft parameter to hold constant during the change. The parameter that the pilot would nor-

mally want to hold constant in operational use of the aircraft is the one that should be selected. Therefore, if the requirements of MIL-F-8785 do not appear logical or complete, then a more appropriate test should be added or substituted.

In addition to the conditions outlined above, it may be necessary to test for some configuration changes that could logically be accomplished simultaneously. The force changes might be additive and could conceivably be objectionably large. For example, on a go-around, power may be applied and the landing gear retracted at the same time. If the trim changes associated with each configuration change are appreciable and in the same direction, the combined changes should definitely be investigated.

The specifications require that no objectional buffet or undesirable flight characteristics be associated with normal trim changes. Some buffet is normal with some configuration changes, e.g., gear extension; however, it would be considered objectionable in this case if this buffet tended to mask the buffet associated with stall warning. The judgment of the pilot is the best measure of what actually constitutes "objectionable" but anything that would interfere with normal use of the aircraft would certainly be considered objectionable. The same is true for "undesirable flight characteristics." An example would be a strong nose-down pitching moment associated with gear or flap retraction after takeoff.

The specification also sets limits on the trim changes resulting from transfer to an alternate control system. The limits vary with the type of alternate system and the configuration and speed at the time of transfer, but in no case may a dangerous flight condition result. It will probably be necessary for the pilot to study the operation of the control systems and methods of effecting transfer in order to determine the conditions most likely to cause an unacceptable trim change upon transferring from one system to the other. As in all flight testing, a thorough knowledge of the aircraft and the objectives of the test will improve the quality and increase the value of the test results.

6.3 EXAMPLE TEST METHODS

The pre-flight preparation for the trim change test should start with a study of the applicable paragraphs of Military Specification F-8785. By comparing the specification requirements with the expected operational use of the aircraft, it will be possible to determine all the configuration changes and the conditions under which they should be tested.

When all the required changes have been determined, some time should be spent in laying out the sequence in which to test the various items in order to conserve flight time. Most of the configuration changes can be planned so that at the end of one test, the aircraft will be ready for the condition desired on the next test. This will result in a minimum delay. Table X in the specification can be used as an excellent guide in establishing the most advantageous sequence, but this sequence may vary depending on the specific aircraft. In an actual test program, much of the trim change information would probably be obtained during other tests as the aircraft was placed in the required test conditions. For example, the trim change with gear extension might be tested in the landing pattern after completion of another test.

After the required configuration changes and the sequence has been determined, a data card or

cards should be prepared. Many forms are possible but the card should include all the information required by the pilot in flight and should be clearly legible. This test does not lend itself to the standard data card format, therefore a card will have to be made up from a blank one. A suggested form follows:

TRIM CHANGES

STUDENT ________________ INSTRUCTOR __________

DATE _______ A/C No _______ CAMERA No __________

GRND SHOT ______________ TEST ________ ________

TOTAL FUEL _____ CONFIGURATION _________ CG _____

RUN NO	ALT	INITIAL TRIM CONDITIONS				CONFIG CHANGE	HOLD CONSTANT	C.N.	REMARKS
		SPEED	GEARS	FLAPS	POWER				
1	5M	140	UP	UP	P[illegible]	GEAR ON	ALT.		
2	5M	140	ON	[illegible]	PLF	FLAPS ON	ALT.		

Prior to leaving the line a normal ground photo will be taken to insure that all force and position measuring equipment is functioning properly. Following the flight data card, the aircraft should be placed in the desired condition and trimmed for pilot comfort. Excessive time should not be wasted on being exactly trimmed since a change in control forces is the objective rather than the magnitude of the total force. When trimmed and stabilized the oscillograph should be started, (speed 3 is recommended) and then the desired configuration change is made. Actuating the event marker at the same time the configuration change is initiated will facilitate interpretation of the recorded data. The desired parameter (attitude, altitude, rate-of-climb, etc.) should be held constant with force alone for approximately 5 seconds and then normal trimming action employed to relieve the forces. It is important to pick out the most logical parameter to hold constant, and then smoothly hold that parameter constant. The control force changes which occur are often strongly affected by overcontrolling or allowing the supposedly constant parameter to vary. Both extremes must be avoided. While trimming, a qualitative evaluation of the adequacy of the aircraft trim should be made. If it is too fast or too slow or there is insufficient trim available to relieve the forces, a notation should be made on the data card. It should also be possible to trim the control forces to zero throughout the operational envelope of the aircraft. While there is no specific test for this requirement, the pilot should be alerted to note non compliance during other tests. When approximately trimmed, the oscillograph should be turned "OFF" and the aircraft readied for the next test condition.

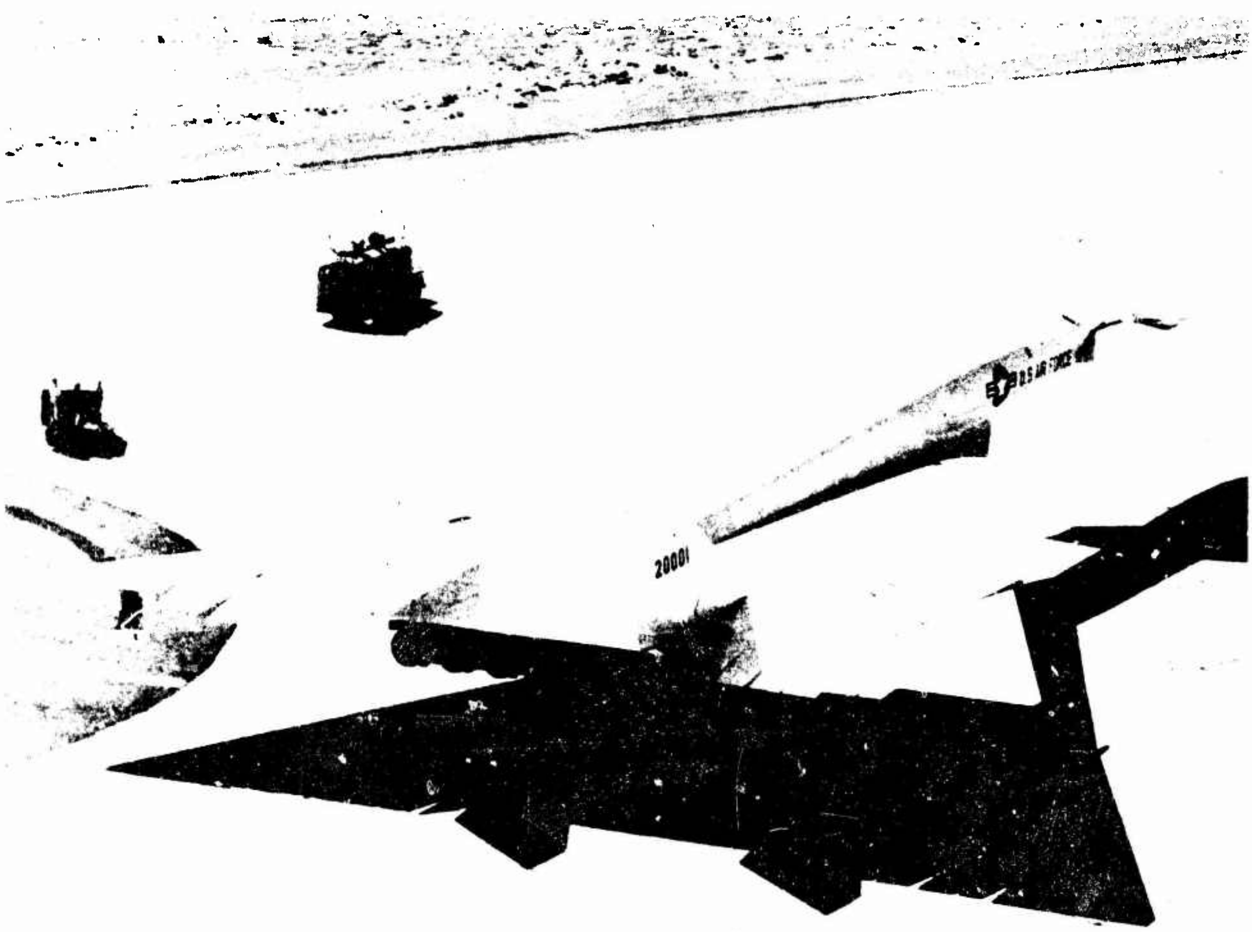
This same general procedure will be repeated until all the points listed on the data card have been accomplished. The test for transfer to alternate control modes will have to be done twice. The first one done with the controls free and the second test done with the pilot maintaining attitude and

zero sideslip throughout. As always, fuel, time, and orientation should be kept in mind.

The oscillograph traces will be used to determine the changes in forces and control deflections associated with each configuration change. If all the force changes are within the specification limits, all that is necessary is a table of values similar to the example below. However, for one configuration change a time history will be included. This should be for a configuration change that exceeded the limits of the specification. If none exceeded the specification, then a time history of a typical change will be made. The time history will include, but not be limited to, airspeed, altitude, load factor, angle of pitch, angle of bank, elevator force, aileron position, rudder position, and rudder force plotted against time. The table will be as shown below.

TABULAR SUMMARY OF RESULTS

RUN No.	ALT	INITIAL TRIM CONDITIONS A/S	GEAR	FLAPS	POWER	CONFIG CHANGE	HELD CONSTANT	F_s Lbs	δ_e Deg
1	SM	195	UP	UP	PLF	GEAR DN	ALTITUDE	-7	-1.5
2	SR	165	DN	UP	PLF	FLAPS DN	ALTITUDE	+16	-1.0
ETC									



CHAPTER 7

LATERAL-DIRECTIONAL FLIGHT TESTS

7.1 INTRODUCTION

7.1 With lateral-directional theory as a background, it is possible to look at the flight test techniques used to investigate the actual lateral-directional static stability of an aircraft.

Basically, the lateral-directional characteristics of an aircraft are determined by two different flight tests; the Sideslip Test and the Aileron Roll Test. The tests do not measure lateral and directional characteristics independently. Rather, each test yields information concerning both the lateral and the directional characteristics of the aircraft.

In this chapter, both the Sideslip Test and the Aileron Roll Test will be covered. As each test is discussed, the required results, as determined by MIL-F-8785 will also be covered. Finally, an example test mission in the T-33 aircraft will be discussed.

7.2 SIDESLIP FLIGHT TEST TECHNIQUE

7.2 The purpose of the sideslip test is to investigate the static lateral and directional stability characteristics of a particular aircraft in each of several configurations. Since the static lateral and directional stabilities are functions of Mach number, angle of attack, elasticity and configuration, it is important to check the aircraft in various configurations at several altitude-airspeed combinations. In so doing, the sense (+) of the lateral and directional stabilities and the characteristics of the side force can be determined throughout the flight envelope.

All equations relating to the static directional stability of an aircraft were developed under the assumption that the aircraft was in a "steady straight sideslip." This is the maneuver used in the Sideslip Test. To develop a ground for discussion, it is appropriate to discuss the basic mechanics the pilot must perform to establish a "steady straight sideslip." First, the aircraft is trimmed at the desired altitude-airspeed combination. The rudder is then depressed and an increment of sideslip is developed. In order to maintain "straight" or constant heading flight, it will then be necessary for the pilot to bank the aircraft in a direction opposite that of the applied rudder. The aircraft is then stabilized in this condition. Thus, the pilot establishes a "steady straight sideslip." To understand the forces and moments at play in this condition, consider figure 7.1. In this figure the aircraft is in a steady sideslip. Thus, the moment created by the rudder, η_{δ_r}, must equal the moment created by the aerodynamic forces acting on the aircraft, η_β. It can be seen, however, that in this condition the side forces are unbalanced and that this will cause an acceleration. The force, F_{y_β}, will always be greater than $F_{y_{\delta_r}}$. Thus, in the case depicted, the aircraft will accelerate, or turn, to the right. In order to stop this turn, it is necessary to bank the aircraft; in this case to the left (figure 7.2). The bank allows a component of aircraft weight, $W\sin\phi$, to act in the y direction and thus balance the previously

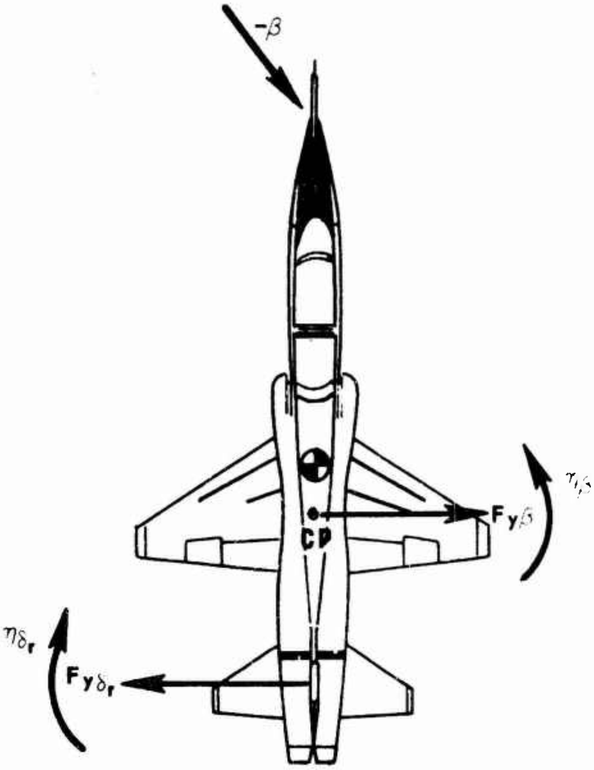


FIGURE 7.1 STEADY SIDESLIP

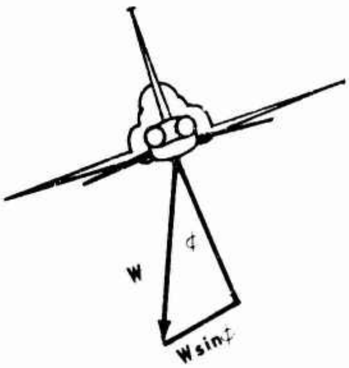


FIGURE 7.2 STEADY STRAIGHT SIDESLIP

unbalanced side forces. Thus, the pilot establishes a "straight sideslip." By holding this condition constant with respect to time, or varying it so slowly in a continuously stabilized condition that rate effects are negligible, he establishes a "steady straight sideslip" - the condition that was used to derive the flight test relationships in static directional stability theory.

By simply establishing a steady straight sideslip, the side force characteristics of the aircraft can be investigated. MIL-F-8785, paragraph 3.3.6.2 states that "an increase in right bank angle must accompany an increase in right sideslip," where right sideslip is defined by the incident airflow approaching from the right side of the plane of symmetry.

One property of basic importance in the sideslip test is the directional stiffness of an aircraft or its static directional stability. To review, the static directional stability of an aircraft is defined by the initial tendency of the aircraft to return to or depart from its equilibrium angle of sideslip when disturbed from the equilibrium condition. In order to determine if the aircraft possesses static directional stability, it is necessary to determine how the yawing moments change as the sideslip angle is changed. Thus, the slope of a line in a plot such as figure 7.3 is of interest. For positive directional stability a plot of C_{n_β} must have a positive slope.

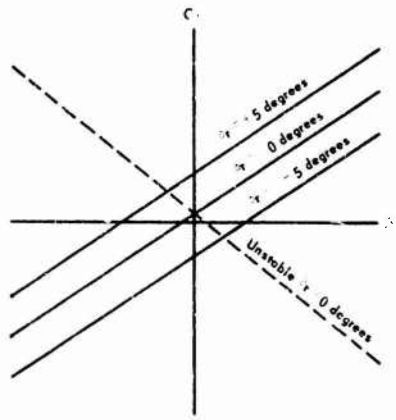


Figure 7.3 WIND TUNNEL RESULTS OF YAWING MOMENT COEFFICIENT vs SIDESLIP ANGLE

Plots like those presented in figure 7.3 would be obtained from wind tunnel data. The aircraft model would be placed at various angles of sideslip with various angles of rudder deflection, and the unbalanced moments would be measured. However, it is impossible to determine from flight tests the unbalanced moments at varying angles of sideslip. It was shown in static directional theory, however, that the amount of rudder deflection required to fly in a steady straight sideslip is considered to be an indication of the amount of yawing moment present tending to return the aircraft to or remove it from its original trimmed angle of sideslip. Thus, in order to determine the sign of the rudder fixed static directional stability, C_{n_β}, a plot is made of rudder deflection required versus sideslip angle.

The apparent stability parameter, $\partial\delta_r/\partial\beta$, for a directionally stable aircraft is shown in figure 7.4. For a stable aircraft, this plot has a negative slope.

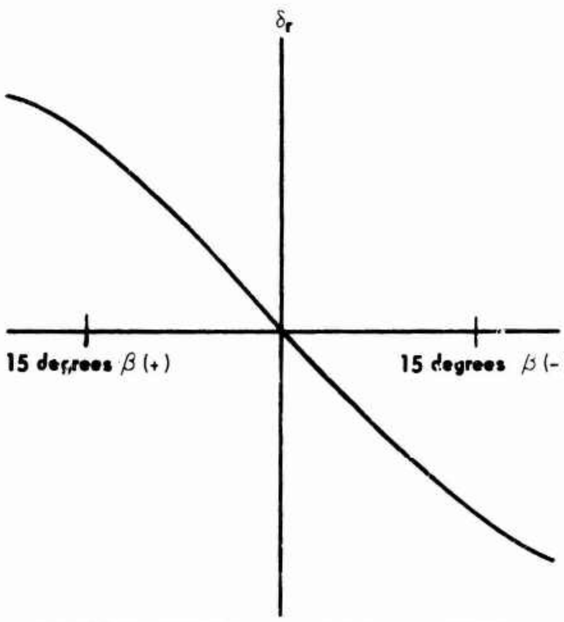


FIGURE 7.4 RUDDER DEFLECTION vs SIDESLIP

MIL-F-8785, paragraph 3.3.6.1 requires that right rudder pedal deflection ($+\delta_r$) accompany left sideslips ($-\beta$). Further, for angles of sideslip between ± 15 degrees, a plot of $\partial\delta_r/\partial\beta$ should be essentially linear. For larger sideslip angles, an increase in β must require an increase in δ_r. In other words, the slope of $\partial\delta_r/\partial\beta$ cannot go to zero.

It should be remembered that drastic changes occur in the transonic and supersonic speed regions. In the transonic region where the flight controls are most effective, a small δ_r may give a large β and thus $\partial\delta_r/\partial\beta$ may appear less stable. However, as speed increases, control surface effectiveness decreases, and $\partial\delta_r/\partial\beta$ will increase in slope. This apparent change in C_{n_β} is due solely to a change in control surface effectiveness and can give an entirely erroneous indication of the magnitude of the static directional stability if not taken into account.

It is now necessary to investigate the control free stability of an aircraft. As discussed in the theory of static directional stability, a plot of rudder force required versus sideslip, $\partial F_r/\partial\beta$, is an indication of the rudder-free static directional stability of an aircraft. It was shown that a plot of $\partial F_r/\partial\beta$ must have a negative slope for positive rudder-free static directional stability. MIL-F-8785 paragraph 3.3.6.1 requires that a plot of $\partial F_r/\partial\beta$ be essentially linear between ± 10 degrees of β from the trim condition. However, at greater angles of sideslip, the rudder forces may lighten but may never go to zero, or overbalance. These requirements are depicted in figure 7.5.

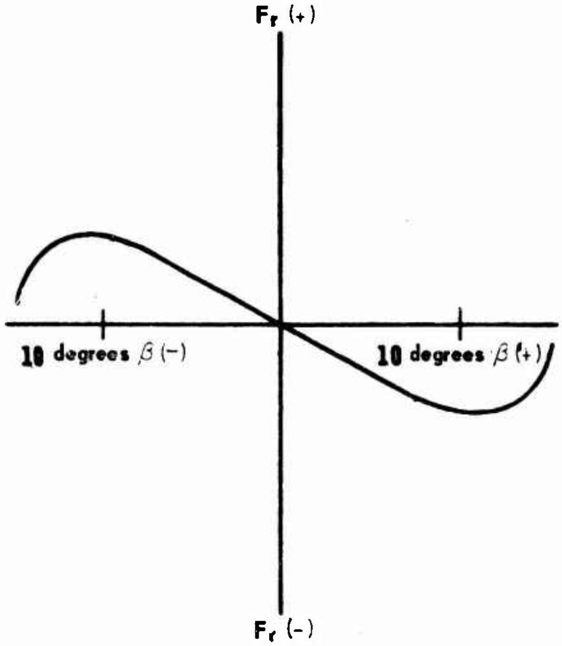


FIGURE 7.5 CONTROL FREE SIDESLIP DATA

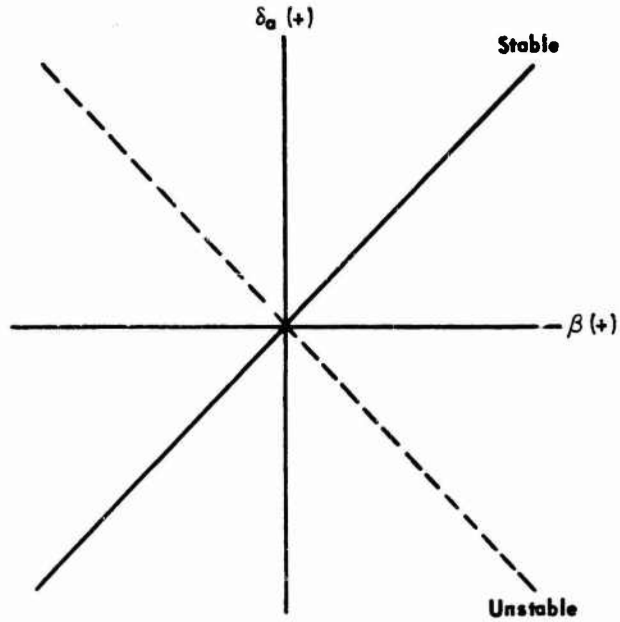


FIGURE 7.6 CONTROL FIXED SIDESLIP DATA

The control force information in figure 7.5 is acceptable in accordance with MIL-F-8785 as long as the algebraic sign of F_r/β is negative. It can be seen that at very large sideslip angles, the slope $\partial F_r/\partial \beta$ may be positive. This is acceptable as long as the rudder force required does not go to zero.

Static lateral characteristics are also investigated during the sideslip test. It was shown in the theory of static lateral stability that $\partial \delta_a/\partial \beta$ may be taken as an indication of the control-fixed dihedral effect of an aircraft, $C_{\ell_{\beta}(Fixed)}$. For stable dihedral effect, it was shown that a plot of $\partial \delta_a/\partial \beta$ must have a positive slope. MIL-F-8785 specifies that right aileron control deflection shall accompany right sideslips and left aileron control shall accompany left sideslips. A plot of $\partial \delta_a/\partial \beta$ for stable dihedral effect is presented in figure 7.6.

MIL-F-8785, paragraph 3.3.6.3b, limits the amount of stable dihedral effect an aircraft will exhibit by specifying that no more than 75 percent of full aileron cockpit control deflection will be required in any of the mandatory sideslip tests required by paragraph 3.3.6. MIL-F-8785 paragraph 3.3.6.3b limits the amount of dihedral effect an aircraft may have. It states that no more than 10 pounds of aileron stick force or 20 pounds of aileron wheel force is allowed for sideslips which may be experienced in operational employment.

The theoretical discussion of control free dihedral effect revealed that $\partial F_a/\partial \beta$ will give an indication of $C_{\ell_{\beta}(Free)}$, and that for stable dihedral effect $\partial F_a/\partial \beta$ is positive. Refer to figure 7.7. MIL-F-8785 paragraph 3.3.6.3 states that the left aileron force should be required for left sideslips and that a plot of $\partial F_a/\partial \beta$ should be essentially linear for all of the mandatory sideslips tested.

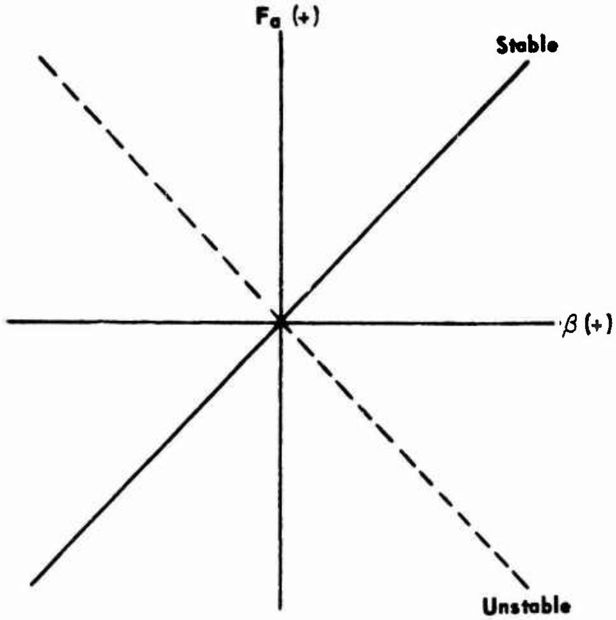


FIGURE 7.7 CONTROL FREE SIDESLIP DATA

MIL-F-8785, paragraph 3.3.6.3a does permit an aircraft to exhibit negative dihedral effect in wave-off conditions as long as no more than 50 percent of available roll control or 10 pounds of aileron control force is required in the negative dihedral direction.

A longitudinal trim change will most likely occur when the aircraft is sideslipped. MIL-F-8785 paragraph 3.2.3.7, places definite limits on the allowable magnitude of this trim change. It is prefered that an increasing pull force accompany an increase in sideslip angle and that the magnitude and direction of the trim change should be similar for both left and right sideslips. The specification also limits the magnitude of the control force accompanying the longitudinal trim change depending on the type of controller in the aircraft (stick or wheel). A plot of elevator force versus sideslip angle that complies with MIL-F-8785 is presented in figure 7.8.

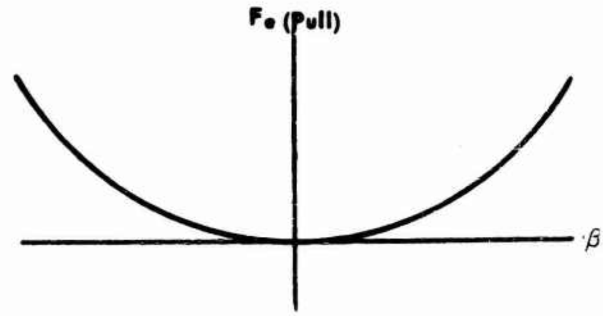


FIGURE 7.8 ELEVATOR FORCE VERSUS SIDESLIP ANGLE

Paragraph 3.3.6 of MIL-F-8785 outlines the sideslip tests that must be performed in an aircraft. The specification requires that sideslips be tested to full rudder pedal deflection, 250 pounds of rudder pedal force, or maximum aileron deflection, whichever occurs first. Often sideslips must be discontinued prior to reaching these limits due to controllability or structural problems.

The following is a complete list of MIL-F-8785 paragraphs that apply to sideslip tests:

3.2.3.7

3.3.6

3.3.6.1

3.3.6.2

3.3.6.3, 3.3.6.3a, 3.3.6.3b

7.3 LIMITATIONS

On student data missions at the Aerospace Research Pilot School it is not possible to conform to all aspects of MIL-F-8785 that concern sideslip tests. Therefore, certain general limitations must be applied to all student sideslip tests.

1. In the cruise configuration, sideslip tests will be conducted at three different altitudes. At each altitude, three different airspeeds will be investigated.

2. In the power approach configuration, sideslip tests will be conducted at 10,000 feet AGL, at the minimum speed for normal final approach.

3. Sideslip tests will be conducted within the limitations outlined in the appropriate aircraft Flight Manual.

4. Within these limitations, the student test program will be set up to investigate all of the requirements concerning sideslip tests that are outlined in the applicable paragraphs of MIL-F-8785.

5. To void excessive data reduction, all results will be plotted against V_i.

Sample data plots of sideslip test results are presented in figures 7.9 and 7.10.

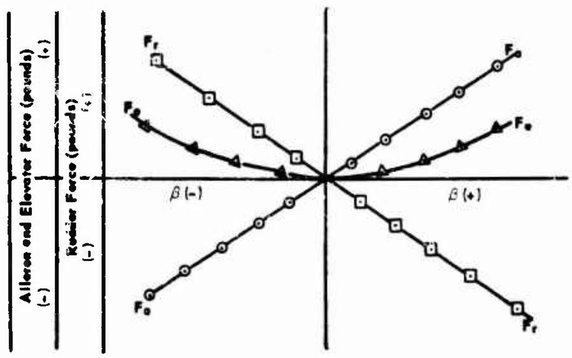


FIGURE 7.9 STEADY STRAIGHT SIDESLIP CHARACTERISTICS CONTROL FORCES VERSUS SIDESLIP

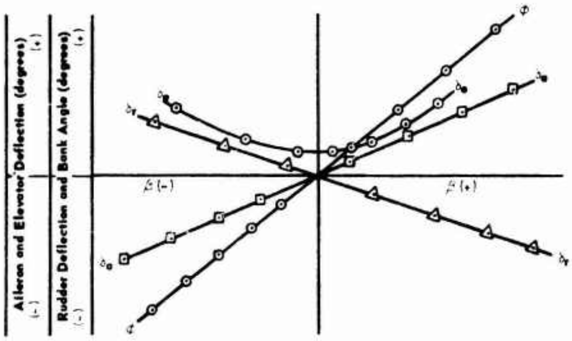


FIGURE 7.10 STEADY STRAIGHT SIDESLIP CHARACTERISTICS CONTROL DEFLECTIONS AND BANK ANGLE VERSUS SIDESLIP

• 7.4 AILERON ROLL FLIGHT TEST TECHNIQUE

Generally, the aileron roll flight test technique is used to determine the rolling performance of an aircraft and the yawing moments generated by rolling. Roll coupling is another important aircraft characteristic normally investigated by using the aileron roll flight test technique. The roll coupling aspect of the aileron roll test will not be investigated at the Aerospace Research Pilot School. However, the theoretical aspects of roll coupling will be covered in a later course.

It is necessary to understand the basic mechanics involved in the aileron roll flight test technique. The aircraft is first trimmed in the desired configuration at the desired altitude-airspeed combination. Then, the lateral control is abruptly placed to a particular control deflection (1/4, 1/2, 3/4, or full). Normally, the desired control deflection is obtained by using some mechanical restrictor such as a chain stop. With the lateral control at the desired deflection, the aircraft is rolled through a specified increment of bank. For control deflections less than maximum, the aircraft is normally rolled through 90 degrees of bank. Because of the higher roll rates obtained at

full control deflection, it is usually desirable to roll the aircraft through 360 degrees of bank when using maximum lateral control deflection. To facilitate aircraft control when rolling through a bank angle change of 90 degrees, start the roll from a 45-degree bank angle. During the roll, a mechanical recorder, such as an oscillograph, is used to record the following information: aileron position, aileron force, bank angle, sideslip and roll rate. Aileron rolls are normally conducted in both directions to account for roll variations due to engine gyroscopic effects. Aileron rolls are performed with rudders free; with rudders fixed; and are coordinated with $\beta = 0$ throughout roll.

Caution should be exercised in testing a fighter type airplane in rolling maneuvers. The stability of the airplane in pitch and yaw is lower while rolling. The incremental angles of attack and sideslip that are attained in rolling can produce accelerations which are disturbing to the pilot and can also cause critical structural loading. The stability of an airplane in a rolling maneuver is a function of Mach number, roll rate, dynamic pressure, angle of attack, configuration, and control deflections during the maneuver.

The most important design requirement imposed upon ailerons or other lateral control devices is the ability to provide sufficient rolling moment at low speeds to counteract the effects of vertical asymmetric gusts tending to roll the airplane. This means, in effect, that the ailerons must provide a minimum specified roll rate, and a rolling acceleration such that the required rate of roll can be obtained within a specified time, even under loading conditions that result in the maximum rolling moment of inertia (e.g., full tip tanks). The steady roll rate and the minimum time required to reach a particular change in bank angle are the two parameters presently used to indicate rolling capability. Pilot opinion surveys reveal that time to roll a specified number of degrees provides the best overall measure of rolling performance.

The minimum rolling performance required of an aircraft is outlined in MIL-F-8785, table VI. This rolling performance is expressed as a function of time to reach a specified bank angle. Table VI is supplemented further by roll performance required of Class IV airplanes in various flight phases. The specific requirements for Class IV airplanes are spelled out in MIL-F-8785, paragraphs 3.3.4.1a, 1b, 1c, 1d. Paragraph 3.3.4.2 and table VII of MIL-F-8785 specify the maximum and minimum aileron control forces allowed in meeting the roll requirements of table VI and the supplemental requirements concerning Class IV aircraft. Paragraph 3.3.2.3 specifies the maximum rudder force permitted for coordinating the required rolls.

In addition to examining time required to bank a specified number of degrees and aileron forces, F_a, it is necessary to examine the maximum roll rate, P_{max}, to get a complete picture of the aircraft's rolling performance. Therefore, in any investigation of aircraft rolling performance, the maximum roll rate obtained at maximum lateral control displacement is normally plotted versus airspeed.

Paragraph 3.3.4.3 of MIL-F-8785 states that there should be no objectionable nonlinearities in roll response to small aileron control deflections or forces. To investigate this area, it is necessary to observe the roll response to aileron deflections less than maximum - such as 1/4 and 1/2 aileron deflections.

MIL-F-8785, paragraph 3.3.4.5 states that it should be possible to raise a wing by using the rudder pedal alone, and that right rudder pedal force should be required for right rolls. Further, it states that with the aileron cockpit control free, it should be possible to produce a roll rate of 3 degrees per second by use of rudder pedal forces of 50 pounds or less. Turn coordination requirements are spelled out in MIL-F-8785, paragraph 3.3.2.4 for steady turning maneuvers.

The other area of prime interest in the aileron roll flight test is the amount of sideslip that is developed in a roll and the phasing of this sideslip with respect to the roll rate. Associated with this characteristic is the roll rate oscillation. These factors influence the pilot's ability to accomplish precise tracking tasks.

The following is a complete list of MIL-F-8785 paragraphs that apply to aileron roll tests:

3.3.2.3

3.3.2.4

3.3.4

3.3.4.1, 3.3.4.1a, 3.3.4.1b, 3.3.4.1c, 3.3.4.1c

3.3.4.3

3.3.4.4

3.3.4.5

3.3.5

3.3.5.1, 3.3.5.1a

3.3.5.2

7.5 SCHOOL TEST LIMITATIONS

The following limitations will apply to all student data missions at the Aerospace Research Pilot School:

1. In the cruise configuration, aileron roll tests will be conducted at three different altitudes. At each altitude, three different airspeeds will be investigated.

2. In the power approach configuration, aileron roll tests will be conducted at final approach speed at 10,000 feet AGL.

3. Aileron roll tests will be conducted within the limitations outlined in the appropriate aircraft Flight Manual.

4. Within these limitations, the student test program will be set up to investigate all of the requirements concerning aileron roll tests that are outlined in the applicable paragraphs of MIL-F-8785(ASG).

5. To avoid excessive data reduction, all results will be plotted against V_i. Sample plots of aileron roll data are presented in figure 7.11.

7.6 DEMONSTRATION MISSION

To unify all that has been said concerning the sideslip and aileron roll flight test techniques, a complete description of an example demonstration mission in the T-33 will be presented.

1. After engine start, and with aileron boost on, visually position the ailerons to approximately 1/4, 1/2 and 3/4 deflections. Mark each position on the instrument panel with masking tape. Prior to leaving the ramp, a ground shot will be taken to ascertain proper functioning of the force and control deflector indicator. The ground shot will consist of a con-

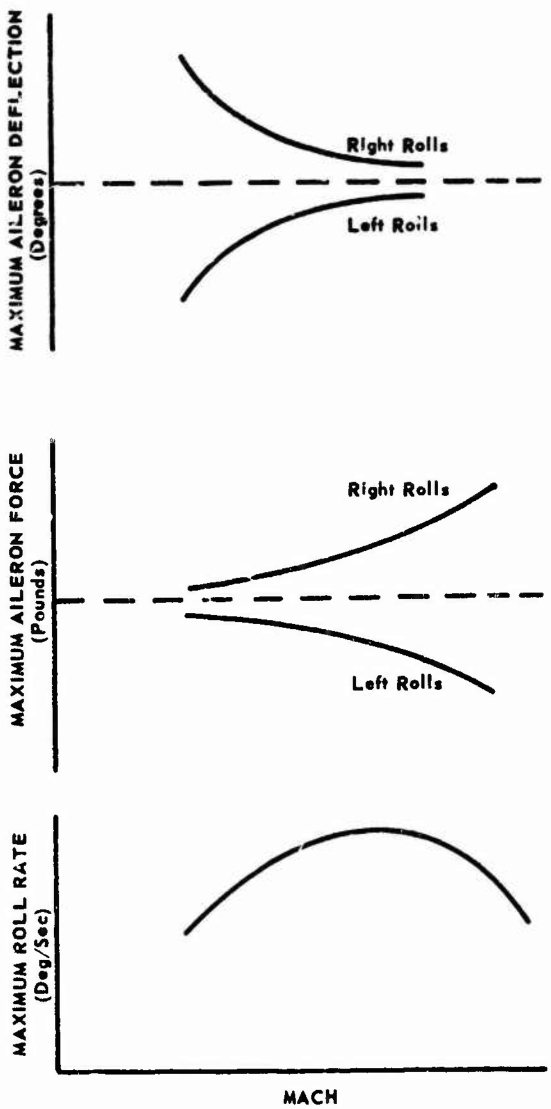


FIGURE 7.11 MAXIMUM AILERON ROLL TEST RESULTS ONE ALTITUDE

tinuous recording of all control deflections and forces from neutral to full positive, to full negative, and then back to neutral. This will help eliminate confusion in reading the film and the oscillograph paper. While taxiing, the pilot should practice making 1/4, 1/2, 3/4, and full aileron deflections.

2. Climb to 20,000 feet and trim the aircraft in the cruise configuration at 250 KIAS using the back side trim technique. Obtain a photo panel shot with the aircraft trimmed in this condition.

3. If a yaw string is available and it is not centered during the trim shot, align it with the longitudinal axis of the aircraft and obtain a photo panel shot. If a sideslip indicator is available for inflight use, this zero sideslip shot can be used to calibrate it. The photo panel shot with zero sideslip should be examined on the ground to ascertain that the zero sideslip indication, as obtained from the calibration book, is correct.

4. During the sideslip test, altitude will be lost if trim power and 250 KIAS are maintained. Therefore, this test will be conducted at 20,000 $\pm$1,000 feet. After the trim shot at 20,000 feet has been obtained, note trim power and climb to 21,000 feet.

5. The instructor pilot will demonstrate the stabilized sideslip flight test technique. Starting from a zero sideslip condition, establish a steady straight sideslip of approximately two degrees while holding trim airspeed. This may best be done by applying a small amount of rudder and then coming in with just enough bank to hold a constant heading. A point on the outside horizon will provide the pilot with the most accurate means of holding a constant heading. The needle of the turn and slip indicator may also be an aid in holding constant heading, zero yawing velocity flight.

Constant trim velocity should be maintained as the sideslip is increased in approximately 2 degree increments until reaching the maximum sideslip obtainable (if no other restriction applies), or until a dangerous flight condition is anticipated. A photopanel shot will be taken at each stabilized sideslip condition. Sideslips in the T-33 will be discontinued at rudder buffet, or at plus or minus 14 degrees of sideslip in the cruise configuration to prevent inadvertent tumbling and possible structural damage. The T-33 is restricted from full rudder deflection sideslips. If no sideslip gauge is available in the cockpit, the following guide may be used: At approximately 12 degrees of sideslip, the standard airspeed indicator will jump approximately 5 knots. The pilot may continue the sideslip investigation approximately two degrees past this point if no other adverse indications are noted. If an airspeed calibration is available for the boom airspeed system at various angles of sideslip, this correction should be used while attempting to hold a constant airspeed. If such a calibration is not available, an indication of the magnitude of the position error due to sideslip can be obtained by placing the aircraft in a sideslip and rapidly coming back to zero sideslip while noting the magnitude of the airspeed change. This correction can be ignored if this change is only one or two knots. It should be immediately apparent whether back or forward stick is necessary to hold a constant airspeed as the sideslip is increased, and thus the correct control movement should be anticipated as the sideslip is increased. Throughout the test, lead all inputs with the rudder. This technique, coupled with slow, deliberate control inputs, will help keep Dutch roll at a minimum. If a Dutch roll should develop, stop it with the rudder; aileron inputs will only reinforce the motion. After the maximum sideslip point is reached, the aircraft should be smoothly returned to trim and then similar sideslip points should be made in the opposite direction. Smoothness in this test, as in all tests, is imperative in order to get good stabilized points quickly. Anticipation of correct control movement is a great aid in establishing good test points quickly. A record should be kept of beginning and final photo correlation numbers as well as a list of any unreliable points.

6. The pilot will practice the stabilized sideslip flight test technique. Maintain altitude at 20,000 ±1,000 feet.

7. The instructor pilot will demonstrate the slowly varying sideslip method. This is an alternate method of obtaining sideslip information. Starting from zero sideslip, continuously increase the sideslip angle at not more than one degree per second while maintaining heading and velocity. A continuous photo record is taken out to the maximum sideslip angle. This method is considerably more difficult to fly properly than the stabilized sideslip method.

8. The pilot will practice the slowly varying sideslip method. Maintain 20,000 ±1,000 feet.

9. At the completion of the sideslip practice, the pilot will return the aircraft to 21,000 feet and 250 KIAS. The instructor pilot will demonstrate the aileron roll flight test technique. Using trim power and holding 250 KIAS, roll the aircraft into a 45 degree bank. Stabilize the aircraft in this condition, holding the rudder pedals fixed to hold trim sideslip. The recording trigger is depressed prior to starting a roll to permit some leader on the oscillograph paper. The stick is rapidly moved to 1/4 aileron deflection, and the aircraft is rolled to 45 degrees of bank in the opposite direction. This control input should be a "step input." The pilot may avoid stick "bobble" by using both hands on the stick. The recording trigger is held depressed throughout the roll. The airspeed should be held as close as possible to the aim V_i throughout the roll. When the roll is complete, rapidly re-establish a 45-degree bank at 250 KIAS. This will prevent needless altitude loss and airspeed excursion. This procedure is repeated in the opposite direction. When the aircraft has been rolled in both directions at 1/4 aileron deflection, repeat the procedure at 1/2 aileron deflection. The full deflection aileron roll is started from wings level flight at 250 KIAS. Apply full aileron deflection in the desired direction of roll. In order to determine exactly how much aileron force it required to hold full aileron deflection, it will be necessary to slowly relax the control force prior to completing 360° of roll. Once on the ground, the oscillograph trace will allow the pilot to match a control force against the point where the aileron deflection first starts to decrease. The full deflection aileron roll will be repeated in the opposite direction. For the sake of demonstration, the instructor pilot may roll in only one direction for each control deflection tested. The instructor will then demonstrate a step aileron input that will accomplish a 45°-45° roll in approximately 6 seconds.

10. The pilot will practice the aileron roll test with rudders fixed. When complete, he will repeat the full deflection roll with rudders free. He will then practice the step aileron input needed to get a 45°-45° roll in approximately 6 seconds.

11. The pilot will descend to 10,000 feet pressure altitude in an area that will allow at least 5,000 feet terrain clearance. The aircraft will be placed in the power approach configuration, i.e., gear down, full flaps, speed brakes up. Obtain a trim shot at 10,000 feet and 120 knots plus fuel using the backside trim technique. Obtain a photopanel shot with the aircraft trimmed in this condition. Also, obtain a photopanel shot in a zero sideslip condition.

12. This test will be conducted at 10,000 ±1,000 feet. After the 10,000 foot trim shot has been obtained, note trim

power and climb to 11,000 feet.

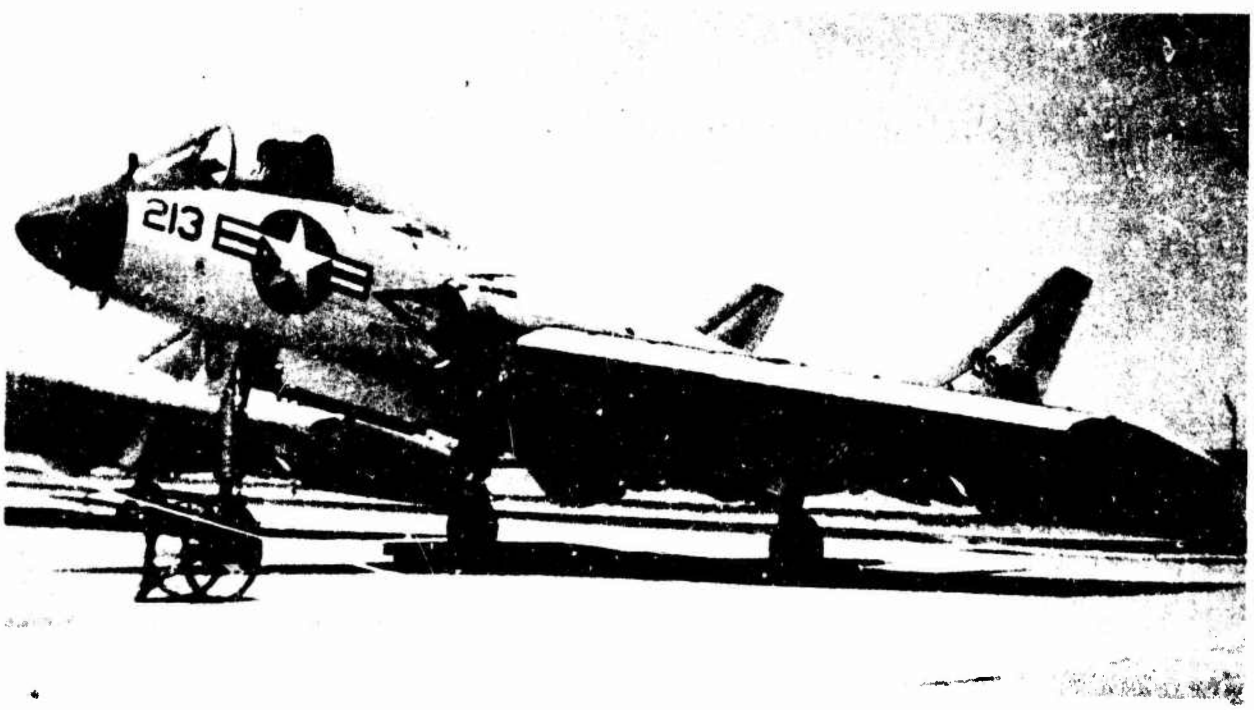
13. The instructor pilot will demonstrate the stabilized sideslip flight test technique in the power approach configuration. Because of the low "q", rudder forces will be very light and care should be exercised to avoid overcontrolling. <u>Sideslips in the T-33 will be discontinued at rudder buffet or at plus or minus 10 degrees of sideslip in the power approach configuration to prevent inadvertent tumbling</u>. Care should be exercised in returning from maximum sideslip to the trim condition. Climb when necessary in order to remain within the allowable altitude band (±1.000 feet).

14. The pilot will practice the stabilized sideslip flight test technique in the power approach configuration.

15. The instructor pilot will demonstrate the aileron roll flight test technique in the power approach configuration. During this low "q" condition, considerable sideslip will develop. Recover from the roll using aileron only. Overcontrolling or putting in incorrect rudder inputs can create a hazardous situation. Therefore, 360-degree rolls will not be accomplished in the power approach configuration.

16. The pilot will practice the stabilized sideslip flight test technique in the power approach configuration with the rudders fixed. When this is completed, he will repeat the 1/2 aileron deflection point with rudders free. Recovery will be made with rudders free.

17. Landing will be made from a simulated flameout pattern set up by the instructor pilot.



CHAPTER 8
ENGINE-OUT OPERATION

● 8.1 INTRODUCTION

The problems associated with an engine failure in a multi-engine aircraft may be classified into two types; control problems and performance problems. The severity of one may greatly overshadow the other in certain aircraft; but in general, the pilot is confronted with a generous portion of both.

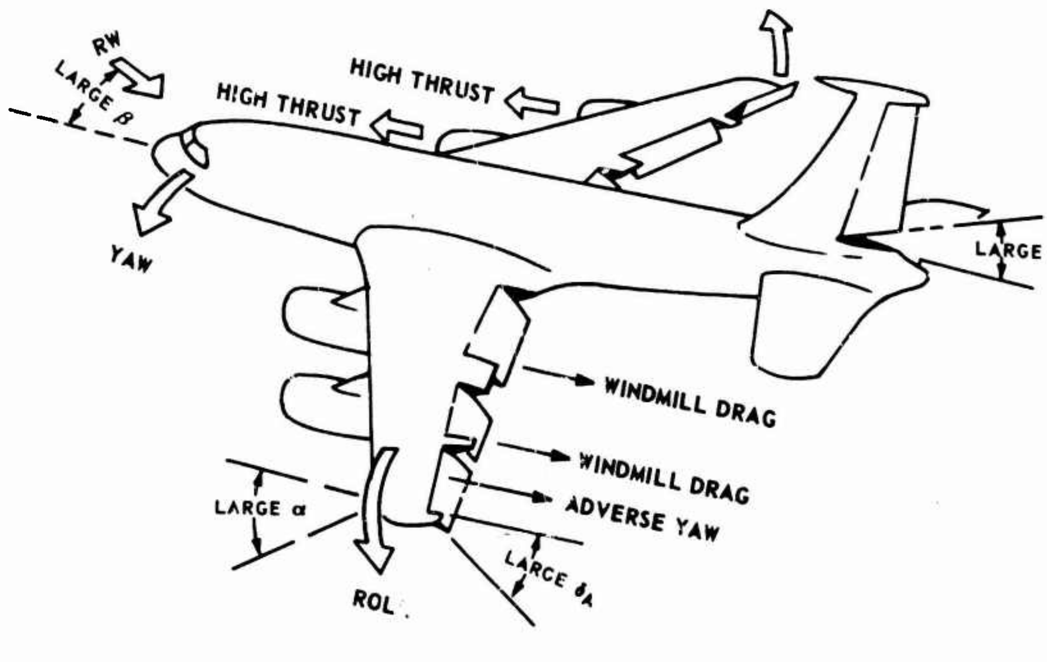
● 8.2 THE CONTROL PROBLEM

The control problem may be simply stated - the pilot must be able to achieve and maintain straight unaccelerated flight following the loss of an engine. Thus the engine-out control problem can be divided into cases; the non-steady state dynamic case and the steady state equilibrium case. The dynamic case begins when an engine fails and terminates when the equilibrium case has been achieved.

When a pilot intentionally shuts down an engine in an aircraft with adequate control authority to maintain equilibrium, the dynamic case is usually not severe and the transients encountered are mild. If, however, an engine fails suddenly on takeoff, or the pilot makes a sudden application of go-around power to asymmetric engines, a potentially divergent rolling and yawing motion can ensue.

These hazardous dynamic situations are caused by a rapid sequence of events, as illustrated

FIGURE 8.1



in the following hypothetical sequence:

1. The aircraft is in a critical flight phase such as takeoff or go-around when a large yawing moment due to asymmetric thrust appears very suddenly. The aircraft yaws rapidly through a large angle.

2. The pilot allows a large sideslip angle to develop because of the high yaw rate and the surprise factor. A rolling moment into the bad engines is generated by the dihedral effect. This rolling moment is augmented by wing blanking on swept-wing configurations and by asymmetric slipstream effects on propeller aircraft.

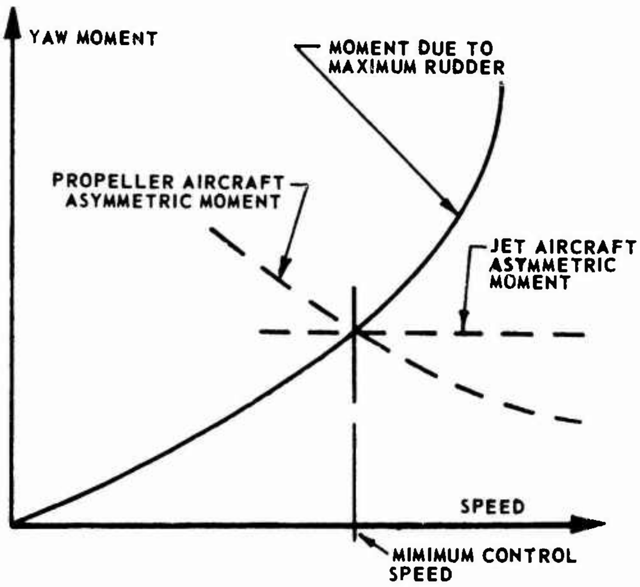
3. As the angular momentum builds up in roll and yaw, larger compensating moments, over and above the steady state requirements, are required to arrest the motion. Large control deflections are required because of the reduced control effectiveness at slow speed, and adverse yaw adds to the forcing moment. If full control is insufficient to achieve equilibrium, a power reduction on the good engines will be required.

4. But a power reduction aggravates an already critical performance problem. Speed is difficult to maintain because of decreased thrust and increased drag. If the down-going wing, which is at a high angle of attack because of the slow speed and the rolling velocity, is allowed to reach stall, the dynamic case may terminate without ever reaching equilibrium.

The severity of such responses is difficult to predict by theoretical analysis, and flight test of critical situations is required to establish safe flight boundaries. Slow speed restrictions due to decreased control effectiveness are most common, although others may exist in the supersonic range due to reduced stability. The dynamic case boundaries are usually (although not necessarily) more restrictive than those due to the equilibrium case.

For every given set of asymmetric conditions there is a speed below which aerodynamic control is insufficient to maintain the equilibrium case. This is called the minimum control speed.

FIGURE 8.2



Obviously, this minimum speed will vary with the prevailing conditions. Not so obvious, however, is the fact that for a given condition the equilibrium case can be maintained with different combinations of bank angle, sideslip angle, and rudder deflection and that the minimum speed will vary according to the combination used.

Engine-out definitions and terminology are not standard throughout the aviation industry and in any discussion it must be clearly understood what the conditions are, and that everyone is talking about the same thing. Several more or less standard definitions are discussed below.

Minimum Control Speed:

It is possible that there will be no minimum control speed for a multi-engine aircraft because it can be controlled up to aerodynamic stall. This is the desired situation. MIL-F-8785 (para 3.3.9.2) specifies that straight flight must be possible during takeoff at any speed above minimum takeoff speed and further specifies the control forces and deflections that may be used to accomplish this. This establishes the minimum control speed required for every possible gross weight condition. It might therefore seem that a minimum control speed is only of academic interest, but there may be instances where a multi-engine aircraft could meet the specifications at takeoff but be operated at a speed in some operational or approach flight phase which would be lower than minimum takeoff speed and hence the asymmetric thrust minimum control speed must still be determined by flight test.

Ground Minimum Control Speed:

Control of asymmetrically powered multi-engine aircraft on the ground also presents a problem that must be considered. If a pilot loses the most critical engine during takeoff roll, he must decide whether to continue the takeoff or abort. MIL-F-8785 (para 3.3.9.1) specifies that the pilot must be able to maintain a path on the runway that does not deviate more than 30 feet from the original path if he decides to continue the takeoff and is above the refusal speed (based on the shortest runway from which the airplane is designed to operate). If the pilot decides to abort, the directional control requirements are still specified but the pilot is allowed to use additional controls such as nosewheel steering and differential braking. Flight (ground) testing is required to show compliance with the specification so a ground minimum control speed can be determined. This is defined as the lowest speed at which directional control can be maintained on the ground when the most critical engine fails during takeoff roll.

8.3 THE PERFORMANCE PROBLEM

Reduced climb performance, service ceiling and range capability accompany an engine failure as a natural result of decreased thrust and increased drag. The effect of engine failure on takeoff performance, however, is a complex subject requiring additional definitions and operational techniques.

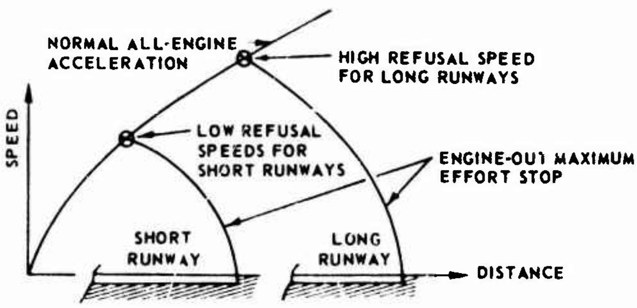
Takeoff Performance:

The basic requirement is simple; at every instant throughout the takeoff roll the pilot must have an acceptable course of action available to him in the event of engine failure. During the first part of the roll, this action will be to abort the takeoff and stop. Beyond a certain point the action will be to continue the takeoff with the engine failed. The dividing point between these courses of action is a function of aircraft performance.

Consider an aircraft at a particular configuration and gross weight starting its takeoff roll on a given day. For a given runway length there is a maximum speed to which it can accelerate on all engines, lose the critical engine and then just complete a maximum effort stop at the far end of the

runway. This speed, called the refusal speed, is relatively high for long runways and relatively low for short ones.

FIGURE 8.3

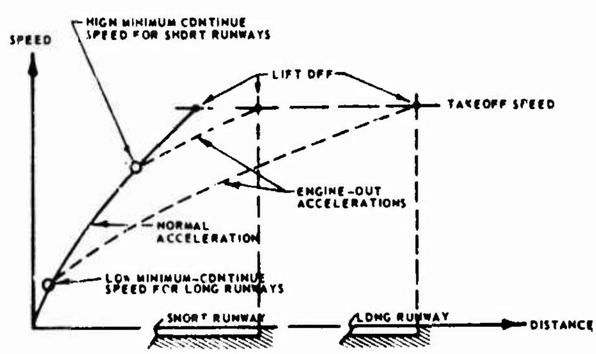


REFUSAL SPEED

REFUSAL SPEED is thus defined as the maximum speed to which the aircraft can make a normal takeoff acceleration, lose the critical engine at that speed and then stop on the remaining runway. Stopping technique and devices to be used must be specified.

Now consider the same aircraft making the same takeoff under identical conditions. For a given runway length there is a minimum speed to which it can accelerate on all engines, lose the critical engine at that speed and then continue the takeoff with the engine failed, getting airborne just at the far end of the runway. This speed (a "minimum-continue" speed) varies with runway length in a manner opposite that of refusal speed, i.e., it is relatively low for long runways.

FIGURE 8.4



The gap between the minimum-continue speed and the Refusal Speed reflects the size of the safety margin provided by a given runway for the particular conditions.

FIGURE 8.5

SAFE TAKEOFF

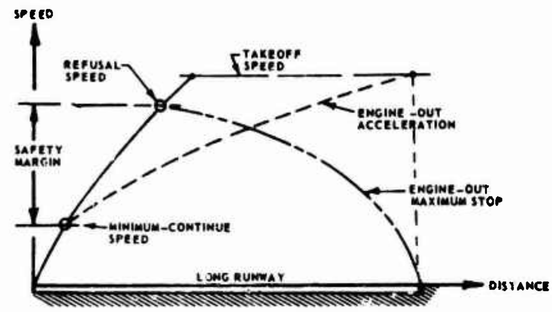


FIGURE 8.6

UNSAFE TAKEOFF

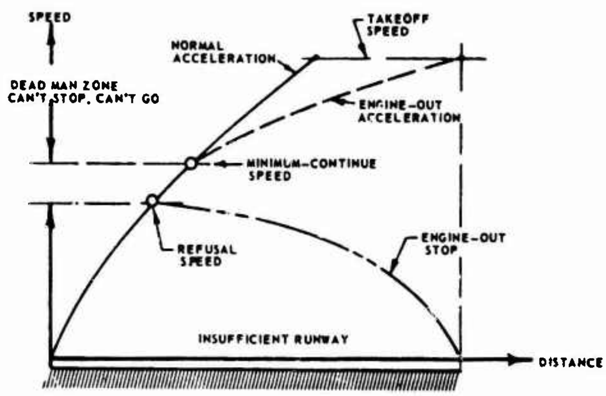
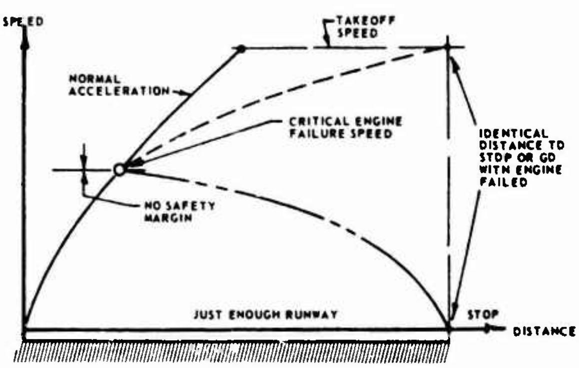


FIGURE 8.7

CRITICAL FIELD LENGTH



Critical Engine Failure Speed:

If the critical engine fails at this speed, the distance required to complete the takeoff is identical to the distance required to stop. The total runway required to accelerate to this speed, lose an engine, and then stop or go is the Critical Field Length.

Initial Climb Performance:

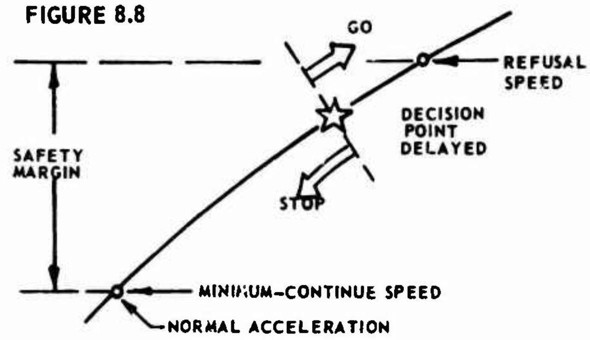
The period between lift-off and attainment of best engine-out climb speed can be very critical. Military multi-engine aircraft are routinely loaded to gross weights that provide as low as 50 feet-per-minute rate of climb with an engine inoperative. Obviously, this level of performance allows little margin for mis-management of attitude or configuration. Flap retraction may have to be accomplished incrementally on a very tight speed schedule to keep sufficient lift for a positive climb gradient without excessive drag. Unexpected characteristics may be encountered in this phase. For example, the additional drag due to doors opening might make it desirable to delay gear retraction until late in the clean-up phase or in another instance, the time available to obtain the clean configuration might be limited by the supply of water injection fluid if dry thrust is insufficient to maintain the climb. Careful flight test exploration of this phase is an obvious requirement.

Decision Speed/Distance:

All the performance discussions above are concerned with what the aircraft will actually do. It still remains for the pilot or operational authority to decide at what particular speed or distance the course of action will change from stop to go in the event of engine failure. This defines the decision point.

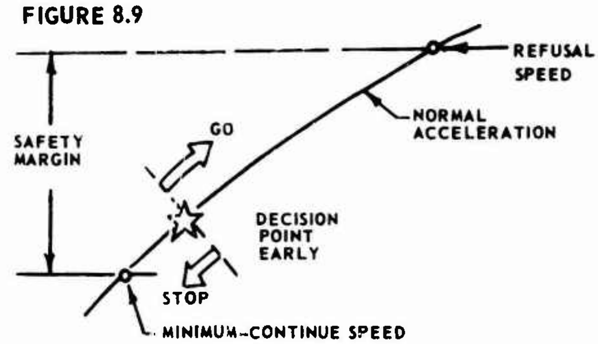
If the initial climb performance is going to be critical on the takeoff in question, the decision point may be near the higher speed end of the safety margin.

FIGURE 8.8



The B-47 illustrates the opposite case. This aircraft has a very poor record for successful aborts and is operated with the decision point relatively early (near the low speed end) of the safety margin.

FIGURE 8.9



Other cases may be decided by the nature of the overrun or the terrain beyond the runway, i.e., is it better to go off the far end almost stopped or almost flying?

8.4 ENGINE-OUT FLIGHT TESTING

Military aircraft are usually designed with relatively low safety margins in order to attain the desired performance - the marginal engine-out climb capability pre-

viously mentioned is an example. In fact, during war emergency operation the gross weight may be so high that engine-out operation is not possible at all. Flight tests of these critical phases, on or near the ground, require a high level of crew skill and proficiency; each point must be carefully planned and flown.

Such tests are a normal part of the Category I and II testing of a new aircraft. They also play a vital part in side-by-side evaluations of assault or VSTOL transports where the ability to carry a useful load in and out of a given landing area is frequently limited by engine-out performance. Individual evaluations to determine if an aircraft meets the contractor's guarantees may also hinge on this area of operation.

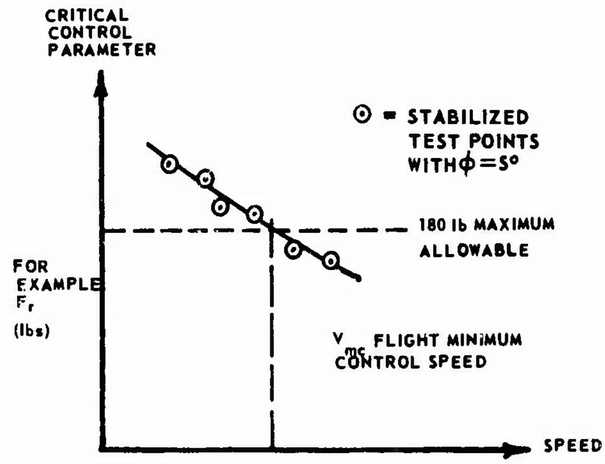
Flight Minimum Control Speed:

It will be shown later in paragraph 8.5 that an aircraft with an engine inoperative can be stabilized in straight (unaccelerated) flight in various combinations of bank angle, sideslip angle, and rudder deflection. For determination of minimum control speed, the maximum bank angle of 5 degrees allowed by MIL-F-8785 will be held constant, and the other two parameters adjusted as necessary to achieve straight flight.

The critical engine is always an outboard engine. For reciprocating aircraft with clockwise rotating propellers (looking forward), the left outboard is critical due to torque. Assuming there is no angular motion of the aircraft to provide gyroscopic couples from rotating engines, left or right is usually not critical on a jet powered aircraft (the distinction may be important for dynamic points, however).

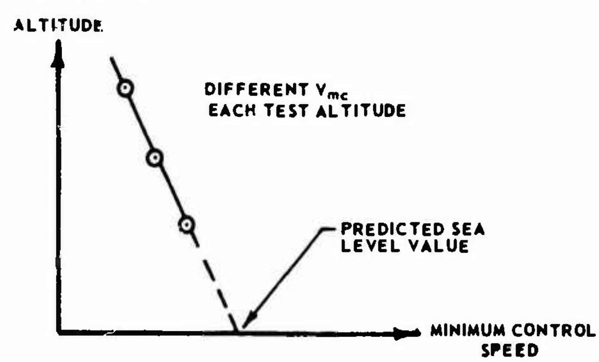
With the aircraft in the specified configuration, and with the critical engine failed, a series of stabilized points are recorded at decreasing speeds. A plot of the critical control parameter (this will most frequently be rudder force or deflection) versus airspeed is made to determine the minimum control speed.

FIGURE 8.10



The minimum control speed usually increases at lower altitude due to increased engine thrust; the test must be accomplished at more than one altitude, including one as low as is safely possible, to provide an extrapolation to sea level.

FIGURE 8.11



Care must be exercised to obtain points that are unaccelerated and well stabilized. The outside

visual attitude is primary for maintaining airspeed, bank angle, and zero yaw rate, using the cockpit instruments for cross check. The ball, which in unaccelerated flight will always be at the bottom of the race, is the primary reference used to eliminate accelerations that result from unbalanced forces in the y direction. These lateral translations are difficult to discern visually.

Dynamic Engine Failure:

The military specifications (MIL-F-8785 para 3.3.9.3) require that a pilot be able to avoid dangerous conditions that might result from the sudden loss of an engine during flight. The method to test compliance with this specification is to stabilize with symmetrical power and dynamically fail the most critical engine. After observing a realistic time delay for pilot realization and diagnosis, the pilot arrests the aircraft motion and achieves the equilibrium engine-out condition. Since it obviously requires more control to arrest the motion than to maintain equilibrium, this dynamic situation must be considered in determining the minimum control speed.

Minimum control speed should not be set by any factor other than insufficient control. If the aircraft stalls before reaching the minimum control speed, a statement that "at this gross weight, the aircraft is controllable down to the stall" is preferable to calling the stall speed the "minimum control speed."

Ground Minimum Control Speed:

The ground minimum control speed will differ from the flight value because of:

1. The inability to use sideslip and the restriction on the use of bank angle.
2. Cross wind components.
3. The additional yaw moments produced by the landing gear, which in turn vary with: the landing gear configuration; the amount of steering used; the vertical loads on each gear; and runway condition.

There are two basic test methods, one involving acceleration and the other deceleration. If the aircraft will decelerate with the asymmetric power condition set up (symmetrical pairs of non-critical engines may also be retarded) the "back-in" method may be used. The test is started at a ground speed in excess of the expected minimum and the power condition is set. As the speed decreases, more aerodynamic control deflection is required; the speed where directional control cannot be maintained is the minimum control speed.

Some high performance aircraft accelerate in the test condition and the acceleration method is required. The asymmetric yawing moment is gradually increased (by throttle manipulation) as increasing speed provides more control. The speed where sufficient control is available to hold the full asymmetric power condition is the minimum control speed. This method requires considerable skill and coordination to obtain good results - the aircraft is essentially at minimum control speed throughout the acceleration.

Both of the methods above determine equilibrium control speeds. When sufficient experience has been obtained, sudden engine cuts are performed to determine if dynamic effects are more restrictive.

In-Flight Performance:

Normal performance flight test methods may be used to deter-

mine the climb, range, and endurance at altitude with engines inoperative.

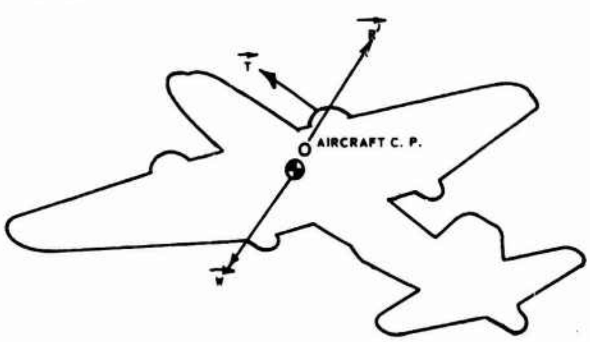
Landing Performance:

Restricted reversing capability and possible higher approach speeds required to maintain minimum safe speeds will affect landing performance. Normal flight test methods are valid, but caution must be exercised in go-around situations.

8.5 EFFECT OF BANK ANGLE ON THE EQUILIBRIUM CASE

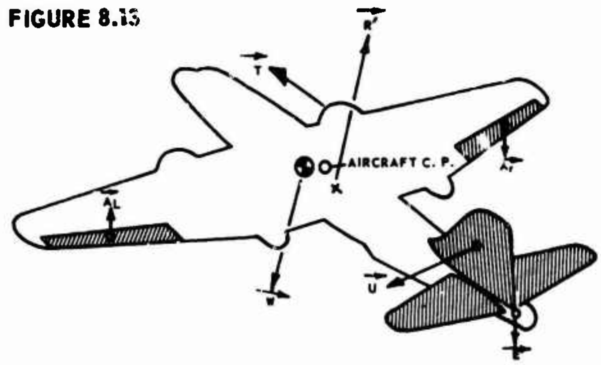
If torque and gyroscopic effects due to rotating engines or propellers are neglected, all the forces acting on an aircraft in flight with an engine inoperative are shown in figure 8.12.

FIGURE 8.12



The vector $\vec{R}$ is the total aerodynamic reaction acting at the aircraft center of pressure. This vector may be thought of as the sum of all the smaller reactions acting on the separate parts of aircraft. For the present discussion it is convenient to handle separately the smaller reactions that act on the ailerons ($\vec{A_r}$ and $\vec{A_L}$), the vertical fin and rudder ($\vec{U}$) and the horizontal stabilizer and elevator ($\vec{E}$), as shown in figure 8.13.

FIGURE 8.13

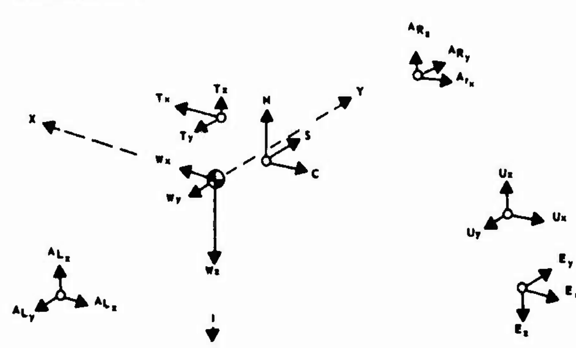


$\vec{R'}$ is now the remaining portion of the total aerodynamic reaction, such that:

$$\vec{R} = \vec{R'} + \vec{A_R} + \vec{A_L} + \vec{E} + \vec{U}$$

$\vec{R'}$ has a point of action that is near, but not necessarily at, the aircraft c.p. Assuming mass to be symmetrically distributed, the weight vector acts through the cg at the origin of the body axis system. Because of the possibility of sideslip, none of the aerodynamic force vectors necessarily pass through a body axis, i.e., they may all produce moments in three directions. When all forces are resolved into components parallel to the body axes, the representation in figure 8.14 is obtained.

FIGURE 8.14



If the restriction of equilibrium (unaccelerated) flight is now imposed, six equations result:

Longitudinal	Lateral-Directional
(1) $F_x = 0$	(4) $F_y = 0$
(2) $F_z = 0$	(5) $L = 0$
(3) $M = 0$	(6) $N = 0$

The longitudinal equations are not critical in the achievement of equilibrium; they are balanced by the usual technique of stabilized points, i.e., variation of pitch angle (θ), angle of attack (α), and elevator deflection (δ_e).

If the forces of figure 8.14 are all moved to the cg and the moments lost by the move are replaced, the six equations can be expanded as shown below.

$$(1)\quad \underbrace{W_x}_{\text{equation is balanced with } \theta} + \underbrace{E_x + U_x + A_{R_x} + A_{L_x}}_{\text{control drag}} + \underbrace{C}_{\text{other drag}} + T_x = 0$$

$$(2)\quad \underbrace{N}_{\text{equation is balanced with } \alpha} + E_z + \underbrace{U_z + A_{R_z} + A_{L_z}}_{\text{very small}} + W_z + T_z = 0$$

$$(3)\quad \underbrace{M/_E}_{\text{equation is balanced with } \delta_e} + M/_T + M/_N + M/_C + \underbrace{M/_U + M/_{A_{R,L}}}_{\text{very small}} = 0$$

The lateral-directional equations are of most interest in achieving equilibrium. The roll equation (5) is usually not critical, although in some cases lack of aileron authority may be the limiting factor.

$$(5)\quad \underbrace{L/_{A_{R,L}}}_{\text{equation is balanced with } \delta_a} + L/_T + L/_U + \underbrace{L/_N + L/_S + L/_E}_{\text{very small}} = 0$$

It now remains to be shown that the side force equation (4) and the yaw equation (6) can be simultaneously balanced using an infinite number of combinations of bank angle (ϕ), sideslip (β), and rudder deflection (δ_r).

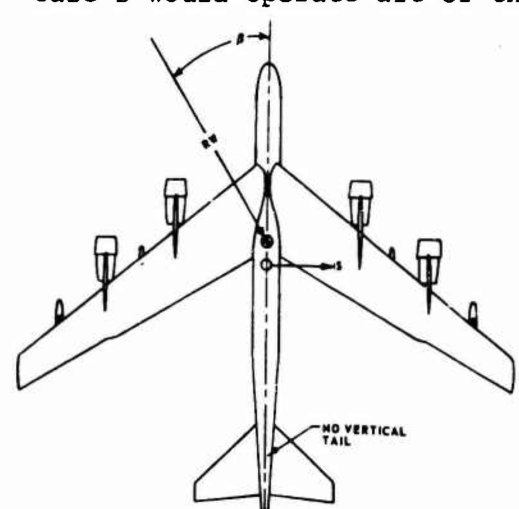
$$(4)\quad \underbrace{W_y}_{W \sin \phi} + U_y + S + \underbrace{E_y + A_{R_y} + A_{L_y}}_{\text{small values combined into S}} = 0$$

$$\boxed{\text{Side Force Equation: } W \sin \phi + U_y + S = 0}$$

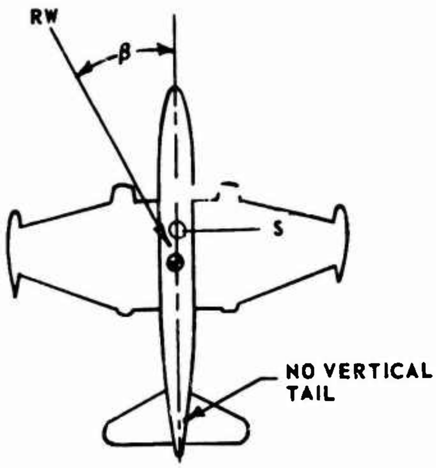
$$(6)\quad \underbrace{N/_T + \underbrace{N/_C + N/_E}_{\text{small}} + \underbrace{N/_{A_{R,L}}}_{\text{adverse yaw}}}_{\text{lumped together as } N_{Forcing}} + N/_U + N/_S = 0$$

$$\boxed{\text{Yaw Equation: } N_{Forcing} + N/_U + N/_S = 0}$$

The point of action of S is related to the directional stability with the vertical tail removed. Certain aircraft, such as B-52 with its long, slab-sided aft fuselage and swept wings, might have some directional stability without the vertical tail installed, in which case S would operate aft of the cg.



Aircraft more generally will be directionally unstable in this condition, and S will operate ahead of the cg.



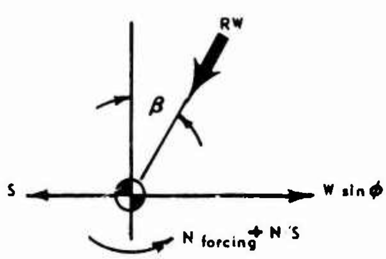
In either case S will have a short arm compared to that of U_y and the sign of $N/_S$ (which in the discussion below will be considered unstable) or the effect of the other simplifying assumptions will not alter the diagrams below.

<u>Equilibrium with $\delta_r = 0$</u>

β from good engine side

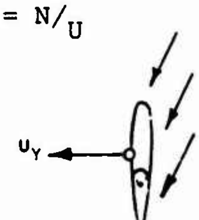
ϕ large

$\delta_r = 0$



$W \sin \phi = U_y + S$

$N_{Forcing} + N/_S = N/_U$

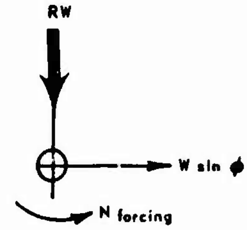


<u>Equilibrium with $\beta = 0$</u>

$\beta = 0$

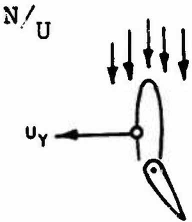
ϕ = reduced

δ_r = moderate



$W \sin \phi = U_y$

$N_{Forcing} = N/_U$

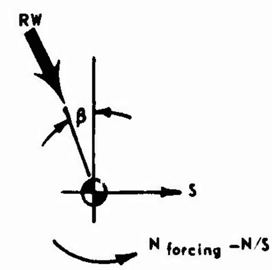


<u>Equilibrium with $\phi = 0$</u>

β from bad engine side

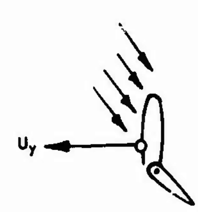
$\phi = 0$

δ_r = large



$U_y = S$

$N_{Forcing} - N/_S = N/_U$



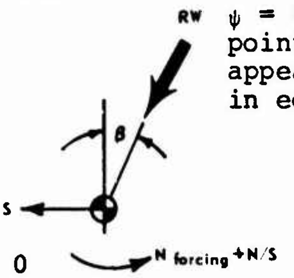
It may be shown by trial and error that the arrangements above are the only ones possible for the conditions specified <u>provided the aircraft is truly following an un-</u>

accelerated flight path. For example, if the $\phi = 0$ condition is attempted with β from the good engine side, equilibrium cannot be obtained.

False $\phi = 0$ Point

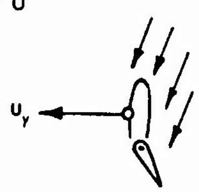
Aircraft is accelerating because of insufficient rudder

Because $\psi = 0$ the point may appear to be in equilibrium



$U_y + S \neq 0$

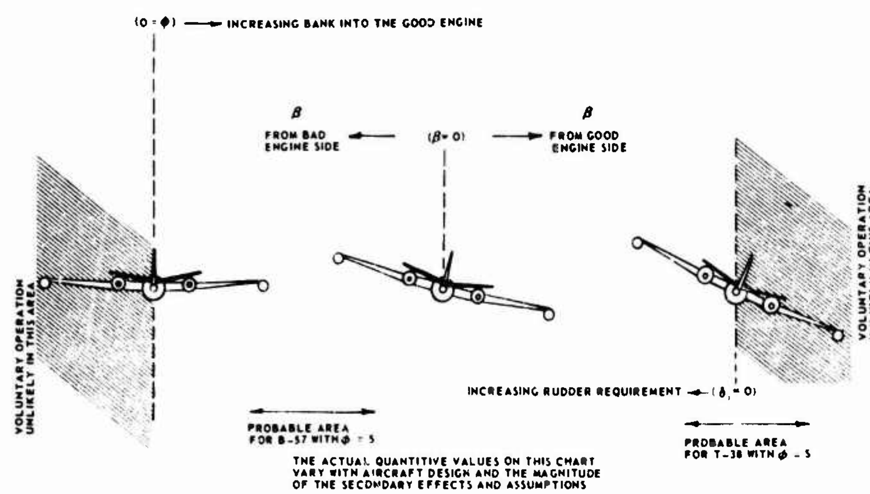
$N_{Forcing} + N/_S = N/_U$



Compared to the true $\phi = 0$ condition, more U_y is required to balance the yaw equation (since $N/_S$ is now added to $N_{Forcing}$) but it is obtained with less δ_r because of the favorable β at the tail. But the side force equation is not balanced ($U_y + S \neq 0$) and the aircraft is actually accelerating to the left. This condition, which can be readily attained in flight if insufficient rudder is used in the $\phi = 0$ condition, is difficult to see visually but can be recognized by a displacement of the ball to the right. If additional right rudder is applied until the ball returns to the bottom of the race, the sideslip will return to the bad engine side.

The relationship between ϕ, β, and δ_r revealed above is summarized in figure 8.15.

FIGURE 8.15



8.6 DEMONSTRATION MISSION

The student will fly an Engine-Out demonstration mission from the rear cockpit of the B-57.

Performance Evaluation - Single-Engine Climb:

1. Clean configuration trimmed for normal climb (300 kt).

2. Single-engine climb to assigned altitude with left engine at idle, right engine at 100%.

3. With time to climb to altitude starting at 5,000 feet and maintaining a constant heading, take stabilized camera readings at:

$\phi = 0°$ holding all forces constant
$\phi = 5°$

$\phi = 5°$ rudder forces only trimmed

$\phi = 5°$ rudder and elevator forces trimmed

$\phi = 5°$ all forces trimmed

Effect of Bank Angle on Yaw Control:

1. Clean configuration stabilized at assigned altitude at a speed slightly above minimum control speed.

2. Holding forces with left engine failed and right engine at 100%, record at trim airspeed:

False $\phi = 0°$ point

$\phi = 0°$

$\beta = 0°$

$F_r = 0$

ϕ into engine idle

Directional Control Test (MIL-F-8785 para 3.3.9.2):

1. The takeoff configuration is trim shot power, the same as required for the previous test.

2. Left engine idle, right engine 100%.

3. Record stabilized points at equal airspeed increments from trim speed to $V_{min.\ cont.}$ first at $\phi = 0°$ and then repeat the test with $\phi = 5°$ and Sensitive Bank Angle Indicator Operating.

Asymmetric Power Test (MIL-F-8785 para 3.3.9.4):

1. Configuration CR

2. Trim to 90 percent noseup

3. Set both engines at 96.5 percent power

4. Airspeed 160 kt

5. Use a sawtooth entry 1,000 feet below the assigned altitude. At the assigned altitude, an engine will be chopped to idle. Recover with no delay using the ailerons, feet on the floor.

6. Record data through recovery (start data 300 feet prior to reaching the assigned altitude).

7. Hand-record ψ_{max} and ϕ_{max} and qualitatively determine if MIL-F-8785 requirements are met.

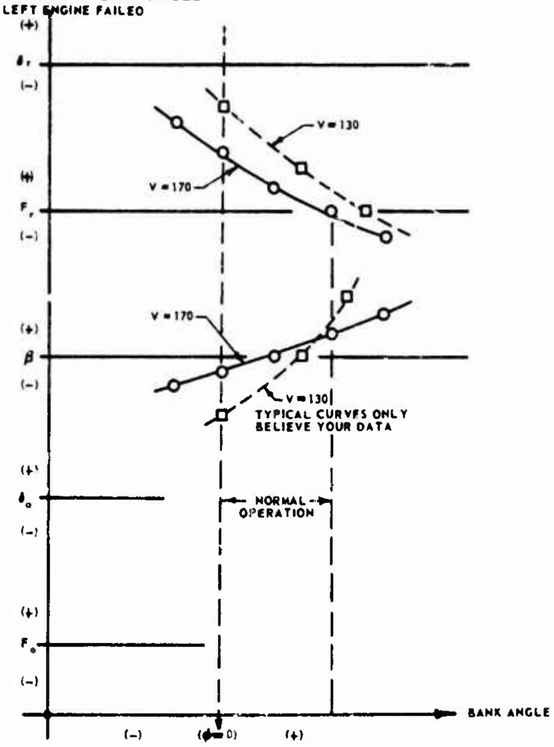
8. Test. Repeat steps 1-7 using 150 KIAS (or 1.3 $V_{O_{min}}$ whichever is higher)

Trim Evaluation (MIL-F-8785 para 3.6.1.1):

1. Configuration - CR

FIGURE 8.16

EFFECT OF BANK ANGLE
LEFT ENGINE FAILED



2. Trim - as set for trim shot

3. Power: Left engine - As required

   Right engine - Idle

4. Airspeed - Flight Manual max - range single-engine cruise speed, 40,000 pounds gross weight, at assigned altitude

5. Altitude - as assigned

6. Test - with $\phi = 0°$

7. Record - holding all forces
   - rudder trimmed; $F_r = 0$
   - aileron trimmed; $F_a = 0$
   - all forces trimmed

Dynamic Engine Failure (MIL-F-8785 para 3.3.9.3):

1. Configuration CR

2. Trim - as required

3. Power - 100 percent both engines

4. Airspeed - 140 KIAS

5. Start 1,000 feet below assigned altitude

6. Test - Climb through to the assigned altitude. The IP will chop an engine. Delay 3 seconds and then recover. Test each of the following recovery methods:
   - ailerons first, then rudder
   - rudder first, then ailerons
   - rudder and aileron simultaneously

7. Qualitatively record:
   - ψ and ϕ at 3 seconds after throttle chop
   - ψ_{max} and ϕ_{max} during recovery
   - Altitude lost in recovery

8. Photo record data throughout recovery

9. Repeat steps 1-8 @ 130 and 120 KIAS

Single-Engine Go Around Evaluation:

1. Configuration - PA

2. Trim - As required

3. Power - as required for descent at 135 KIAS,

rate of descent 500 feet per minute with both engines operating.

4. Altitude - 8,000 feet

5. Test - Qualitatively evaluate with an engine failed and ϕ - 5° maximum
   - Power required to maintain a 500 foot per minute descent with flaps up.
   - Power required to level off
   - Accelerate with gear up to 160 KIAS and go around.

6. Repeat Test 5 with the flaps down.

<u>Single-Engine Traffic Pattern and Landing</u>:

1. Qualitative analysis of single-engine traffic and landing qualities.

2. Test - Consult the Technical Order for proper single engine traffic and landing procedures (pay particular attention to WARNINGS CAUTIONS and NOTES).

3. Record qualitative comments.

<u>Single-Engine Taxi Evaluation</u>:

1. Record qualitative comments.

● 8.7 DATA

<u>Data to Be Recorded</u>:

The photopanel will be used for all stabilized points (parameters as prescribed in <u>ARPS Special Instrumentation Requirements</u>). Normally a continuous oscillograph record of dynamic points would be obtained. The student should record any applicable qualitative comments deemed necessary.

<u>Data Presentation</u>:

Plots similar to figures 8.10 and 8.11 will be presented in the report. Discussion of the minimum control speed and any other applicable findings should be included. A plot similar to figure 9.16 should also be prepared to illustrate the effect of bank angle.

Several methods of asymmetric data presentation are presently being developed. They are worthy of note. As aircraft size increases; gross weight and bank angle become important considerations in determining minimum control speeds. To normalize this consideration a plot of $C_L \sin \phi$ versus Thrust Moment is used. Temperature, pressure altitude, and dynamic pressure also effect the minimum control speed. To normalize these effects, (colder or hotter than standard day) a plot has been developed that depicts Thrust Moment versus Dynamic Pressure.

CHAPTER 9

DYNAMIC STABILITY

9.1 PURPOSE

The purpose of the dynamic stability flight test is to investigate an aircraft's primary modes of motion. This investigation will ascertain the acceptability of these modes - frequency and damping being the characteristics of primary importance.

9.2 AIRCRAFT MODES OF MOTION

The characteristic modes of motion of a modern aircraft are becoming of more interest as flight regions expand. An aircraft that has its mass primarily distributed along the fuselage and is designed for high speed flight could foster undesirable conditions during certain flight regions. The dynamic response of an aircraft to various pilot control inputs is important in evaluating its handling qualities. The aircraft may be statically stable yet its dynamic response could be such that a dangerous or impossible flight characteristic results. The aircraft must have dynamic qualities that will permit the design mission to be accomplished. One of the test pilot's prime responsibilities is to evaluate these handling qualities with respect to the expected mission.

An airplane usually has five major modes of free motion. (Phugoid, short period, rolling, Dutch roll and spiral.) This chapter will deal with two longitudinal modes first (phugoid and short period) then two lateral-directional modes (Dutch roll and sprial). The rolling mode is covered in Chapter VII.

There are several different forms that the modes of motion may take. Figure 9.1 shows four possibilities for aircraft free motion; a pure divergence, a pure convergence, a damped or an undamped oscillation. The aircraft, being a rather complicated dynamic system, will move in a manner that is a combination of several different modes at the same time. One of the problems of flight testing is to initiate the excitation input so that the various individual modes can be picked out and analyzed on an individual basis.

An airplane usually has two major longitudinal modes of free motion. One is the long period mode or "phugoid" which is essentially a variation in airspeed and pitch angle at nearly constant angle of attack. Its period is of the order of 20 seconds to 2 minutes. The other mode is of short period and is characterized by an oscillation of angle of attack and pitch angle at nearly constant airspeed. Its period is usually less than four seconds.

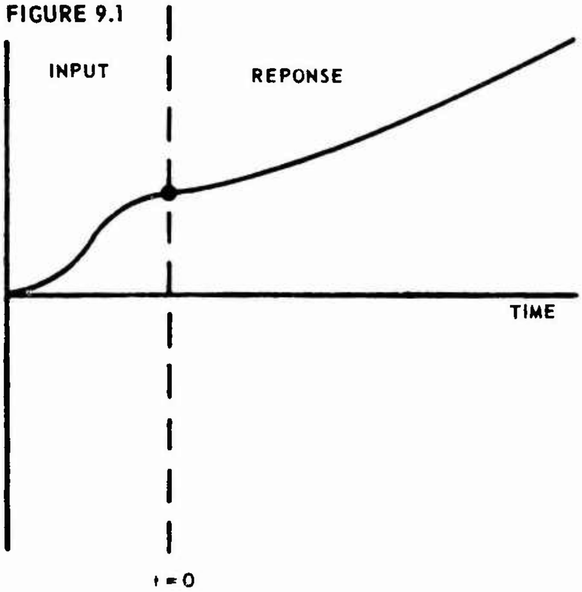
The phugoid mode is generally not considered an important flying quality because its period is usually of sufficient duration that the pilot has little difficulty in controlling it. However, under certain conditions it is possible for the damping to degenerate sufficiently so that the phugoid mode becomes important. The phugoid is characterized by airspeed, altitude, pitch angle, and rate variations while at essentially constant angle of attack.

The short period mode is an important flying quality because its period can approach the limit

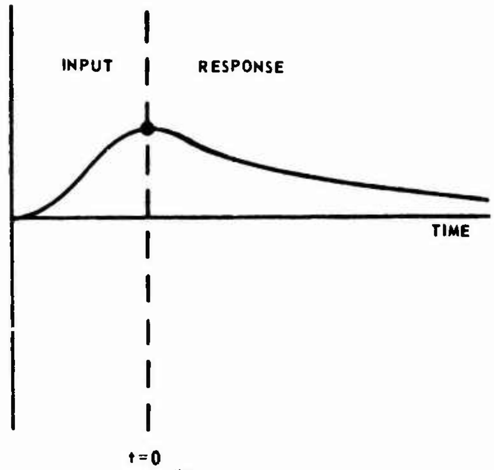
of pilot reaction time and it is the mode which a pilot uses for longitudinal maneuvers in normal flying. The period and damping may be such that the pilot may induce an unstable oscillation if he attempts to damp the motion with control movements. Hence, heavy damping of this mode is desirable. In most airplanes the short period mode is sufficiently damped, but some airplanes must be fitted with artificial damping devices. These airplanes should be flight tested with dampers on and off. The short period is characterized by pitch angle, pitch rate, and angle of attack change while essentially at constant airspeed and altitude.

Damping is described in terms of damping ratio or number of cycles to damp to a specified fraction of initial amplitude. Although heavy damping of the short period mode is desired, investigations have shown that damping alone is insufficient for good flying qualities. In fact, very high damping may result in poor handling qualities. It is the combination of damping and frequency of the motion that is important.

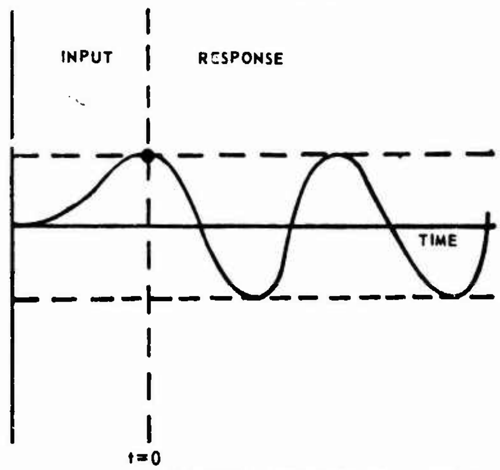
FIGURE 9.1



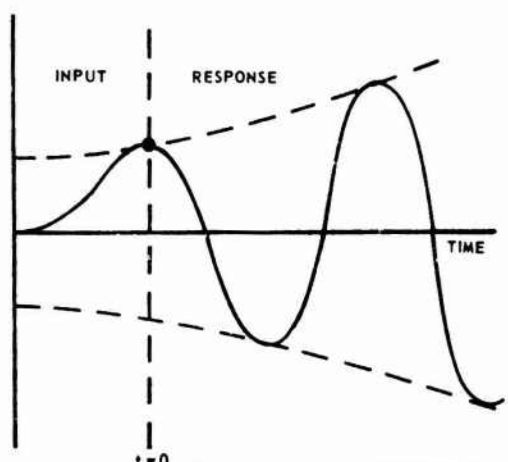
(a) PURE DIVERGENCE



(b) PURE CONVERGENCE



(c) OSCILLATION WITH ZERO DAMPING



(d) OSCILLATION WITH NEGATIVE DAMPING

The longitudinal modes should be flight tested for open-loop as well as closed-loop stability, since open-loop longitudinal modes can also be important. In open-loop motion, the elevator and control system is free to move (pilot does not hold the control) so that its motion is coupled with the longitudinal stick-fixed modes. The influence of the free elevator depends upon the magnitude, frequency and phase of elevator motion.

Tests for short period stability should be conducted from level flight at several altitudes and Mach numbers. Closed loop short period stability tests should be made also at various normal accelerations in maneuvering flight. This stability, when coupled to the pilot, is especially important to tracking and formation flying.

9.3 MILITARY SPECIFICATION REQUIREMENTS

MIL-F-8785 specifies that an aircraft's short period response, controls fixed or free, shall meet the requirements of frequency damping and acceleration sensitivity established in para. 3.2.2.1a, 3.2.2.1b, and figure 1. Residual oscillations shall not be greater than 0.05g at the pilot's station nor more than ± 3 mils of pitch excursion for category A Flight Phase tasks.

9.4 EXAMPLE TEST METHODS

The phugoid mode may be examined by stabilizing the airplane at the desired flight conditions and trimming the control forces to zero. Increase or decrease the airspeed by some small increment by the proper control pressure. For stick-fixed stability return the control to neutral and hold it fixed. For stick-free stability, return the control to neutral and then release it. After the control is released or returned, it may be necessary to maintain wings level by light lateral or slight directional pressure. Damping and frequency of phugoid motion may be changed appreciably by the presence of small bank angles (5 to 15 degrees). It may be very difficult to return the control to its trimmed position if the aircraft control system has a very large friction band. In such a case, the airspeed increment may be obtained by an increase or decrease in power and returning it to its trim setting or extending a drag device. In either case the aircraft configuration should be that of the trim condition at the time the data measurements are made.

To examine the short period mode, stabilize the airplane at the desired flight condition, (altitude, airspeed, normal acceleration). Trim the control forces to zero (for one g normal acceleration). Abruptly deflect the longitudinal control to obtain a change in normal acceleration of about one-half g. For stick-fixed stability, return the control to neutral and hold fixed. For stick free stability, release the control after it is returned to neutral (normally conducted only from one g flight). The aircraft response should be examined for positive and negative changes in normal acceleration. If the aircraft is equipped with artificial stabilization devices the test should be conducted with this device off as well as on. A note of caution: The abruptness and magnitude of the control input must be approached with due care! Use very small inputs until it is determined that the response is not violent. A suggested technique is to apply a longitudinal control doublet (a small positive displacement followed immediately by a negative displacement of the same magnitude followed by rapidly returning the control to the trimmed position). Start with small magni-

tudes and gradually work up to the desired excitation.

An input that is too sharp or too large could very easily excite the aircraft structural mode or produce a flutter that might seriously damage the airplane and/ or injure the pilot.

Data Required:

For trim conditions, pressure altitude, airspeed, weight, cg position, and configuration should be recorded.

The test variables of concern are, airspeed, altitude, angle of attack, normal acceleration, pitch angle, pitch rate, control surface position, and control position.

Reduction and Presentation of Data:

Time histories of stick-fixed and stick-free oscillations should be presented. A complete analysis would present damping ratio and frequency as a function of flight condition. If the motion were non-oscillatory divergent, the instability could be represented by the time required to attain a certain parameter value from a trimmed condition.

Short period mode investigations have shown that frequency as well as damping is important in a consideration of flying qualities. This is so because at a given frequency, damping alters the phase angle of the closed-loop system (which consists of a pilot coupled to the airframe system). Phase angle of the total system governs the dynamic stability.

A. Phugoid:

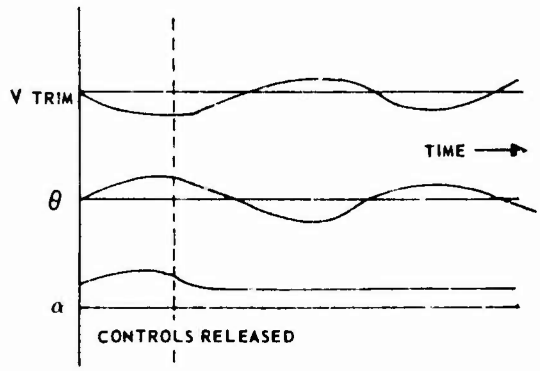
Stabilize the aircraft at the test altitude and the test airspeed. Smoothly increase the pitch angle until the airspeed reduces 10 to 15 knots below the trim airspeed. Very smoothly return the control column to the trim position and release all pressure. When the pitch angle reverses start timing. Record the maximum airspeed as the nose comes through level flight. The nose of the aircraft will continue to come up, reverse, and start down. Record the minimum velocity as the nose again passes through level flight. As the pitch angle reverses again mark the time. Continue the maneuver through 3 cycles.

Slight turbulence or imperfect lateral trim may result in wing roll during the pitch oscillations. If this should occur, then control the bank with smooth and light rudder pressures being careful not to excite aircraft Dutch roll.

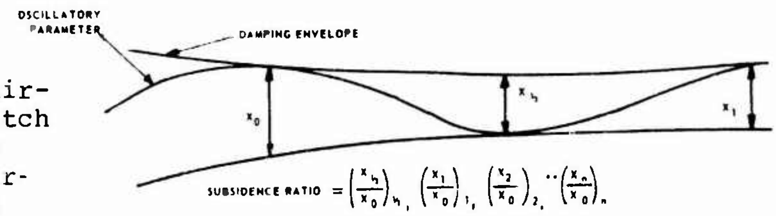
Data Reduction, Phugoid.

1. Plot a time history to include 3 cycles of the phugoid. Label airspeed, pitch rate and angle of attack.

FIGURE 9.2



2. Determine the frequency of the oscillation. Plot cycles versus time in a working plot.



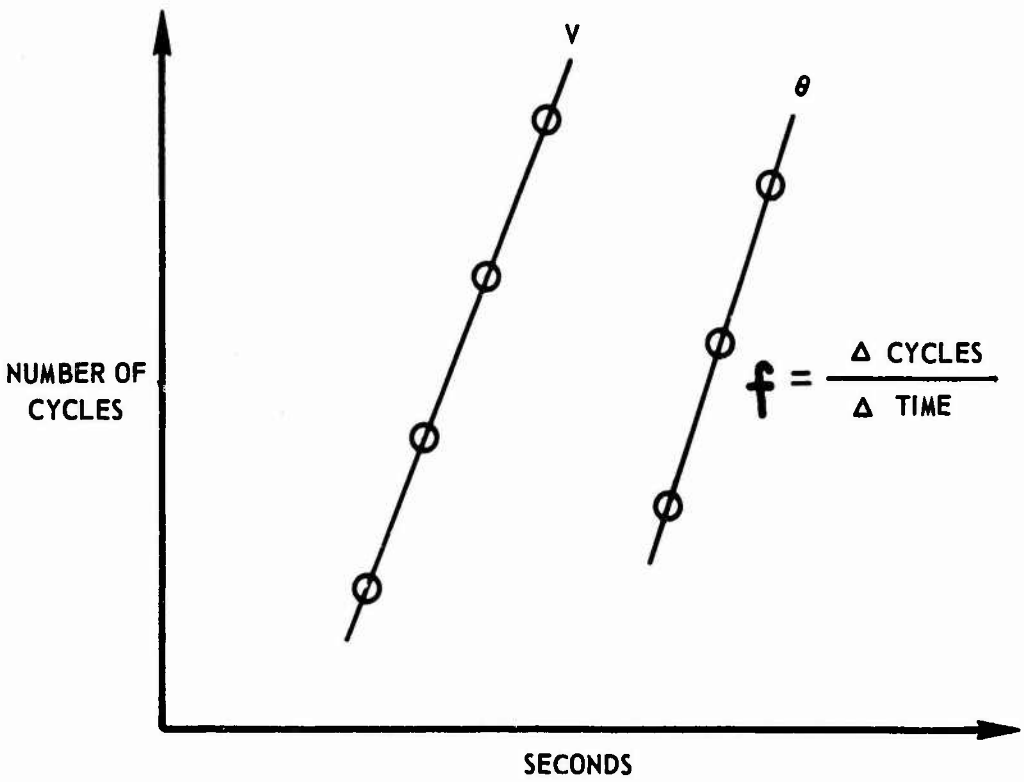
3. Determine the phugoid damping ratio (ζ). Sketch the damping envelope on the oscillograph trace. Measure the width of the envelope at the peak values of the oscillation. Form the subsidence ratios (X_m/X_0). From figure 9.4 or 9.5 find the damping ratio for each subsidence ratio. Average these damping ratios. If the subsidence ratio is greater than 1.0 then use the inverse of that subsidence ratio. The damping ratio thus determined will be negative.

4. Determine the phugoid undamped nautral frequency (ω).

$$\omega_n = \frac{2\pi f}{\sqrt{1 - \zeta^2}}$$

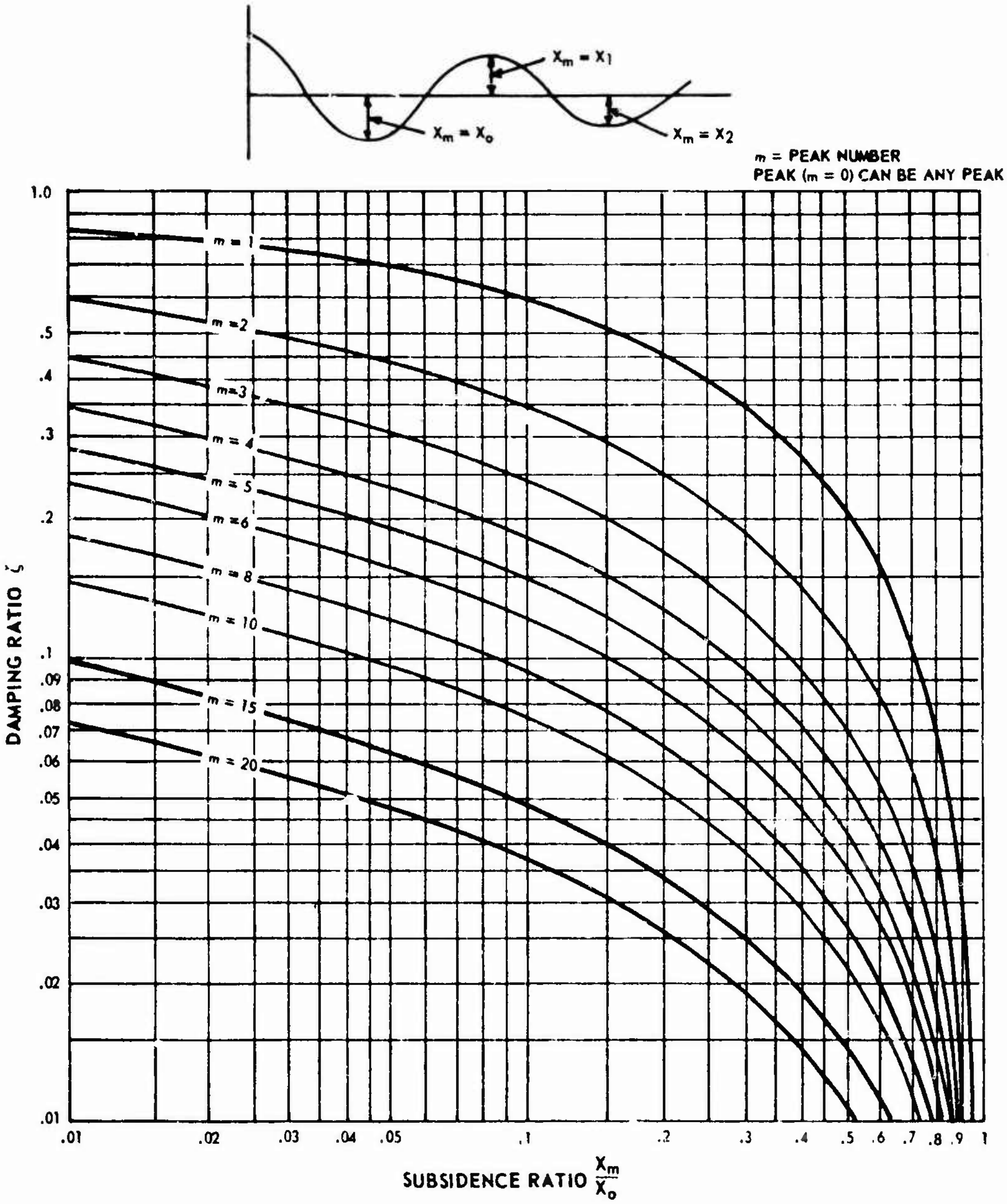
$$f = \frac{\Delta \text{ cycles}}{\Delta \text{ time}}$$ from figure 9.3

FIGURE 9.3



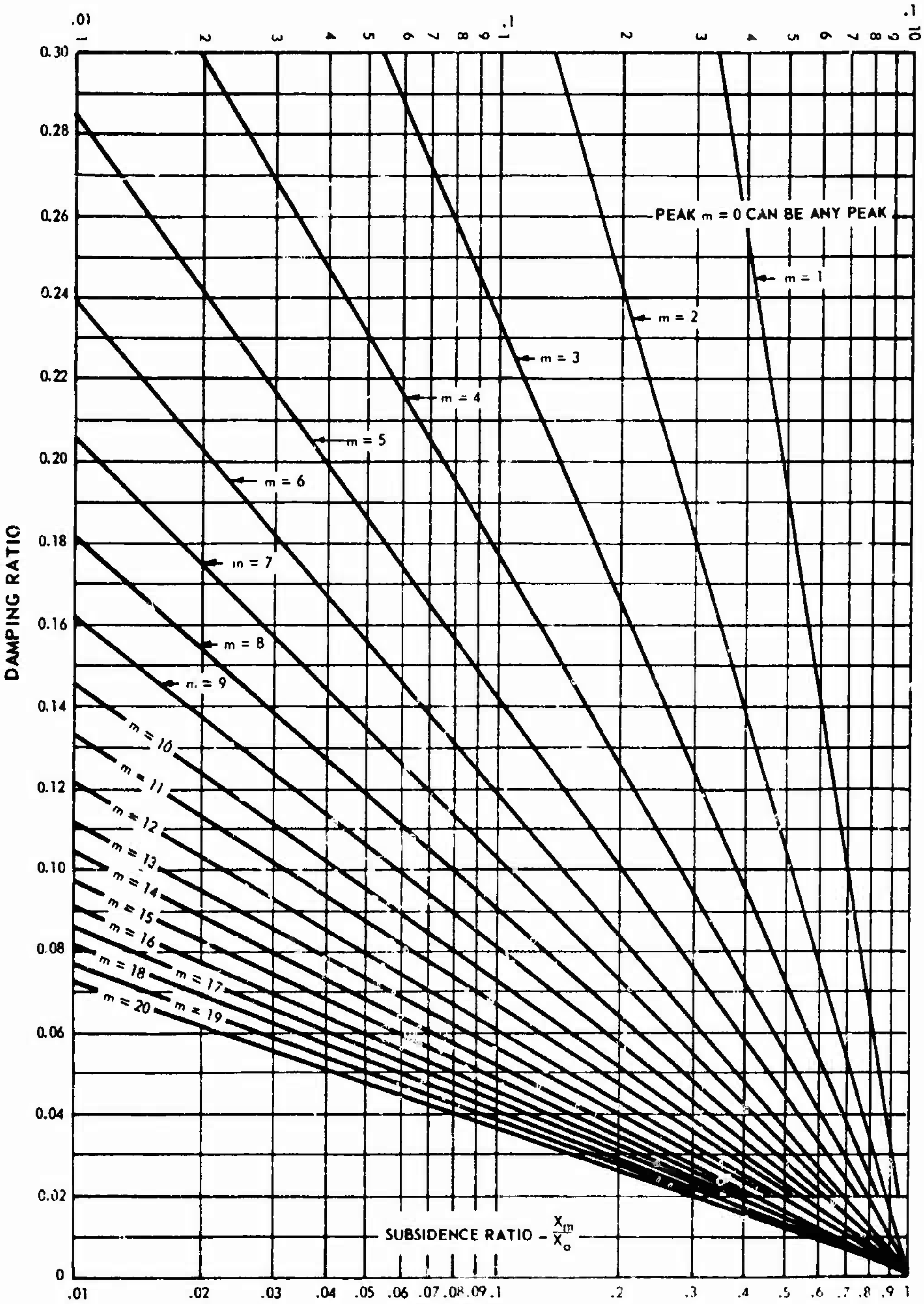
9.5

FIGURE 9.4



$X_m = X_1$
$X_m = X_o$
$X_m = X_2$
m = PEAK NUMBER
PEAK (m = 0) CAN BE ANY PEAK
DAMPING RATIO ζ
1.0
.5
.4
.3
.2
.1
.09
.08
.07
.06
.05
.04
.03
.02
.01
m = 1
m = 2
m = 3
m = 4
m = 5
m = 6
m = 8
m = 10
m = 15
m = 20
.01
.02
.03
.04
.05
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
SUBSIDENCE RATIO $\frac{X_m}{X_o}$

FIGURE 9.5



DAMPING RATIO AS A FUNCTION OF SUBSIDENCE RATIO

5. Plot phugoid frequency and damping ratio versus Mach number.

FIGURE 9.6

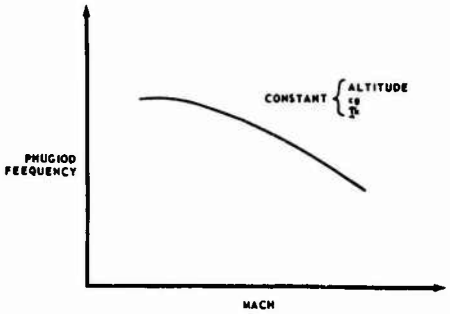
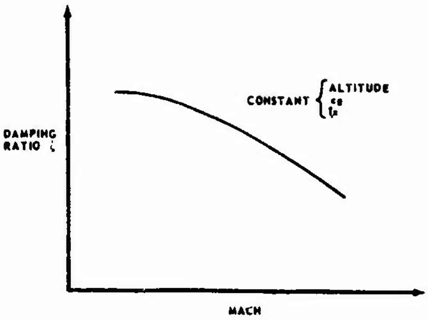
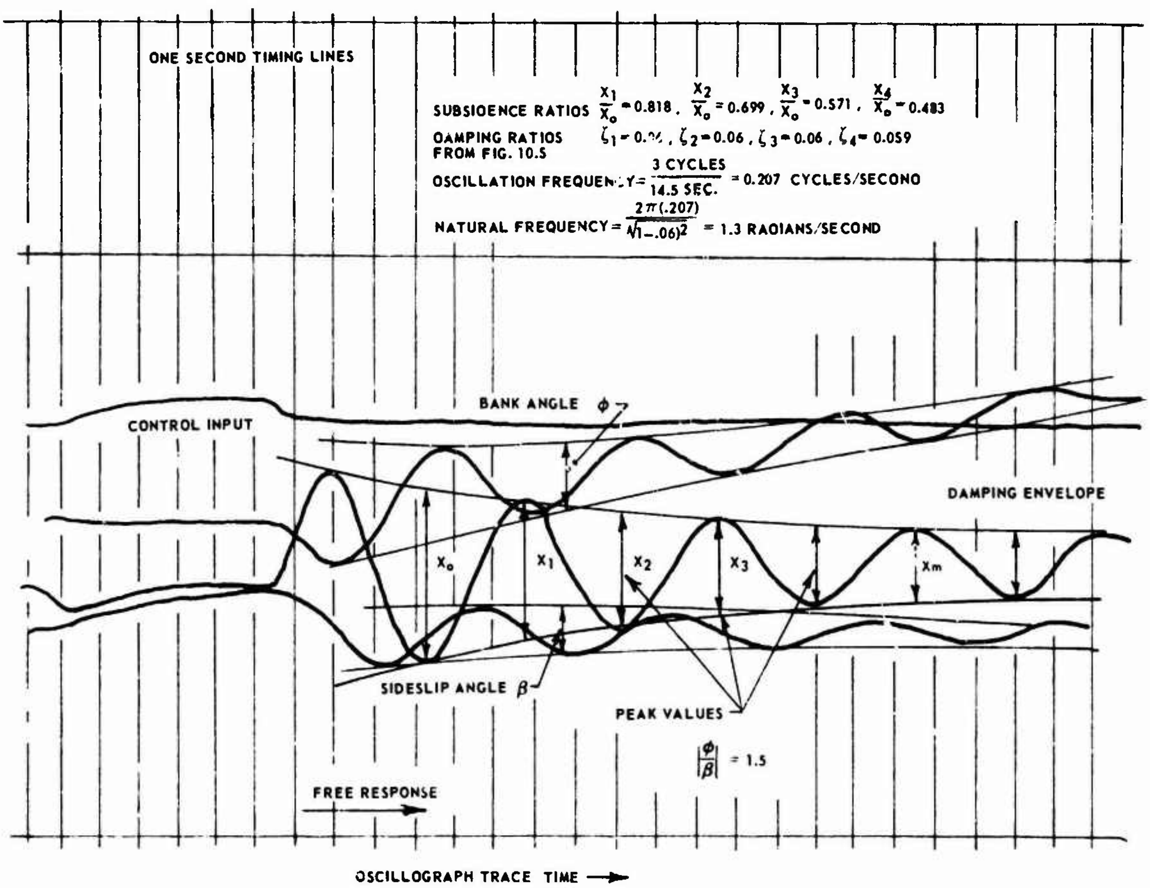


FIGURE 9.7



EXAMPLE NO. 1 OATA REDUCTION LIGHTLY OAMPEO OSCILLATION



6. In the test results include a short discussion on the effect the phugoid mode has on the aircraft handling qualities. This discussion should be presented with respect to an intended mission for the aircraft. Quantitatively compare the damping ratios with the requirements of MIL-F-8785.

B. Short Period:

Stick-Fixed.

Stabilize the aircraft at the test altitude on the test airspeed. Select oscillograph speed 4 and start recording. Smoothly but abruptly pull back on the control column; push it forward, and then rapidly return it to the trimmed position and hold it there. When the aircraft transient motion stops, stop recording data. Reverse the order of the input pulse and repeat. The airspeed and altitude should remain essentially constant during this maneuver. The data should be taken when the input pulse is approximately one-half g. This input pulse should be started small and gradually increased as the pilot's technique improves and if the aircraft response is satisfactory.

Stick-Free.

Stabilize the aircraft at the test altitude on the test airspeed. Select oscillograph speed 4 and start recording. Smoothly but abruptly pull back on the control column, push it forward, return it to approximately neutral and release. When the aircraft transient motion stops, stop recording data. Reverse the input order and repeat.

Data Reduction, Short Period.

1. Plot a time history of the aircraft response for closed-loop and open-loop. Label elevator deflection, control deflection, load factor, and angle of attack. Examine the phase angle between the stick movement and the actual elevator deflection for compliance with MIL-F-8785.

2. Determine the short period damping ratio (ζ). If the short period response is oscillatory and the damping ratio 0.5 or less proceed as outlined for the phugoid mode. If the damping ratio is between 0.5 and 2.0 then use figure 9.9. Select the point on the response curve at which the response is free. Divide the amplitude into the values 0.736, 0.406, and 0.199. Measure time values t_1, t_2 and t_3. Form the time ratios t_2/t_1, t_3/t_1, and $t_3 - t_2/t_2 - t_1$. Enter figure 9.9 at the Time Ratio side and find a damping ratio for each time ratio. For this damping ratio find a frequency time product for $(\omega_n t_1)$ $(\omega_n t_2)$ and $(\omega_n t_3)$. Average the damping ratios.

FIGURE 9.8

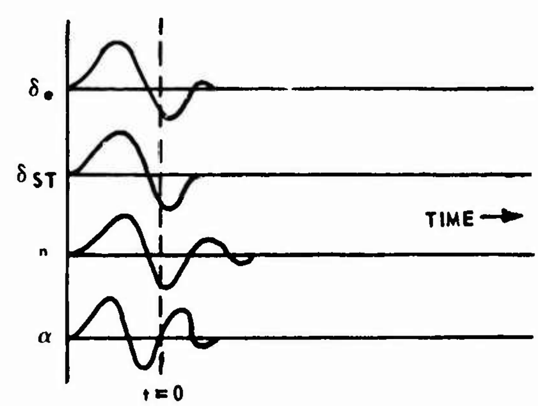
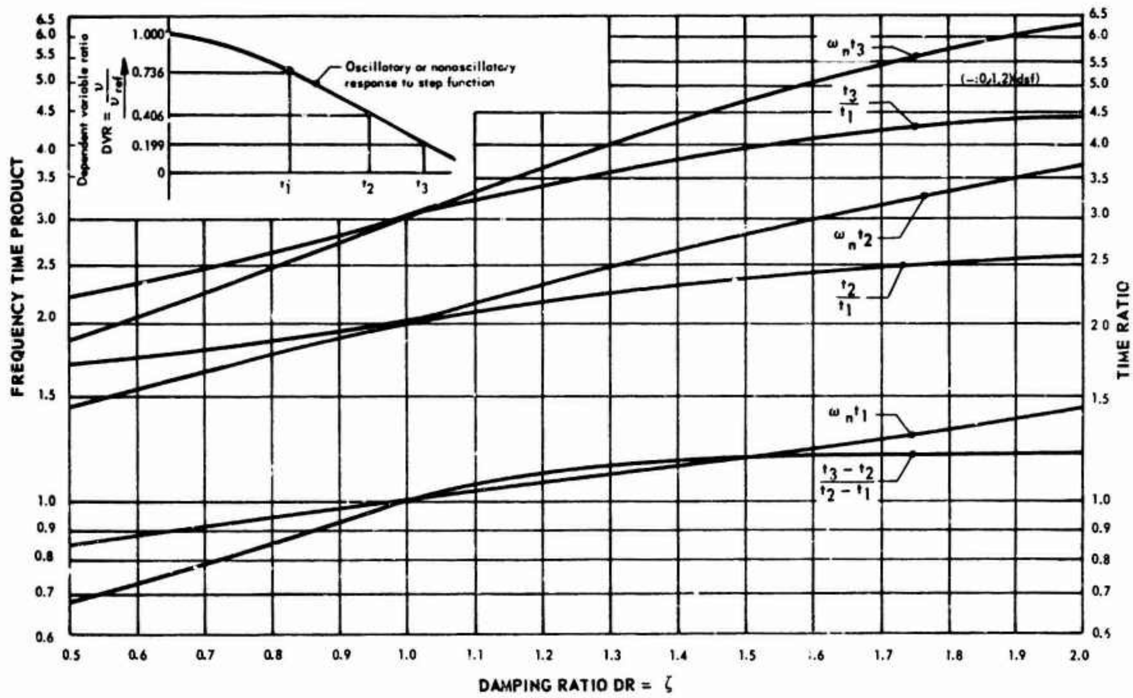


FIGURE 9.9



3. Determine the short period natural frequency (ω_n)

$$\omega_n = \frac{\omega_n t_3}{t_3}$$

$$\omega_n = \frac{\omega_n t_2}{t_2}$$

$$\omega_n = \frac{\omega_n t_1}{t_1}$$

Average the natural frequency.

4. Plot short period natural frequency and damping ratio versus Mach number.

FIGURE 9.10

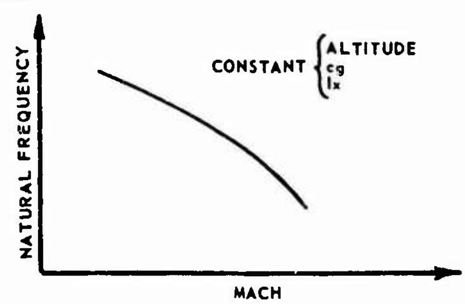
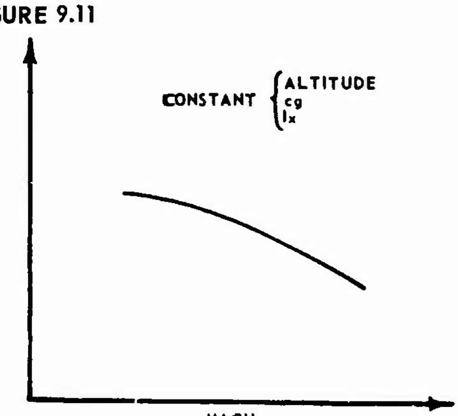


FIGURE 9.11



5. From the oscillograph trace, form the ratio, n_z/α. This is determined from the peak angle of attack which produced the peak "g." See where ω_n versus n_z/α is located in figure 1, MIL-F-8785.

6. In the test results include a short discussion on the effect of the short period characteristics have on the handling qualities of the aircraft.

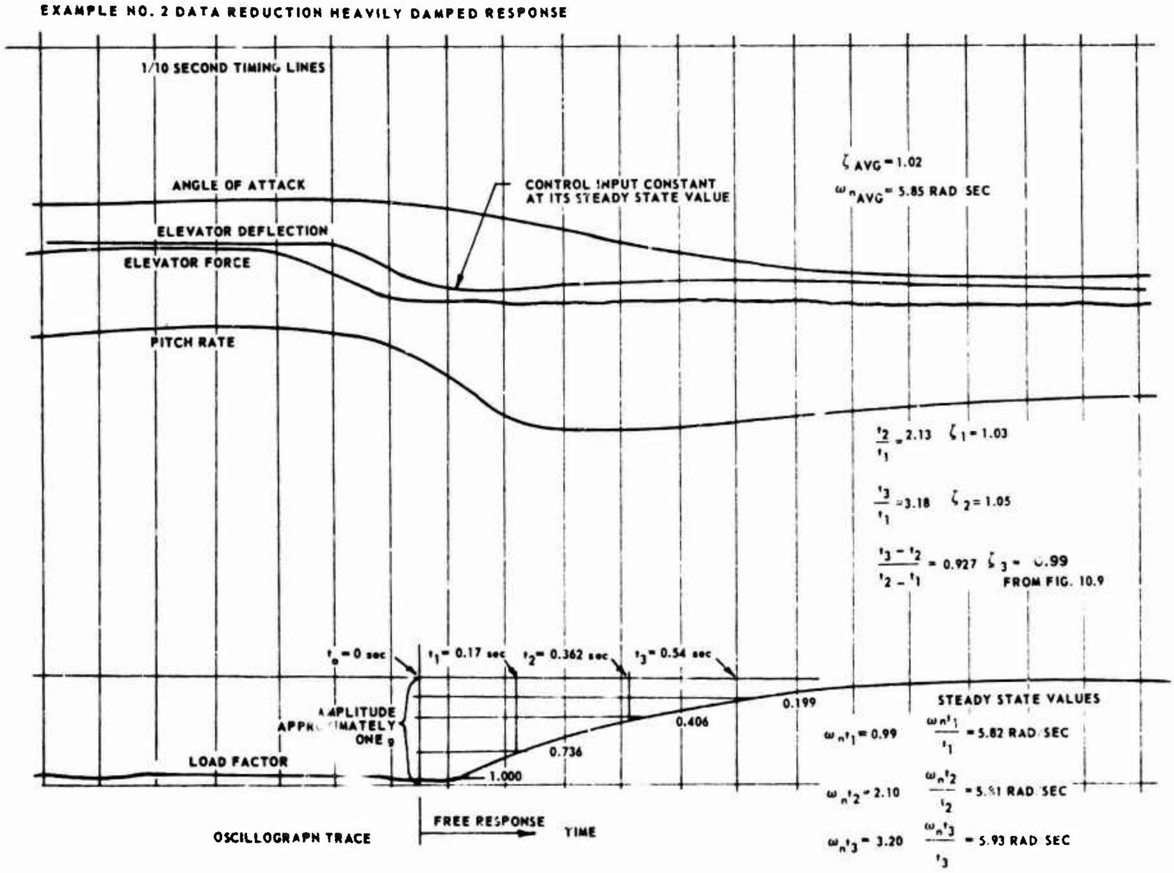
FIGURE 9.12

9.5 LATERAL-DIRECTIONAL DYNAMIC STABILITY

Dutch Roll:

The lateral-directional oscillations involve roll, yaw, and sideslip. The stability of the Dutch roll mode varies with airplane configuration, angle of attack, Mach number, and damper configuration. Dynamic stability of the lateral-directional modes is governed primarily by the static lateral and directional stabilities ($C_{\ell\beta}$ and $C_{n\beta}$), damping in roll and yaw ($C_{\ell p}$ and $C_{n r}$), and moments of inertia. The presence of a lightly damped oscillation adversely affects aiming accuracy during bombing runs, firing of guns and rockets and precise formation work such as in-flight refueling.

EXAMPLE NO. 2 DATA REDUCTION HEAVILY DAMPED RESPONSE



Stability of the oscillations is represented by the damping ratio; however, the frequency of an oscillation is also important in order to correlate the motion data with the pilot's opinion of handling qualities.

Military Specification Requirements:

Section 3.3 of MIL-F-8785 specifies in figure 2 the requirements for lateral directional handling qualities. It also states the residual oscillation that may be allowed for Category A Flight Phases.

Example Test Methods:

Release from Steady Sideslip.

Stabilize the airplane in level flight at test flight conditions and trim forces to zero. Establish a steady straight-path sideslip angle. Rapidly neutralize controls. Either hold controls for control-fixed or release controls for control free response. Start with small sideslip in case the aircraft diverges.

Rudder Pulse (Doublet).

Stabilize the airplane in level flight at test flight conditions and trim. Rapidly depress the rudder in each direction and neutralize. Hold at neutral for control-fixed or release rudder for control free response. For aircraft which require excessive rudder force in some flight conditions, the rudder pulse may be applied through the augmented directional flight control system.

Aileron Pulse.

Stabilize the airplane in level flight at test flight conditions and trim. Hold aircraft in a steady turn of 10 to 30 degrees of bank. Roll level at a maximum rate reducing the roll rate to zero at level flight. CAUTION . . . Such a test procedure must be monitored by an engineer who is thoroughly familiar with the inertial coupling of that aircraft and its effect upon structural loads and non-linear stability.

Data Required.

For trim condition, pressure altitude, airspeed, weight, cg position, and aircraft configuration should be recorded. The test variables of concern are: bank angle, sideslip angle, yaw rate, bank rate, control positions, and control surface positions.

Reduction and Presentation of Data.

Flight test data will be obtained as time histories. When determining the damping ratio the roll rate parameter usually presents the best trace.

Nonlinearities in the aircraft response may hinder the extraction of the necessary parameters. These can be induced by large input conditions. Small inputs balanced with instrument sensitivity give the best result.

moderately damped high frequency oscillation may be less satisfactory than a lightly damped low frequency oscillation. If the frequency is higher than pilot reaction time, the pilot cannot control the oscillation, and in some cases may reinforce the oscillation to an undesirable amplitude. Since it is the damping frequency combination which influences pilot opinion more than damping alone, some effort should be made to correlate this combination with pilot opinion of the lateral-directional oscillation.

At supersonic speeds, stability decreases with increased Mach number and altitude for constant g. An evaluation should proceed cautiously to avoid posible divergent responses that can result from nonlinear aerodynamics.

Control-Free Dutch Roll.

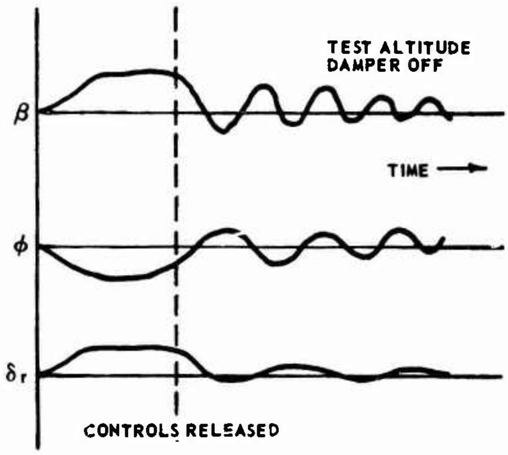
Stabilize the aircraft on test altitude and airspeed. The yaw damper and rudder power should be on. Select oscillograph speed 3 and start recording. Smoothly establish a steady straight sideslip using rudder and aileron. Release the controls. Start counting and timing oscillations when the aircraft nose reaches its extreme position from where it was released. Stop recording when the oscillation stops or after 5 to 8 cycles. Use caution and avoid any excessive sideslip angles.

Restabilize the aircraft with the yaw damper off. Select speed 3 on the oscillograph and start recording. Establish a steady straight sideslip and release controls. Start counting and timing cycles when the nose reaches its extreme position from the point of release. Stop recording after 5 to 8 cycles. Repeat the test with the rudder power off. Use caution in this configuration and avoid any excessive sideslip angles.

Data Reduction.

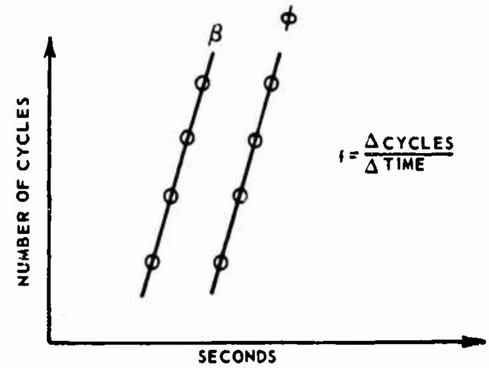
1. Sketch 5 cycles of the Dutch roll in each configuration, (damper and rudder power on, damper off, dampers and rudder power off). Label sideslip, bank angle and rudder deflection.

FIGURE 9.13



2. Determine the frequency of the oscillation. Plot cycles versus time on a working plot.

FIGURE 9.14



3. Determine the Dutch roll damping ratio (ζ) and natural frequency (ω_n) in the same manner as the phugoid mode.

4. Plot Dutch roll damping ratio and natural frequency versus Mach number for each configuration (damper on, damper off, rudder power off).

FIGURE 9.15

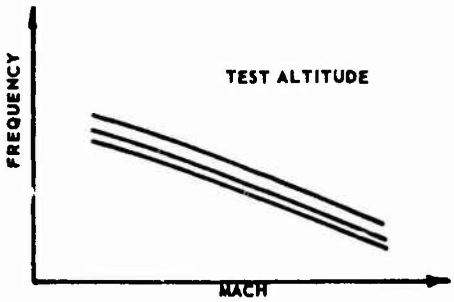
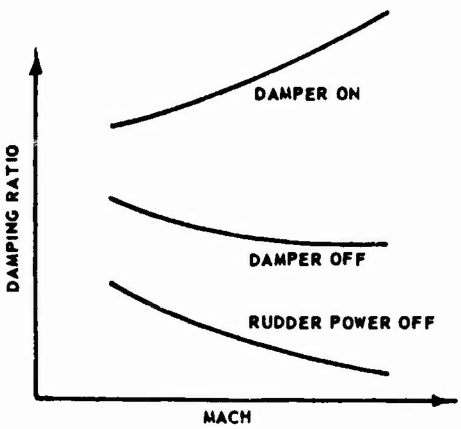


FIGURE 9.16



5. Compare the ω_n and ζ so obtained with figure 2 of MIL-F-8785 and determine compliance.

9.14

9.6 SPIRAL MODE

The spiral mode is, in general, relatively unimportant as a flying quality. However, a combination of spiral instability and lack of precise lateral trimmability may be bothersome to the pilot. This problem will be evaluated as a whole due to the difficulty in separating the effects.

The divergent motion is non-oscillatory, and is most noticeable in the bank and yaw responses. If an airplane is spirally divergent, it will, when disturbed and not checked, go into a tightening spiral dive. This divergence can be easily controlled by the pilot if the divergence is not too great.

Military Specification Requirements:

Spiral Stability is specified in MIL-F-8785 in table V. This table established limiting terms to double amplitude when the aircraft is put into a 20-degree bank and the controls freed.

Example Test Methods:

Trim the aircraft for hands-off flight, insuring that particular attention is given to lateral control and the ball being centered. Roll into a 20-degree bank in one direction, release the controls and measure the time it takes to reach 40 degrees of bank if the bank angle tends to diverge. Repeat the maneuver in a bank to the opposite side.

Data Required:

Aircraft configuration, weight, cg position, altitude and airspeed should be recorded. The test variables are bank angle, sideslip angle, control position, and control surface position.

Excitation of the spiral mode only is difficult because of

its relatively large time constant. Any practical input using control surfaces would usually excite other modes as well. If a deficiency in lateral trim control exists, it is often difficult to determine what portion of the resultant motion following a disturbance is caused by the spiral mode. This flight test is used to determine if a combined problem of lateral trim and spiral stability exists. If test results show a definite divergence in hands-off flight, the problem exists.

Spiral divergence, on its own, is of little importance as a flying quality, which is well within the control capability of the pilot. The ability to hold lateral trim in hands-off flight for 10 to 20 seconds is important.

Spiral Mode:

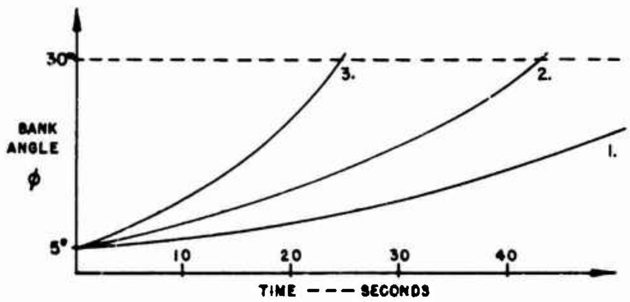
Stabilize the aircraft at test altitude and airspeed. Select oscillograph speed 2 and start recording. Establish a 20-degree bank and release controls. Time the motion to a 40-degree bank angle or 40 seconds elapse, whichever is shorter. Stop recording. Establish an opposite 40-degree bank and repeat. Repeat this procedure with the yaw damper off and then with the rudder power off.

Data Reduction:

1. Sketch a time history of the bank angle.

2. Average the time to double amplitude for right and left banks at each test condition. Compare with table V of MIL-F-8785.

FIGURE 9.17



3. Briefly discuss the spiral mode characteristics with respect to an intended mission.

● 9.7 DEMONSTRATION MISSION

The purpose of this mission is to demonstrate and practice the techniques used to investigate an aircraft's dynamic modes of motion. The aircraft used will be a B-57E.

Procedures:

1. Takeoff and climb to 20,000 feet. Practice rudder and stick inputs.

2. Obtain a trim point at 300 KIAS, 20,000 feet, in the CR configuration.

3. The IP will demonstrate the control inputs used to excite the short period mode. Vary frequency of input from structural frequency to less than the natural frequency. Inputs demonstrated will be singlets and doublets, stick-fixed and stick-free.

4. Practice inputs and observe aircraft motion. (Maintain 300 ±10 knots and 20,000 feet ±2,000 feet.)

5. The IP will demonstrate the methods of exciting the Dutch roll mode.

6. Practice inputs and observe aircraft motion. Estimate ϕ/β ratio, period of oscillation and number of overshoots.

7. Investigate spiral mode from a 20-degree bank angle. Note the effect that the out of trim condition has on results.

8. Obtain a trim point at 200 KIAS, 35,000 feet, in the CR configuration.

   NOTE: Maintain airspeed and altitude within ± 3 knots and ± 500 feet from trim points for remainder of mission.

9. Excite and observe the short period mode.

10. Investigate the Dutch roll motion with dampers off and with the rudder power on and off. Note ϕ/β ratio, overshoots, and period. Use Caution - may be divergent.

11. Obtain a trim point at M = 0.79 at 33,000 feet, in the CR configuration.

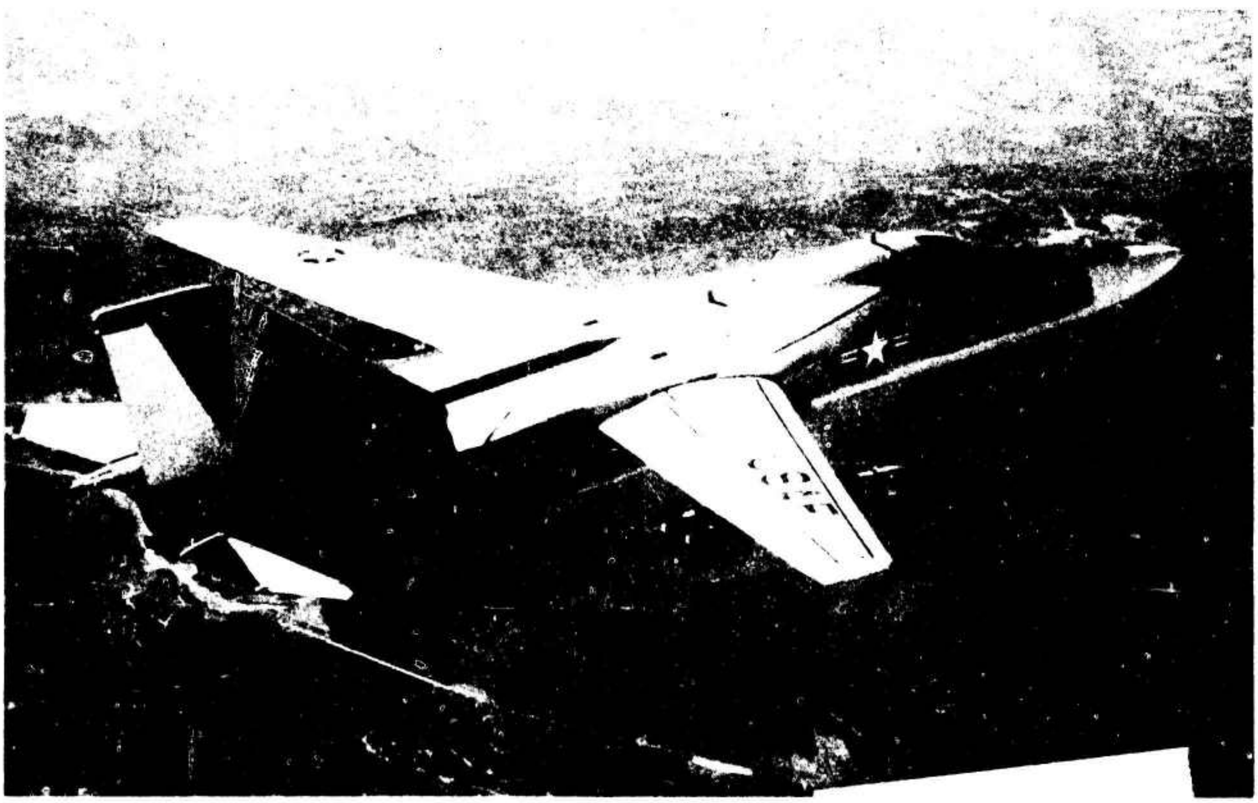
12. Excite the phugoid mode with a 3- to 4-knot Δ airspeed. Note the period and damping present.

13. Repeat No. 12 using a 10- to 12-knot Δ airspeed. Note the divergence due to M_u.

14. Investigate the short period mode.

15. Investigate the Dutch roll mode.

16. Land on the spot.



9.16

CHAPTER 10

QUALITATIVE FLIGHT TESTING

10.1 PURPOSE

The purpose of the qualitative flight test is to determine the maximum amount of information in the minimum amount of flying time in order to evaluate an aircraft with respect to its entire mission or some specific area of interest.

Qualitative flight testing has essentially the same purpose as quantitative flight testing, i.e., to determine how well the aircraft flies and how well it will perform its design mission. To accurately evaluate an aircraft from quantitative data requires analysis of large amounts of precisely measured data. The best a pilot can hope to do on a qualitative evaluation is to measure a limited amount of quantitative data. Thus, the test pilot's opinion on the acceptability of the aircraft is the important result and measured quantitative data when available is used primarily to support this opinion. Quantitative values of stick forces measured with a hand gage, for example, should be included in the report to support the pilot's opinion of acceptability. Estimations of values of stick force can be made if no reliable measurements are available or, qualifying terms such as "heavy", "medium", or "light" can be used to describe the forces. The point is that the difference in evaluating an aircraft qualitatively and quantitatively is a matter of degree. "Use what you've got." Pilot opinion supported by measured data is primary in qualitative testing while the reverse is true in quantitative testing. The general rule is to first decide how well the aircraft does its job and then use the quantitative data you can get to support your opinion.

10.2 PILOT OPINION

Naturally, all pilots will not have exactly the same opinion regarding the acceptability or unacceptability of a particular aircraft characteristic. No two people think exactly alike. However, the opinions of pilots with similar experience and background will usually not differ greatly, particularly with respect to the capability of an aircraft to perform a specific mission. In other respects, such as cockpit arrangements, the opinions may vary more markedly. For this reason, it is important for the qualitative test pilot to be as objective as possible in his evaluation. Guides which specify military requirements, such as MIL SPEC 203E and MIL-F-8785(ASG), should be used wherever possible to establish acceptability. However, it should be kept in mind that mere compliance with a set of requirements does not necessarily yield a satisfactory aircraft. The primary question is "will it do the job?", not "does it meet the specifications."

10.3 MISSION PREPARATION

A very limited amount of flight time is normally available for a qualitative evaluation. To acquire the information necessary to write an accurate and comprehensive report on an aircraft in this limited time requires a great deal of pre-flight study and planning.

The pre-flight preparation for a qualitative test is extremely important. It is almost impossible to put in too much time in planning for the flights. The amount of information acquired in the air will be directly proportional to the amount of preparation put in on the ground. A pilot who doesn't know what he is looking for is not likely to find it, and to know exactly what

to look for in the evaluation requires considerable knowledge of the aircraft and its mission.

The precise mission of the aircraft is important in determining what specific investigations should be made in the evaluation. All fighters, for instance, do not have the same mission, and the characteristics of particular importance may not be the same. The roll characteristics of an air superiority fighter would be more important than for a long range strategic fighter, and the specific test plan should take this fact into account. Expected outstanding characteristics or weaknesses should also receive particular emphasis. Of course, the evaluation must be conducted within the cleared flight envelope of the aircraft, and the amount of flight time available may limit the number of altitudes, airspeeds and tests that can be investigated. However, concentration on the extremes of altitudes, airspeeds, etc., and the areas dictated by the primary mission will provide the best approach to the test planning.

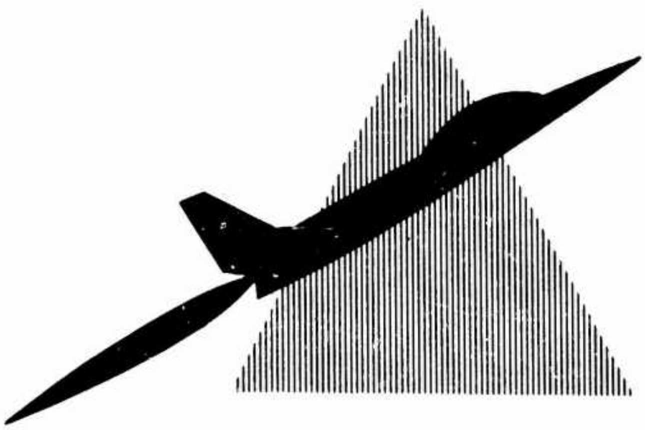
An outline of the test to be conducted and the various altitudes, airspeeds, and configurations to be used will aid in organizing the flights and planning the flight data cards. The points included in the outline should be compatible with the time available for the evaluation but it is always wise to overplan the flight and include more than seems possible to accomplish in the allotted time. Leave yourself the option of skipping the less important parts of your plan if time or fuel run short. The sequence of tests should be such that as little time as possible is wasted. With proper planning a continuous flow from one investigation to the next is possible.

10.4 FLIGHT DATA CARDS

Before planning the flight data cards, as much as possible should be learned about the aircraft. Study the pilot's handbook if one is available, discuss the aircraft with the engineers, or with other pilots who have flown it, and get adequate cockpit time. The more the pilot knows about the aircraft and the more comfortable he is in it, the more thorough will be the evaluation. A pilot who doesn't know the aircraft procedures, both normal and emergency or who has to spend most of his time in the air looking for controls or switches will not be able to do much evaluating.

The flight data cards should be self explanatory and should include all the points it is desirable to investigate during the flight. They should be designed so that a minimum of writing is required in the air because time will not be available to write down more than a word or two about each point. Remember, however, to provide places in the flight plan to write down these necessary comments. Numerous forms for the data cards are possible but completeness and legibility are essential.

The following are some possible formats for planning check lists and flight data cards:



AIRCRAFT QUALITATIVE EVALUATION CHECK LIST

I. Location ____________ Test Crew: Pilot ____________
Date ____________ Co-Pilot ____________
Aircraft ____________ Flight Mechanic ____________
Runway Length ____________ Observers ____________
Weather ____________
Runway Temperature ____________
Press. Altitude ____________
Surface Winds ____________
5M ____________ Climb Wind ____________
10M ____________ Freezing Level ____________
15M ____________
20M ____________
25M ____________

Weight and Balance: Operating Weight ____________
Fuel Weight ____________
MAC ____________ % Gross Weight ____________
T.O. Gross Weight ____________
Est. Fuel Used ____________
MAC ____________ % Est. Landing Weight ____________

Performance

T.O. Distance ____________ Refusal Speed ____________ T.O. Speed ____________
Minimum Control Speed ____________ Abort Landing Distance ____________
Climb Schedule: 4M ______ 6M ______ 8M ______ 10M ______
12M ______ 14M ______ 16M ______ 18M ______
20M ______ 22M ______ 24M ______ 26M ______
28M ______ 30M ______
Cruise (Max Range) Hi ______ Vi ______ Pwr ______

Operating Limitations

Gear Down ______ Flaps: 10 % ______ 50 % ______ 100 % ______
Landing Light ______ Cargo Doors ______ Para-defl ______
Dive Speeds: 30M ______ 25M ______ 20M ______ 15M ______
10M ______ 5M ______
Load Factor ______ Weight ______
Engine: Sync RPM ______ Slave RPM ______ Overspeed ______
TIT: T.O. ______ (MRP) Normal ______ Other ______
Airstart Vi ______ Torque ______ Max. ______ Cont. ______
Remarks: ______

Systems Operation:

DC Generators ______ Eng. AC Generators ______ Eng. ______
Booster Hyd ______ Eng. Utility Hyd ______ Eng. ______
Other:

Auxiliary Equipment Operation:

Auto-pilot ____________
Other:

NOTES:

II. PRE-OPERATION EVALUATION

A. Support Equipment
  1. Power Unit
     Type
     Capacity
  2. Other

B. Cargo Compartment
  1. Entrance
  2. Egress
  3. Systems Accessibility
  4. Other

C. Flight Deck
  1. Crew Stations
     a. Pilot
        Seat Adjustment
        Clearance
        Vision
        Rudder
        Pedal Adjustment
        Restrictions
        Other

     b. Copilot

     c. Flight Mechanic

     d. Navigator

  2. Instrument Panel
     a. Flight Instruments
        Grouping
        Readability
        Adequacy

     b. Engine Instruments
        Grouping
        Readability
        Adequacy

     c. Warning Lights
        Placards
        Switches
        Controls

  3. Pedestal
     a. Engine Controls
        System Controls
        Switches
        Guards
        Placards
        Lights
        Feel Identification
        Accessibility
        Confusion Factor
        Arrangement

     b. Remarks

  4. Overhead Panel
     a. Engine Controls
        System Controls
        Switches
        Guards
        Lights
        Placards
        Accessibility
        Feel Identification
        Confusion Factor
        Arrangement

     b. Remarks

  5. Side Panels
     a. Switches
        CSs
        Lights

     b. Remarks

  6. Flight Controls
     a. Rudder
        Break-out Force
        Travel
        Adjustment
        Clearance
        Slop
        Friction

     b. Elevator
        Break-out Force
        Travel
        Slop
        Friction
        Clearance

     c. Control Wheel
        Aileron Break-out Force
        Travel
        Slop
        Friction
        Clearance
        Grip
        Switches

  7. General Comments

10.4

5. Vibration
    a. Noise
    b. Air vent deflectors
    c. Ventilation/heating
6. Control Required To Maintain Proper Taxi Speed
7. Remarks

D. Pre-Take-Off (line up at even 1,000 feet and check W/V)
1. Flight Control Check With Boost Operating
    a. b/o force
    b. rate
    c. deflection
    d. slop
    e. friction
2. Flaps Set __________ Trim Set ______________
3. Engine Power Check
    a. Acceleration
        Idle __________ to __________ (MRP) ______ Sec.
        Asymmetric ______________
        Overshoot ______________
    b. Stabilized conditions: OAT ______________

Eng	%RPM	Torque	TIT	Throttle Pos
1				
2				
3				
4				

4. Brakes Hold At MIL PWR
5. Fuel reading ____________ lbs. W/V __________ ________ kts.

E. Take-Off. (Use flight data on knee board)
1. Start Time Form BRAKE RELEASE TO START CLIMB __________
2. Brake Release Action
3. Directional Control. Rudder Effective __________ kts.
4. Elevator Effective (nose wheel off) __________ kts.
5. Aileron Control ____________ kts.
6. T.O. Distance ____________ ft. Lift-Off Speed __________ kts.
    Time ________ sec.
7. Control Force __________ Pitch __________ Trim __________
8. Trim-Out - Raise Gear
    Time ____________ sec.
    Yaw ____________
    Trim ____________
9. Trim-Out - Raise Flaps
    Time ____________ sec.
    Trim ____________
10. Acceleration to MINIMUM CONTROL SPEED
11. Acceleration to Climb Speed (1,000 ft)
12. Visibility and Pitch Angle ________
13. Remarks:

10.5

F. Climb (M N________°, 90° to W/V).

1. Visibility________
   Pitch Angle________
2. Record: FUEL at START CLIMB________

TIME	Hi	Vi	R/C	Ti	%RPM	TORQUE	TPT	Wf
	4M							
	6M							
	8M							
	10M							
	12M							
	14M							
	16M							
	18M							
	20M							
	22M							
	24M							
	26M							
	28M							
	30M							
	32M							

FUEL at LEVEL-OFF________

3. Check Cabin Pressurization:
   10M________
   15M________
   20M________
   25M________
   30M________ Note any fluctuations or surges.
4. Cabin Heat Adequacy
   a. Nesi glass
5. Remarks

G. Cruise

1. Vmax
   a. Hi________
   b. Vi________
   c. OAT________
   d. Flt. Controls________
   e. RPM________
   f. Torque________
   g. TIT________
   h. Wf________
   i. FUEL________

2. Dynamics (Hi__________Vi__________) Note Control Position
   a. Phugoid
      1. Trim__________$V_{i_{in}}$__________V_{max}__________V_{min}__________
      2. Sec/cyc__________•__________Damping__________
   b. Porpoise Mode. input__________cycls__________ampl.__________
   c. Spiral stability
      1. RT Ø 10°__________°/__________sec.
      2. LFT " "__________°/__________sec.
      3. Remarks:
   d. Dutch Roll
      1. RT sideslip s/c__________Roll__________Yaw__________
         Damping__________(1)__________(2)__________(3)__________
      2. LFT sideslip s/c__________Roll__________Yaw__________
         Damping__________(1)__________(2)__________(3)__________
      3. (1) Norm (2) Damper Off (3) Rudder Power Off.
   e. Short Period
      1. Fixed (1.0g) Damping__________
      2. Fixed (-1.0g) " __________
      3. Free (1.0g) " __________
      4. Free (-1.0g) " __________
      5. Remarks:

3. Maximum Range Data
   a. Hi__________Vi__________OAT__________FUEL__________
   b. RPM__________Torque__________TPT__________Wf__________
   c. Remarks:

4. Systems Check: Hi__________Vi__________
   a. Engine shut-down, No.__________
      1. Time to feather__________Control force__________
      2. Procedure, etc:
   b. Engine restart
      1. Time to Normal power__________Surge__________Trim__________
      2. Procedure, etc:
   c. Anti-icing/de-icing system
      1. Full operation effect on engines__________
      2. Nesi glass
         Other
      3. Remarks:
   d. GTU/ATM operation__________
   e. Pressurization/heating__________
   f. Other:

5. Emergency Descent, Hi__________Vi__________(Initial)
   a. Time from cruise to start descent__________
   b. Procedure: G and F__________Clean__________Pressurization__________
   c. Time__________from CR to Hi__________at Vi__________
   d. Visibility__________Pitch__________Control__________
   e. Remarks:

6. Static Longitudinal Stability and Performance Hi ________
   a. Acceleration check Trim at Max Range Vi ________
      1. Decel to Vi _____ Control force *(Trim setting) _____
      2. Speed/Pwr Vi _____ Rpm _____ Tq _____ TIT _____ OAT _____
         Speed/Pwr Vi _____ Rpm _____ Tq _____ TIT _____
      3. Acceleration, (RESET TRIM), Time/10 kts (MRP) Initial Vi _____
         10 _____
         20 _____
         30 _____
         40 _____
         50 _____
         60 _____
         70 _____
         80 _____
         V/S _____ ft/min. Control forces/gradient
      4. Remarks: FUEL _____
   b. Trim Changes: Hi _____ Vi _____
      1. Control boost off _____ on _____
      2. Runaway Trim: Elev _____ Ail _____ Rud _____
         5 sec delay (build-up) _____
   c. Turning Performance and Aileron Rolls. Cruise. (Build-up).
      FULL DEFLECT
      1. 60° Ø, Time 360° _____ V_{max} _____ Hi _____
      2. 45° Lft - 45°Rt (FIX) Time for 90° _____
      3. 45° Rt - 45° Lft (FIX) Time for 91° _____
      4. 60° Ø, Time 360° _____ Vi _____ Hi _____
      5. 45° Lft - 45°Rt (FIX) Time for 90° _____
      6. 45° Rt - 45° Lft (FIX) Time for 90° _____
      7. 60° Ø, Time 360° _____ Vi _____ Hi _____
      8. 45° Lft - 45° Rt _____ (FIX) Time for 90° _____
      9. 45° Rt - 45° Lft (FIX) Time for 90° _____
      POWER APPROACH
      10. 45° Lft - 45° Rt (FIX) Time for 90° _____
      11. 45° Rt - 45° Lft (FIX) Time for 90° _____
   d. Spiral Stability PA Hi _____ Vi _____ Pwr _____
      1. Rt Ø 10° _____ o/ _____ sec. (1/2 - 2).
      2. Lft 10° _____ o/ _____ sec. (1/2 - 2).
   e. Phugoid (HI CL) _____
   f. Sideslips, TRIM (L) Hi _____ Vi _____
      1. Rt _____ °, Fr _____ Fa _____ Fs _____ dr _____ da _____ de _____
      2. Lft _____ °, Fr _____ Fa _____ Fs _____ dr _____ da _____ de _____
      TRIM (CR) Hi _____ Vi _____
      3. Rt _____ °, Fr _____ Fa _____ Fs _____ dr _____ da _____ de _____
      4. Lft _____ °, Fr _____ Fa _____ Fs _____ dr _____ da _____ de _____
      5. D.E. with rudder (Pick up wing) _____
      6. Remarks: FUEL _____

7. Stalls, Gross Weight _____ Hi Trim _____
   a. CR 1.0g TRIM Vi _____ Vw _____ Vs _____ Hi _____
   b. CR 2.0g TRIM Vi _____ Vw _____ Vs _____ Hi _____
   c. Remarks:
   d. PA 1.0g TRIM Vi _____ Vw _____ Vs _____ Hi _____
   e. PA 1.5g TRIM Vi _____ Vw _____ Vs _____ Hi _____

10.8

8. Asymmetric Power - Hi ____________
   a. Climb configuration (MRP, Climb Vi, Trimmed-out)
      NTC ________ Feather ____________ No. 1 Eng. Rudder Free, 2 sec.
      Decel to 1.4 Vsl ____________ kts. Ø and sideslip
      (Cond. permitting check 2 out on one side)
   b. T.O. Configuration at V_{max} Gear and T.O. Flaps (168 kts.)
      Fail 1 and 2 and decelerate holding Ø = ZERO.
      V_{imin} __________. Check Ø = 5° and SIDESLIP = ZERO.
   c. AT Min control speed fail 3 and 4, Fr ____________ Fa ________
      Fs __________ TRIM OUT HANDS OFF AT 1.2 Vsl ____________
   d. Remarks:

9. Boost OFF Operation Hi ____________ Vi ____________ Pwr ________
   a. Asymmetric Control 1 and 2 idle, 3 and 4 MRP
   b. Response ________ Fr ________ Fa ________ Fs ____________
   c. Remarks:

10. Descent
    a. CR Configuration Vi ____________ V/S ____________
       1. Visibility ____________ Attitude ____________
       2. Engine operation at idle ____________
       3. Pressurization, systems, etc. ____________
       4. Remarks:
    b. L Configuration Vi ____________ V/S ____________
       1. Visibility ____________ Attitude ____________
       2. Engine operation at idle ____________
       3. Remarks:

11. Trim Changes Trim at Placard Speed, PLF
    a. Flaps to 50% Vi ____________ Hi ____________ PLF/Trim
    b. Gear DOWN Vi ____________ Hi ____________ PLF/Trim
    c. Flaps to 100% Vi ____________ Hi ____________ PLF/Trim
    d. Power to IDLE Vi ____________ Hi ____________ Trim
    e. Idle to HRP Vi ____________ Att ____________ Trim
    f. Gear UP Vi ____________ V/S ____________ Trim
    g. Flaps UP Vi ____________ V/S ____________ Trim

12. Asymmetric Power Go-around
    a. ______ Out, Pa Vi ________ Hi ________ Pwr ________
    b. Fr ________ Fa ________ Fe ________ Response and Control
    c. Remarks:

13. General Comments Prior to Completion of Flying.

H. Approach and Landing
   1. Pre-landing check: Operating Weight ____________
      Alt Setting ____________ Fuel Weight ____________
      W/V ____________ Landing GR WT ____________
      Runway ____________ Best Flare Speed ____________
      (Pilot Pwr and Steer) Touchdown speed ____________
      (Copilot Ailerons) V_{SL} ____________
   2. Traffic pattern:
      a. Visibility ____________ Control
      b. Power response ________
      c. Remarks:
   3. Landing:
      a. Flare ____________ Response ____________ Control ____________
      b. Float ____________ Characteristics in ground effect ____________
      c. Touchdown ________ Nose-wheel off ________ Grd Idle ________
         Reverse ________ Brakes ________ Steering ________
      d. Directional control with ailerons ____________
      e. Stopping distance ____________
   4. Remarks:

I. Post-flight and Shut-down
   1. Normal procedures. Ease and time to accomplish ____________
   2. Coordination ____________
   3. Fuel ____________
   4. Flight Time ____________
   5. Squawks ____________

J. Re-evaluate Cockpit and A/C in General

10.9

Fighter type aircraft - Two hour flight - Plan more than can be accomplished.

EXTERNAL INSPECTION TOD START________

TOD FINISH________

Remarks:

COCKPIT EVALUATION

1. Ease of Entry ladder______________

   Steps______________

2. Location of Instruments and Controls
3. Adjustment of Seat and Controls
4. Comfort
5. Ease of Identification of:

   Switches

   Controls

   Emergency Devises

   Warning Lights
6. Egress - ground and Airborne

BEFORE STARTING CHECKS TOD______________

Remarks

Complexity:

STARTING ENGINES Fuel______ TOD______________

Complexity:

Ground Support:

Equipment______________

Personnel______________

BEFORE TAXI CHECKS TOD______________

Estimated Break-out Force

Longitudinal +______# - ________#

Lateral +______# - ________#

Directional +______# - ________#

Trim rate (Longitudinal) Aft________Sec

Fore________Sec

Flap Extension________sec Retraction__________sec

TAXIING Fuel________ TOD______________

RPM req to move______________

Visibility

Steering N.W.S.

Brakes

Visibility

Power required______rpm, fuel/flow__________pph

Runway temp________°F. P.A. ______________ft.

TAKEOFF Fuel________# TOD______________

Do brakes hold in MIL PWR Yes No

Symmetry of brake release

Directional control

Rudder effective speed___________knots

Ease of rotoation

Lift-off speed______________knots

Estimated T/O distance__________feet

Gear up time______sec Flaps up time______________sec

Trim changes Landing gear + - ________#

Flaps + - ________#

Are placards hard to exceed? Yes No

Visibility during T/O and Initial Climb

Adequacy of T/O trim setting:

Speed stability during acceleration:

CLIMB Fuel________# TOD______________

Control during climb

Longitudinal

Directional

Lateral

Climb Schedule

5000 ft.	.89IMN	550
10000 ft.	.89IMN	510
15000 ft.	.90IMN	470
20000 ft.	.905IMN	430
25000 ft.	.910IMN	390
30000 ft.	.915IMN	360
35000 ft.	.92 IMN	320
39000 ft.	.92 IMN	

LEVEL OFF FUEL__________# TOD__________

EASE

Attitude Change____________________ o

****

CRUISE 90 % RPM .86IMN (recommended cruise)

Start Fuel__________# TOD__________

Linear

Sideslip: Cl_β Hvy Med Lt Yes No

Cl_β Hvy Med Lt Yes No

Dutch Roll Period__________sec

Damping Hvy Med Lt

Cycles to Damp__________

CRUISE cont. 39,000 ft. .86IMN

PIO Tendency Yes No

Short Period Cycles to Damp__________

Period__________sec

Do controls have dynamic tendency?

Yes No

Aileron Rolls: t_{90}

R L Adv. Yaw

1/2 deflection _____sec_____sec

Full deflect. _____sec_____sec

************************DAMPERS OFF**********************

Linear?

Sideslip: Cl_β Hvy Med Lt Yes No

Cn_β Hvy Med Lt Yes No

Dutch Roll: Period__________sec

Damping Hvy Med Lt

Cycles to damp__________

PIO Ter... Yes No

Sho... Cycles to damp__________

Period__________sec

10.13

********************************DAMPERS ON*******************

Finish: Fuel________# TOD________________

Speed brake trim change Hvy Med Lt

Extend Push Pull

Retract Push Pull

MANEUVERING FLIGHT .90 IMN 39-35,000 ft.

Fuel________#

Initial buffet____________g

Heavy buffet____________g n_{max}____________g

Stick force Hvy Med Lt

Linear Yes No

****

ACCELERATION TO 1.20 IMN at 35,000 ft. (trim .90 IMN)

Start: Fuel________# TOD________________

NB Light L________sec R________sec

NB Trim Change______# Push Pull

Stick force gradient____________________

Transonic trim change__________________

Finish fuel____________# TOD________________

****

CRUISE 1.15 TMN 35,000 ft.

Start: Fuel____________# TOD________________

						Linear?	
Sideslips:	C_{ℓ_β}	Hvy	Med	Lt		Yes	No
	C_{ℓ_β}	Hvy	Med	Lt		Yes	No

Dutch Roll: Period____________sec

Damping Hvy Med Lt

Cycles to Damp____________

PIO Tendency Yes No

CRUISE cont 1.15 IMN 35,000 ft.

Short Period: Cycles to Damp ______________ -

Period __________________ sec

*****************************DAMPERS OFF*************************

					Linear	
Sideslip	C_{l_β}	Hvy	Med	Lt	Yes	No
	C_{n_β}	Hvy	Med	Lt	Yes	No

Dutch Roll:

Period ____________ sec

Damping Hvy Med Lt

Cycles to Damp ______________

PIO Tendency Yes No

Short Period: Cycles to Damp _____________

Period _________________ sec

*****************************DAMPERS ON*************************

Aileron Rolls	t_{90} R	L	Adverse Yaw
½ deflection _____	sec _____	sec _____	
Full deflect _____	sec _____	sec _____	

Finish Fuel ___________ TOD _______________

****

SPEED BRAKE TRIM CHANGE 1.15-1-1.10 IMN

Hvy Med Lt

Extend Push Pull

Retract Push Pull

MANEUVERING FLIGHT 1.10 IMN 35-30,000 ft.

Fuel _______ #

Initial buffet ________ g Heavy buffet ________ g

n_{max} ________ g

Stick force Hvy Med Lt

Linear? Yes No

DECELERATION TO 210 knots 30,000 ft. (Long Stat)

Stick Force gradient________________________

****

CRUISE 210 knots 30,000 ft.

Start: Fuel__________# TOD_______________

						Linear?	
Sideslips:	C_{l_β}	Hvy	Med	Lt		Yes	No
	C_{n_β}	Hvy	Med	Lt		Yes	No

Dutch Rolls Period_____________sec

Damping Hvy Med Lt

Cycles to Damp_________________________

PIO Tendency Yes No

Short period: Cycles to Damp_________________________

Period_____________sec

**************************DAMPERS OFF***********************

						Linear?	
Sideslips:	C_{l_β}	Hvy	Med	Lt		Yes	No
	C_{n_β}	Hvy	Med	Lt		Yes	No

CRUISE 210 knots at 30,000 ft.

Dutch Roll: Period_____________sec

Damping Hvy Med Lt

Cycles to Damp_________________________

PIO Tendency Yes No

Short Periods: Cycles to Damp_________________________

Period_________________________________

Finish: Fuel__________# TOD_______________

**************************DAMPERS ON************************

AILERON ROLLS t_{90} Adverse Yaw

1/2 Deflection R________sec L________sec____________________

Full deflect R________sec L________sec____________________

****

MANEUVERING FLIGHT at 210 knots

Fuel____________#

Initial Buffet____________g Heavy Buffet__________g

"max" ______________g

Stick force gradient: Hvy Med Lt

****

STALLS Cruise Configuration 25,000 ft.

Fuel__________#

Cr V_w____________knots V_s____________knots

GLIDE V_w____________knots V_s____________knots

Remarks

POWER APPROACH CONFIGURATION

Gear extension____________sec

Flap extension____________sec

Asymmetric power at 155 knots

MIL RWR Rudder Force Hvy Med Lt

MAX TWR Rudder Force Hvy Med Lt

Trimability MIL________________________MAX

STALLS: Fuel___________

V_w___________knots V_s ____________knots

Remarks:

Trim at 160 knots

Linear?

Sideslip: C_{l_β} Hvy Med Lt Yes No

C_{l_β} Hvy Med Lt Yes No

Dutch Roll Period ________________ sec

Damping Hvy Med Lt

Cycles to Damp __________

PIO Tendency Yes No

Short Period Cycles to Damp __________

Period ________________ sec

**************************DAMPERS OFF**********************

Dutch Roll Period ________________ sec

Damping Damping Hvy Med Lt

Cycles to Damp __________

PIO Tendency? Yes No

Short Period: Cycles to Damp __________

Period ________________ sec

**************************DAMPERS ON**************************

AILERON ROLLS t_{90} Adverse Yaw

1/2 Deflection R_____sec L_____sec __________

Full Deflect R_____sec L_____sec __________

****

ACROBATICS

Loop

Immelman

Barrel Roll

INSTRUMENTS

Holding at 2C,000 Ft.		250 knots	90-92 %
Penetration	S/B	270 knots	90 %
Initial	Clean	220 knots	94 %

Low Cone gear, 86 %, flaps, 155 knots

LANDING

Normal traffic pattern 60 % flaps

Single engine go-around closed pattern

Full stop Full flaps

Touchdown speed_______knots' marker____________

****

TAXIING Fuel________# TOD________________

Engine acceleration Idle to mil__________sec

Turning radius___________feet

Re-evaluate cockpits

ENGINE SHUTDOWN

Check servicing for turn-around

Time_________

Oil__________qts

Hydraulic fluid_______qts

LOX____________liters

This data card is also for a fighter type aircraft - a one hour mission to evaluate the aircraft for a pilot training mission.

TOD ________________________________ beside A/C

START Procedure

F Flow ____________ RPM ____________ F Flow ___________

Before Taxi Check

TOD ____________________

TAXI

Power to roll ____________________ Brakes S NS

Nosewheel steering Turn Rad. ________________

NWS Off Brake turn ___________________

Canopy Operation

Visibility

TOD ________________________________

LINE UP

Brakes Mil Pwr _______________

Pump one brake

Engine Acc Time ______________

RPM ____________ EGT _______________ FF ____________

Throttle friction S NS

FUEL L ____________ R ________________

TOD ____________________

10.20

TAKEOFF

Brake release

A/B light

NWS rel at Rudder Eff A/S__________

CONTROL FORCES L M H__________lbs

NW LIFT OFF__________

T.O. ROLL__________ft A/S__________

GEAR UP__________sec. FLAPS UP__________sec

Trim Changes__________

Noises

Press. Sys

Accelereleration Rotation

CLIMB

Schedule .9 to 35M

Control

Trim

Visibility

Dampers

35M Time__________ Fuel L__________ R__________

Throttle Mil Level Off

TOD__________

10.21

SUPERSONIC

A/B Light time___________

TRIM CHANGES

STABILITY

DAMPERS	PULSE	CYCLE	TIME
ON	Elev		
	Rud		
OFF	Elev		
	Rud		

45' Roll

ONE ENGINE IDLE

Wind Up Turn to g Max.

A/S__________ "g"__________

Stick force gradient

Buffet

FUEL L__________ R____________

TOD______________________________________

10.22

TURNING PERFORMANCE 300 Kts_______________sec

Zoom to Slow A/C

PWR STALL WARN__________ STALL__________

230 Kts. Flight Roll

STABILITY

DAMPERS	PULSE	CYCLE	TIME
ON	Elev		
	Rud		
OFF	Elev		
	Rud		
Sideslip	6' Apx.		

CUT ONE ENGINE

EMERGENCY GEAR EXTENSION_______________sec

AIRSTART

170 knots Flaps Down

Aileron Power

Cycle gear Flaps up TRIM

FUEL L__________ R__________

TOD______________________________

DIVE 450 Kts 12M

CLOVERLEAF

BARREL ROLL

IMMELMAN

Level at 20 M inbound to VOR

200 Kts F FLOW____________

250 Kts F FLOW____________

300 Kts F FLOW____________

HIGH CONE

240 Kts. Gear Flaps Dive Brakes

1 g stall

200 Kts.

STABILITY Check

STALL RIGHT TURN 190 Kts

Clean up A/C 275 Kts. turn to ILS

350 Kts. Speed Brakes Decelerate

ISL Gear, Flaps, D/C 170 Kts

TOD____

18.24

SINGLE ENGINE GO-AROUND

SINGLE ENGINE TOUCH AND GO

RE-ENTER

PITCH OUT

NO FLAP LANDING

TRIM CHANGES

TAXI

AFTER LANDING CHECK

SHUTDOWN

10.5 GENERAL TECHNIQUES

The cockpit evaluation can normally be made while getting cockpit time prior to the first flight. MIL SPEC 203E specifies the standard cockpit arrangement for the various types of aircraft in considerable detail and should be used as a guide in making the cockpit evaluation. However, a summary of some of the points to note may prove helpful. These include: ease of entry, comfort, adjustment of seat and controls, location of basic flight instruments, size and legibility of instruments, accessibility of switches and controls, ease of identification of switches and controls, location and identification of emergency switches and controls, methods of escape (both on the ground and airborne), and general impression of cockpit layout.

Several points should be observed and recorded during the start and while preparing the aircraft for flight. These should be weighed against the aircraft's mission requirements. An all-weather interceptor, for example, should be capable of fast, uncomplicated starts to meet its alert and scramble requirements. Starts for other types may not be so critical; however, no starting procedure should be unnecessarily complex or confusing. Evaluation of the start should include: complexity of start, time to prepare for start, time to start, external power and ground support equipment required, ground personnel required, and time from start to taxi. The system checks and normal procedure requirements from start to taxi should also be evaluated.

An evaluation of the ground handling characteristics can be made while taxiing. How much power is required to start moving and to taxi at the desired speed? Is braking action required to prevent taxiing too fast? Is the visibility adequate? Is the directional control satisfactory? Is the braking action satisfactory? What is the turning radius of the aircraft? Does the aircraft require any auxiliary equipment such as removable wheels, escape ladders, etc? Is there any problem with clearing obstacles with any part of the aircraft?

The takeoff distance may be difficult to determine without assistance from outside personnel, but an estimate should be made using whatever aid is available such as runway distance markers. Use the recommended takeoff procedure, don't try to make a maximum performance takeoff. The normal ground roll will be of more interest than the minimum possible. Some of the other points to note in the takeoff include: ability of brakes to hold in military power, directional control during ground roll, rudder effective speed, nose lift-off speed, visibility after nose up and during initial acceleration and climb, force required to raise nose, any over-controlling tendencies, airborne speed, adequacy of recommended takeoff trim settings, time to retract gear and flaps, trim changes with retraction of gear and flaps, any tendency to exceed gear or flap speed limitations, effectiveness of trimming action during acceleration, and any distracting noises or vibrations.

The in-flight techniques differ very little from the techniques used in flying quantitative tests. However, it generally is not necessary to be as precise in holding airspeeds and altitudes. To do so would only waste time because differences caused by variations of a few hundred feet in altitude or a few knots in airspeed will not be qualitatively discernible so far as qualitative information is concerned. This is not an indorsement for being lax in flying

the aircraft. Just don't waste time with preciseness that will not contribute to the evaluation of the aircraft. If speeds are critical, such as in the climb or in the pattern, then hold them as closely as possible. Otherwise, use good judgment in determining how close to an aim condition it is necessary to be and fly accordingly.

If the climb rate of the aircraft is relatively slow, it may be possible to get some stability information in the climb, i.e., stick pulses, sideslips, etc. Most present day fighter aircraft climb so rapidly that this may not be practical. If so, just record climb performance information (time, fuel, and indicated speed) at intervals of approximately 5,000 feet. Start the time at brake release. Intercept the climb schedule at a comfortable altitude and attempt to fly the recommended schedule precisely. Continue the climb only as far as is compatible with the objective of the flight. Unless climb performance is of primary importance, this will probably be to the altitude selected for the first series of investigations. General aircraft characteristics should be observed during the climb. How difficult is it to maintain the recommended climb schedule? Are the control responses smooth?; too fast?; too slow?; compatible? Is visibility adequate? Is there any buffet?; vibration or excessive noise? Are the ventilation and pressurization systems satisfactory? Are the normal procedures required complicated or excessively distracting? If dampers or other artificial stability devices are provided, check the applicable characteristics with them "ON" and "OFF".

The altitude selected for the first series of stability investigations may be at the tropopause since this is where the aircraft will probably have its best performance. However, if the designed operating altitude is considerably above this level it may be advisable to select an altitude at or near the aircraft's operating altitude. The stability maneuvers performed will be essentially the same at all the altitudes and airspeeds selected. These should be sufficiently spaced to assure descernible qualitative differences in the aircraft's characteristics.

The stability characteristics investigated should include: longitudinal and directional static stability, longitudinal and directional dynamic stability, aileron rolls, and maneuvering flight at several different airspeeds and altitudes. An investigation of the transonic trim changes also should be made. All the dynamic characteristics should be checked with the stability augmentation devices, if any, both "ON" and "OFF". With proper planning these investigations can be made in a minimum amount of time. The longitudinal static stability can be checked while accelerating to V_{max}, for instance. Once at V_{max}, the aircraft can be trimmed for approximately hands-off flight and the static directional stability checked by entering a steady sideslip out to maximum rudder deflection (if the aircraft is cleared to that limit). The periods of the dynamic modes can be timed using a stop watch or counting the seconds. Estimate the cycles to damp completely or to one-half amplitude as the case may be for all the modes.

Approach the aileron rolls cautiously. Make several partial deflection rolls before making any full deflection rolls. The time to reach 90 degrees of roll and the time to roll 360 degrees can be estimated using a stopwatch or again by counting the seconds. It is advisable to make rapid reversals of

ailerons and other rolling maneuvers if these can be expected in operational use of the aircraft. The rolling characteristics should also be checked in accelerated flight as well as 1 g flight.

After completion of investigations at V_{max}, a windup turn to limit load factor can be made to check the maneuvering stability of the aircraft. Then zoom back to the original altitude and repeat these investigations at the second airspeed. The other altitudes and airspeeds can be checked in the same manner. Any differences resulting from the altitude or speed changes should be noted.

Stalls should be approached with caution if the aircraft is cleared for such a maneuver, and investigated in all configurations and types of entry. Determine the approximate stall warning margin, what defines the warning and the stall, and the aircraft characteristics in the stall and the recovery. If possible, determine the best method of breaking the stall and altitude loss in recovery from several points in the stall.

If possible, check the tactical mission capability of the aircraft. Simulated dive bombing runs or labs maneuvers could be made for a strategic fighter; for example. All the information obtainable will be helpful in writing an accurate and comprehensive report.

Fly the traffic pattern as recommended and, if fuel permits, make a go-around on the first pass. Note the power response, power required in the pattern, airspeed control and sink rate, trim changes with gear and flap extension, trimming action, buffet with gear extension, and general aircraft feel in the pattern. On the go-around, recheck the trim changes with gear and flap retraction and with drag device reaction. Don't forget to look at engine out characteristics if time and fuel permit. On the first landing in the aircraft it is probably not advisable to attempt to get the minimum landing roll. Make a normal touchdown and use normal braking action (use the drag chute if provided). Note the touchdown speed, the effects of any crosswind, directional control, nose lowering speed, etc. As with the takeoff, the normal landing roll is of more importance than the minimum possible.

While taxiing back to the parking area, review the flight, re-evaluate the cockpit, and attempt to determine whether the aircraft will perform its design mission and is safe and comfortable to fly. The opinion with everything fresh in mind is probably the most accurate possible. Continue this review of the flight immediately after leaving the aircraft. Put everything remembered about the flight and the impressions of the aircraft down on paper. Do this immediately and before talking to anyone about the airplane or the flight. Waiting or discussing points with other people may alter first hand impressions or cause important aspects of the flight to be forgotten.

10.6 DATA REDUCTION

10.6 The data reduction will consist of writing a comprehensive report on everything learned about the aircraft. A narrative form is normally used for qualitative reports. Comparisons with other aircraft can be used to assist in describing the aircraft. Care should be taken however, to insure that only aircraft familiar to most readers are used for comparison. Otherwise the comparison will mean nothing to them.

Keep in mind the purpose of the qualitative evaluation while writing the report. Mere figures are normally not enough to describe the stability of the aircraft, particularly on a qualitative evaluation since the data obtained are very limited. Analyze the aircraft characteristics in light of its ability to perform its design mission, give opinions of the aircraft's ability to do the job and support these opinions with the facts obtained on the evaluation flights. Comment on anything personnally disliked but be objective in condemning any shortcomings. Recommendations for specific changes in the aircraft are to be included in the report. The exact manner in which the aircraft should be fixed should not be specified or recommended. The test pilot's job is to evaluate the existing hardware and state what should be changed. It is then the manufacturer's responsibility to determine how to make the necessary changes.

APPENDIX A

SPINNING REQUIREMENTS FOR AIRPLANES

1. SCOPE

This specification contains the spinning requirements for piloted airplanes.

1.2 Application

For all airplanes proposed or contracted for subsequent to the effective date of this specification, the spinning characteristics shall be demonstrated in accordance with the provisions contained herein unless specific deviations are authorized by the procuring activity. Additional requirements for spinning of particular airplanes may be specified in the contract.

1.3 Classification

For purposes of this specification, airplanes shall be divided into the categories given below:

Category 1 - Liaison and observation airplanes, and other light airplanes specifically designated by the procuring activity.

Category 2 - Primary training airplanes.

Category 3 - All other high-maneuverability types such as fighter, interceptor, general purpose attack, and trainers for these airplanes, except those classified in Category 3a.

Category 3a - Airplanes of the type of Category 3 specifically designated by the procuring activity. (These aircraft will be judged according to their missions. Normally, aircraft used solely as all-weather interceptors and low-load factor airplanes used for tactical ground support will fall into this category.)

Category 4 - All low-maneuverability types such as horizontal bomber, cargo, transport, patrol, early-warning, and trainers for these airplanes.

2. APPLICABLE DOCUMENTS

2.1 The following specifications, standards, drawings, and publications, of the issue in effect on date of invitation for bids, form a part of this specification:

SPECIFICATIONS

Military

MIL-T-7378	Tank, fuel, aircraft, external, removable, general specifications for
MIL-F-8785	Flying qualities of piloted airplanes.

(Copies of specifications, standards, drawings, and publications required by contractors in connection with specific procurement functions should be obtained from the procuring activity or as directed by the contracting officer.)

3. REQUIREMENTS

3.1 General

With the exception noted in 3.5, it shall be possible at any combination of weight and loading normally attainable in flight to recover readily from incipient and fully developed spins without exceeding either the limit airspeed or the limit normal acceleration of the airplane. Compliance with this requirement normally may be shown by performing the spin tests of table I, section 4.

3.1.2 Right and Left Spins

All required spins shall be performed in both directions except in the cases noted below:

3.1.2.1 At the discretion of the procuring activity and the pilot, if early spins in both directions appear identical, the remaining spins need be performed in one direction only.

3.1.2.2 At the discretion of the procuring activity, if early spins demonstrate consistently poorer recoveries in one direction, the most critical direction only may be used for the remaining spins.

3.1.3 Configurations

The following terms used in this specification have the meanings given below:

a. Aft Loading[1] - The normal service loading which results in a combination of weight and center of gravity producing minimum static longitudinal stability (ordinarily the lightest gross weight at which the most aft center of gravity can be obtained in a given configuration at a normal service loading.)

b. Fuselage Loading[1] - That normal service loading which results in the minimum algebraic value of the inertia yawing moment parameter $(I_x - I_y)/mb^2$. (I_x and I_y are moments of inertia about the x and y body axes, respectively; m is mass of airplane; b is wing span.)

c. Wing Loading[1] - That normal service loading which results in the maximum algebraic value of the inertia yawing moment parameter $(I_x - I_y)/mb^2$.

3.1.4 Standard Recovery Procedure

For erect spins, standard recovery procedure shall consist of briskly reversing the directional control to its full deflection (or maximum force limitation) against the spin, and returning lateral control to neutral, followed approximately one-half turn later by a brisk forward movement of the longitudinal control, as required. For inverted spins, standard recovery procedure shall consist of briskly neutralizing all three controls. Once applied, the applicable control positions for recovery shall be held until the spin stops; recovery to level flight shall then be made in the normal manner.

[1] On airplanes equipped to carry external stores, all stores normally carried except Types II and IV (expendable) tanks shall be considered in determining the loadings given above to the extent that loads higher than structural limits will not be imposed during the spin tests. When stores cause an appreciable change in the aerodynamic characteristics of the airplane, additional or substitute loading may be required. External stores are defined in Specification MIL-T-7378.

3.1.5 Control Forces in Recovery

Control forces for recovery shall not exceed the values specified in Specification MIL-F-8785 and repeated below for reference:

Rudder (directional control)	250 pounds
Elevator (longitudinal control)	75 pounds
Aileron (lateral control)	35 pounds

3.2 Category 1 Airplanes

It is preferred that airplanes in this category be incapable of spinning. In no case shall there be any tendency to spin in moderately smooth air unless essentially full pro-spin control deflections are applied in the stall.

3.2.1 In the required clean configuration spins of table I, section 4, the airplane shall recover within 1 1/2 turns after initiation of action to start recovery; recovery from landing configuration spins shall be within one turn. Standard recovery procedure is desired; if nonstandard recovery procedure is required, the optimum recovery procedure shall be determined. Special emergency spin-recovery devices are not acceptable for service use in Category 1 airplanes.

3.2.2 Altitude loss during recovery in the required spins of table I shall not exceed 800 feet, measured from the initiation of action to start recovery until regaining level flight.

3.3 Category 2 Airplanes

Airplanes in this category shall be capable of spinning; however, in no case shall there be any tendency to spin in moderately smooth air unless essentially full pro-spin control deflections are applied in the stall.

3.3.1 In the required clean configuration spins of table I, section 4, the airplane shall recover within 1 1/2 turns after initiation of action to start recovery; recovery from landing configuration spins shall be within one turn. These recoveries shall be made using the standard recovery procedure specified in 3.1.4.

3.4 Category 3 and 3a Airplanes

It is preferred that airplanes in these categories be incapable of spinning.

3.4.1 In the required clean configuration spins of table I, section 4, the airplane shall recover within two turns after initiation of action to start recovery; recovery from landing configuration spins shall be within one turn. Standard recovery procedure is desired; but in any case, the optimum recovery procedure shall be determined.

Special emergency spin-recovery devices will be accepted for service use only if it can be shown to the satisfaction of the procuring activity that they are necessary and produce less performance penalty than redesign of the airplane to meet the spinning requirements.

3.4.2 The spin-recovery requirements of 3.4.1 shall apply only to the low altitude spins and not to the high altitude spins of table I, section 4. Characteristics of the high altitude spins, however, shall be reported in accordance with 4.5.

3.5 Category 4 Airplanes

These airplanes are not normally required to be spun, and no recovery requirements are specified.

Particular care shall be taken in the design of these airplanes that inadvertent spins are unlikely.

3.6 Government-Loaned Property

When provided for in the contract, spin recovery parachutes, release mechanisms, and cockpit controls for the chutes for use in the tests required herein will be furnished by the Government on loan to the contractor upon his request.

4. QUALITY ASSURANCE PROVISIONS

4.1 Acceptance Test

All of the tests required herein for the spin testing of airplanes are classified as acceptance tests. The necessary sampling techniques and testing methods are specified in this section.

4.2 Test Conditions

Spins shall be conducted in the configurations and under the conditions specified in table I.

4.3 Sampling Tests

4.3.1 One airplane representative of the production configuration shall be subjected to the tests specified in table I. Tests shall be conducted by the contractor under the supervision of the procuring activity.

4.3.2 Entry into the required spins may be made with any necessary use of the controls but, once started, the spin shall thereafter be maintained with the directional and longitudinal controls in the full pro-spin position and the lateral control neutral. Except as noted in table I, the erect, clean-configuration spins shall be repeated (but the requirements for number of turns and altitude loss in recovery need not apply) with lateral control in the steady spin held full with the spin and full against the spin. Characteristics of all spins shall be reported in accordance with 4.5.

4.3.3 If no spins result using the entry procedures of table I, or if for some reason such as engine flame-out during stall approach such entries are not feasible, other entries (such as from a zoom or the top of a loop) shall be attempted. The requirements of section 3 shall then apply regardless of the type of entry.

4.4 Emergency Spin-Recovery Provisions

On all spins to show compliance with this specification, spin chutes, spin-recovery rockets, or other devices acceptable to the procuring activity shall be provided to effect recovery in case of failure of other means. Such devices shall be designed to give sufficient forces and moments for recovery in two turns when controls remain in the full pro-spin position.

4.5 Spin Report

After completion of the test required herein, the contractor shall submit a report to the procuring activity.

4.5.1 The report shall contain the following information for each test: gross weight, general arrangement of loading, center of gravity, moments of inertia, locations of principal axes, gear and flap positions, positions of cowl flaps, etc., starting altitude, method of entry, power condition, turns of spin executed before applying recovery controls, nature of the steady spin, time per turn, altitude loss per turn, control positions and maximum

forces during recovery, and altitude loss in recovery.

4.5.2 The report shall contain the following information for each test: time histories (starting before initiation of spin and continuing through recovery to level flight) of control positions and forces, airspeed, altitude, normal and longitudinal acceleration, angles and rates of pitch, roll and yaw, angles of attack and sideslip.

4.5.3 The report shall describe the emergency spin-recovery device including load calculations.

4.5.4 The report shall include a description of instrumentation.

4.5.4.1 Satisfactory instrumentation shall be provided: for example, gyros not subject to gimbal lock, satisfactory calibration of airspeed, altitude, angle of attack, and sideslip instrumentation for lag and interference effects, satisfactory count of number of turns during spin and recovery.

5. PREPARATION FOR DELIVERY

5.1 Marking and method of delivery of the spin report shall be as contractually required for flight test reports.

6. NOTES

6.1 Intended Use

This specification contains the spinning requirements for piloted airplanes and shall form one of the basis for determination by the procuring activity of airplane acceptability. The specification shall serve as design and flight test requirements, and as criteria for use in spin calculations and spin tunnel tests and evaluations.

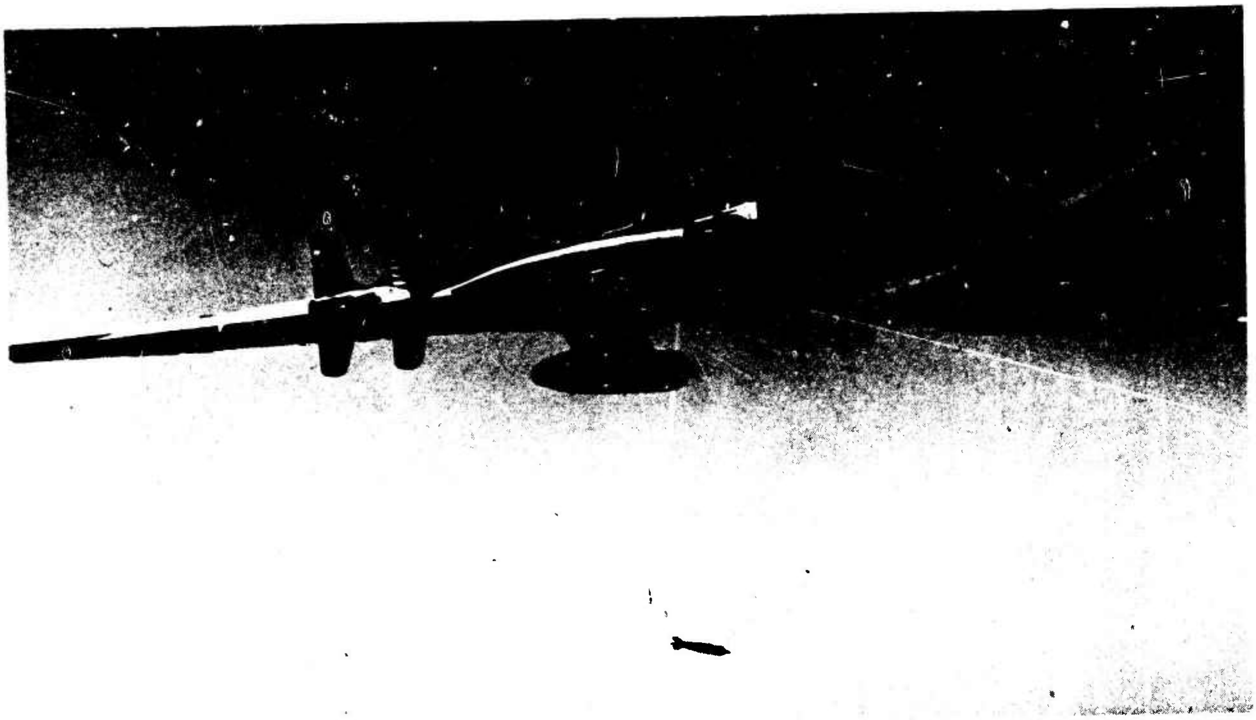
PATENT NOTICE: When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

TABLE 1
REQUIRED SPIN TESYS

TABLE 1

Type of Spin	Entry	Loading	Number of Turns Before Start of Recovery			
			Category 1	Category 2	Category 3	Category 3a
Erect spin, clean configuration	From power-off[1] straight erect stall	Aft Fuselage Wing	5 5 5	5 5 5	5 - -	3 - -
	From 2.5 "g" turn; power (thrust) for constant altitude turn on through first turn	Aft Fuselage Wing	5 - -	5 - -	5 5 5	3 3 3
Erect spin, landing configuration	From power off[1] straight erect stall	Aft	1	1	1	-
Inverted spin clean configuration	From power off[1] straight inverted stall	Aft	-	2	2	2
High altitude erect spin, clean configuration	From straight erect stall at combat ceiling; power (thrust) for level flight on through first turn.	Aft Fuselage Wing	- - -	- - -	5* 5* 5*	5* 5* 5*

1 throttle closed or at flight idle setting.
* one spin only, with ailerons neutral; required only if combat ceiling is more then 10,000 feet above starting altitude for other spins.



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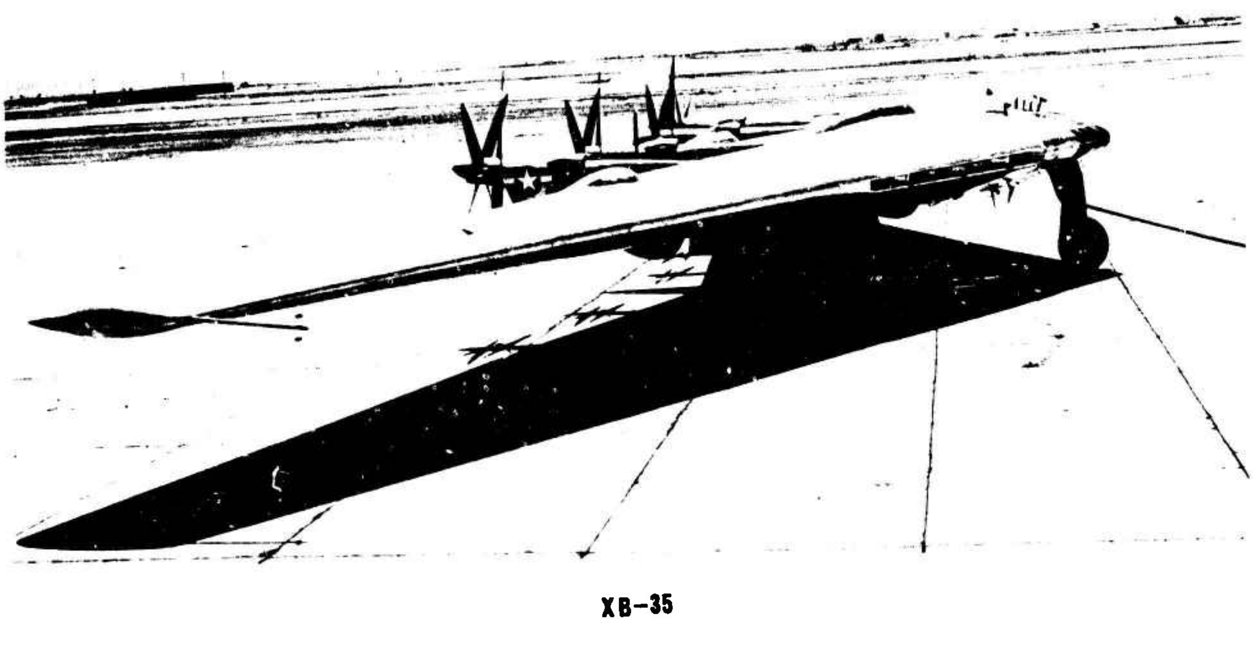

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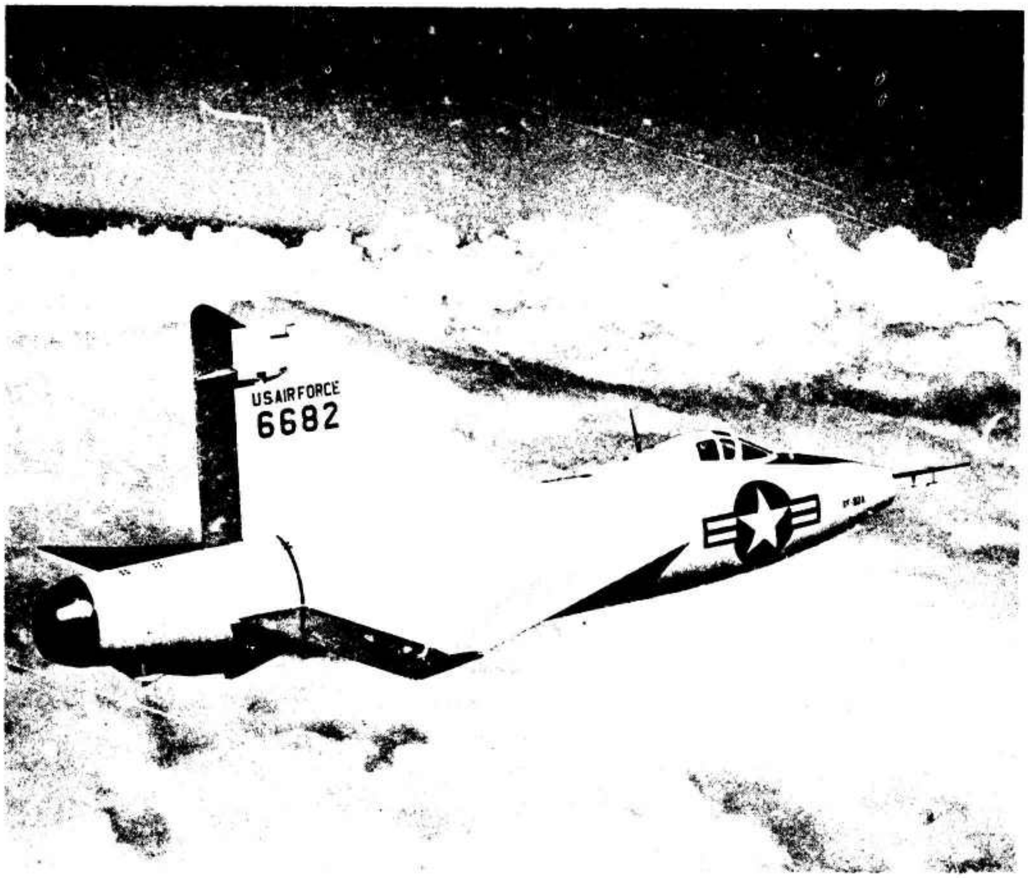

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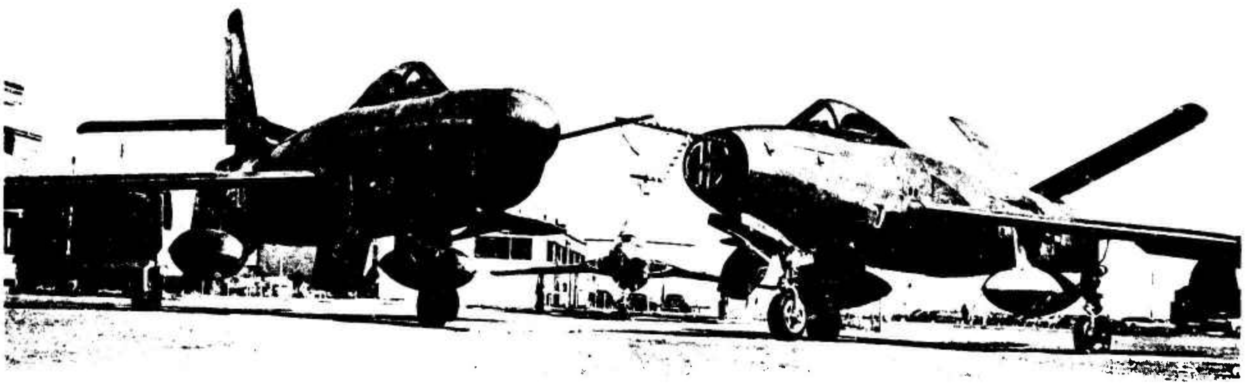

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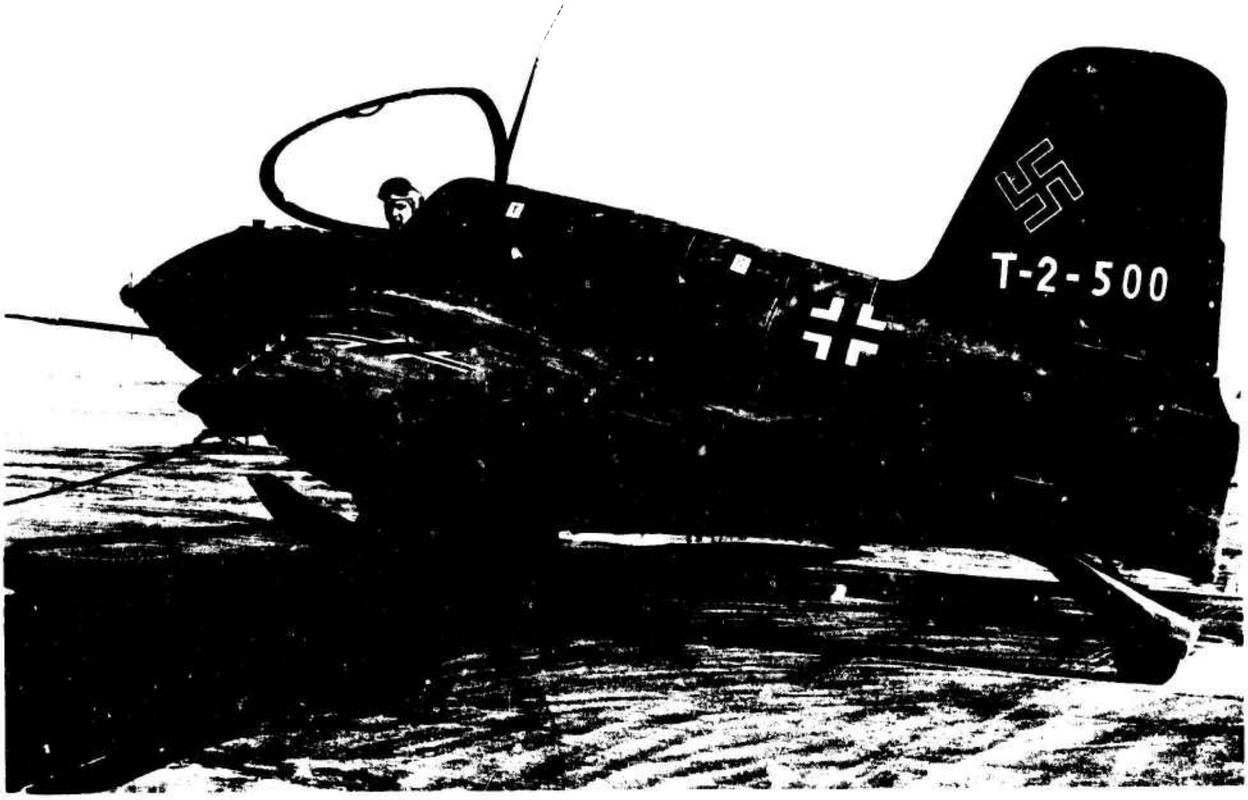

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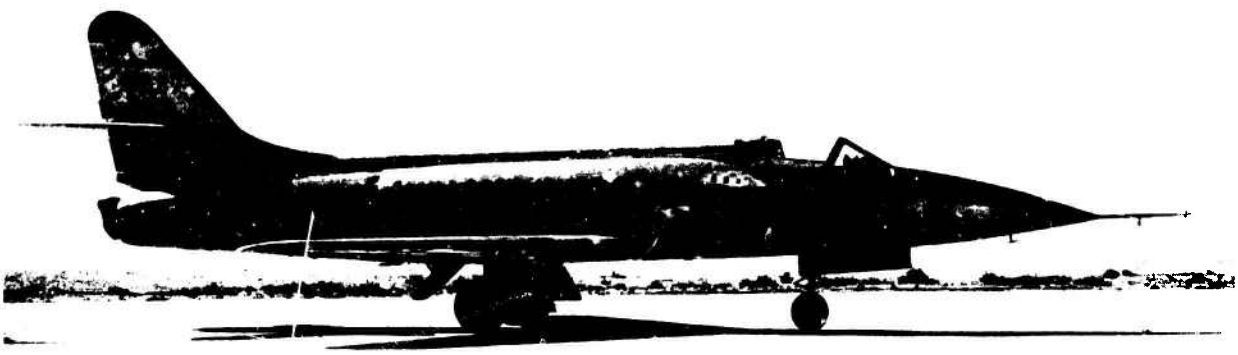

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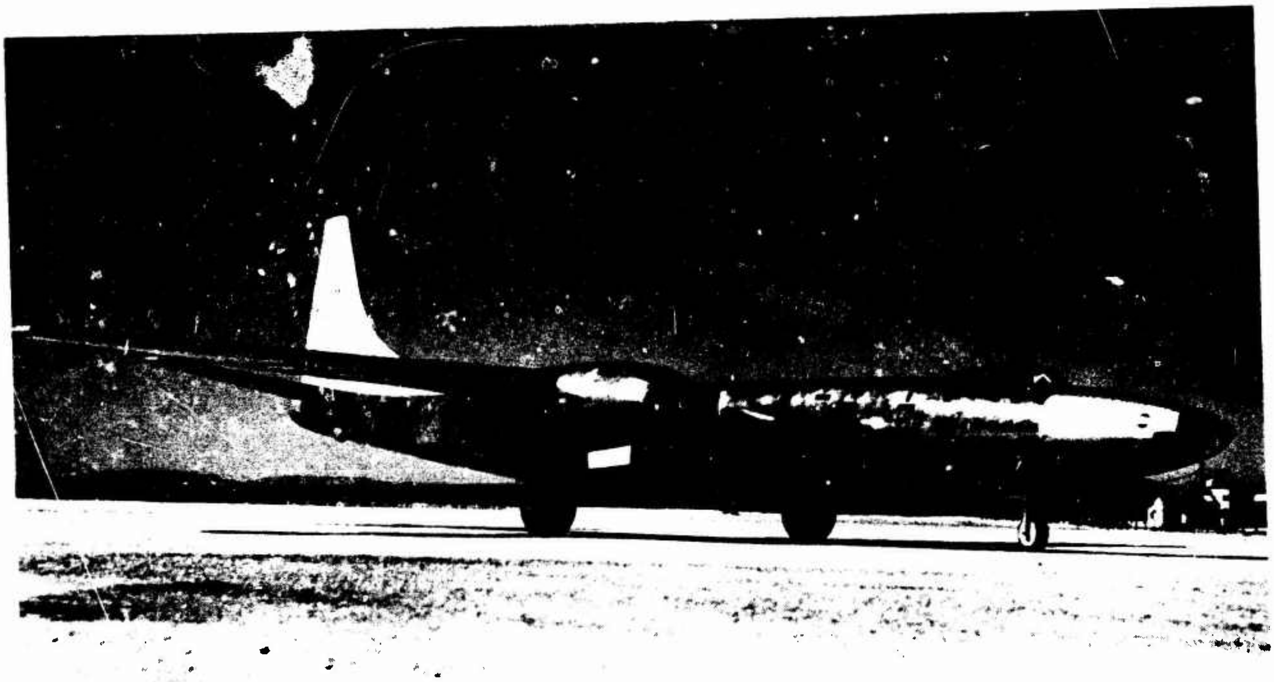

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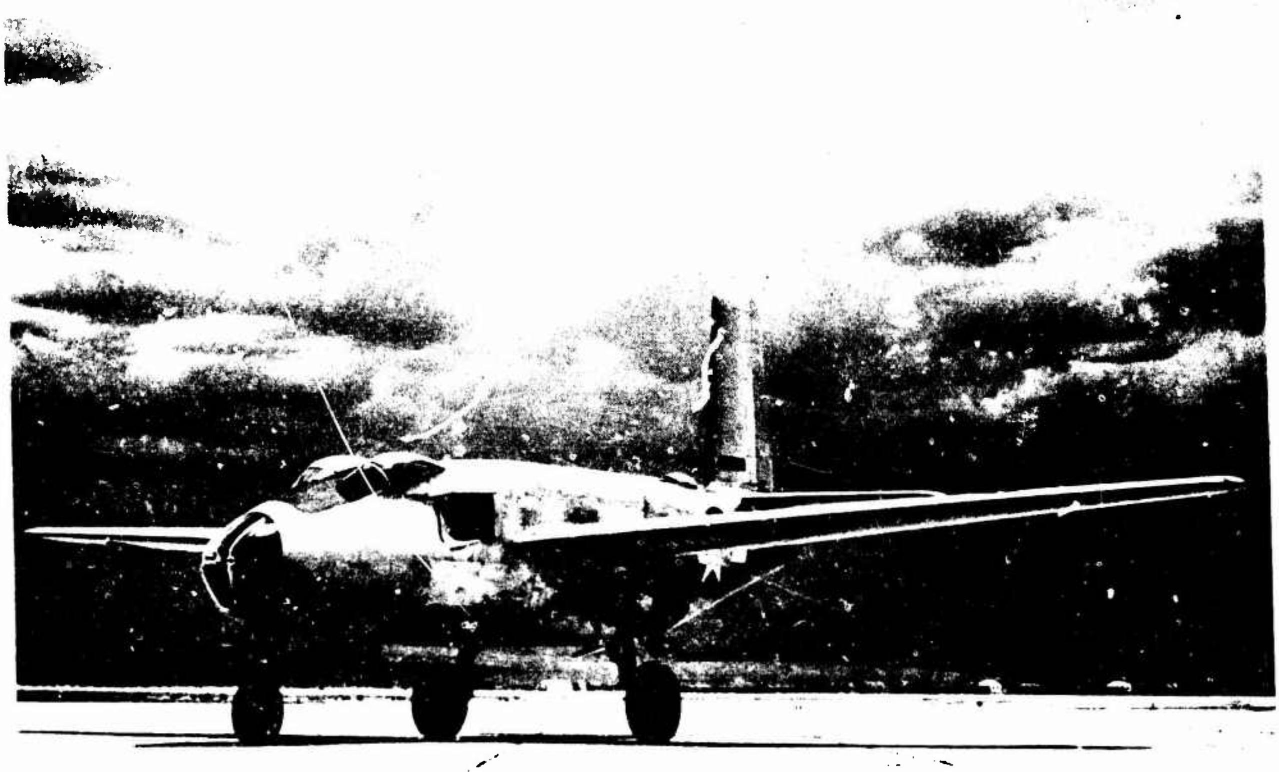

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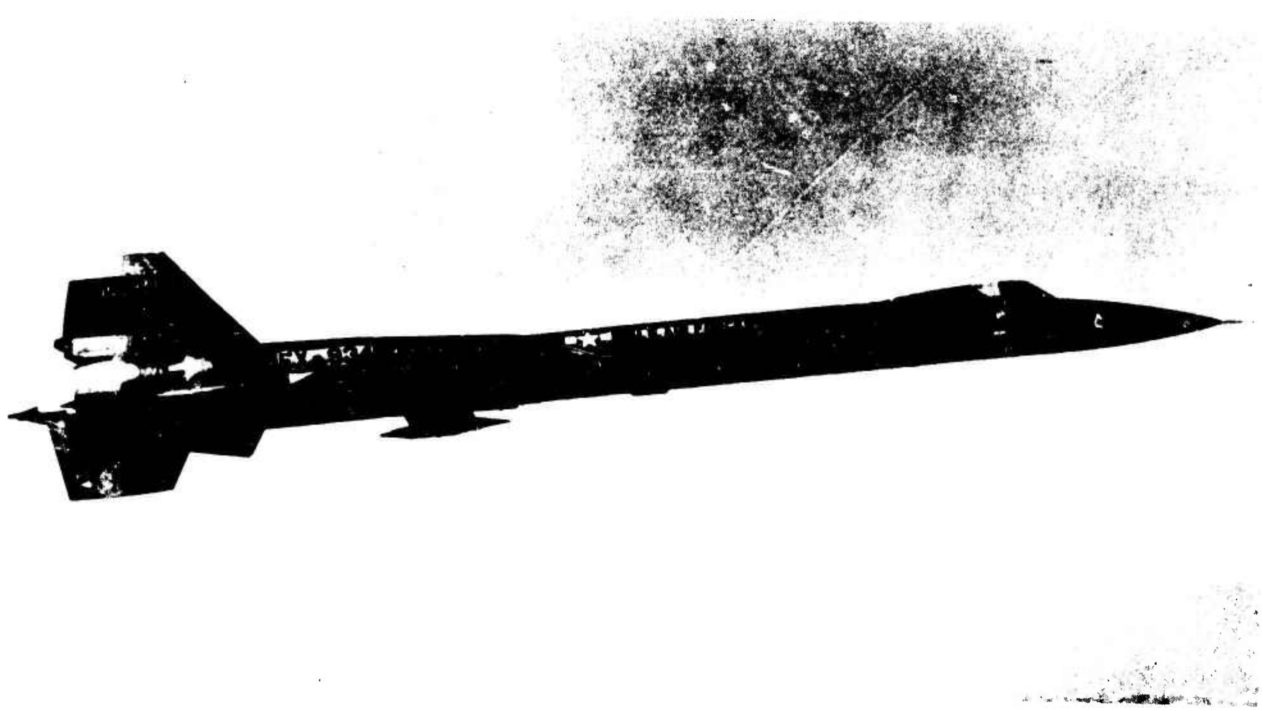

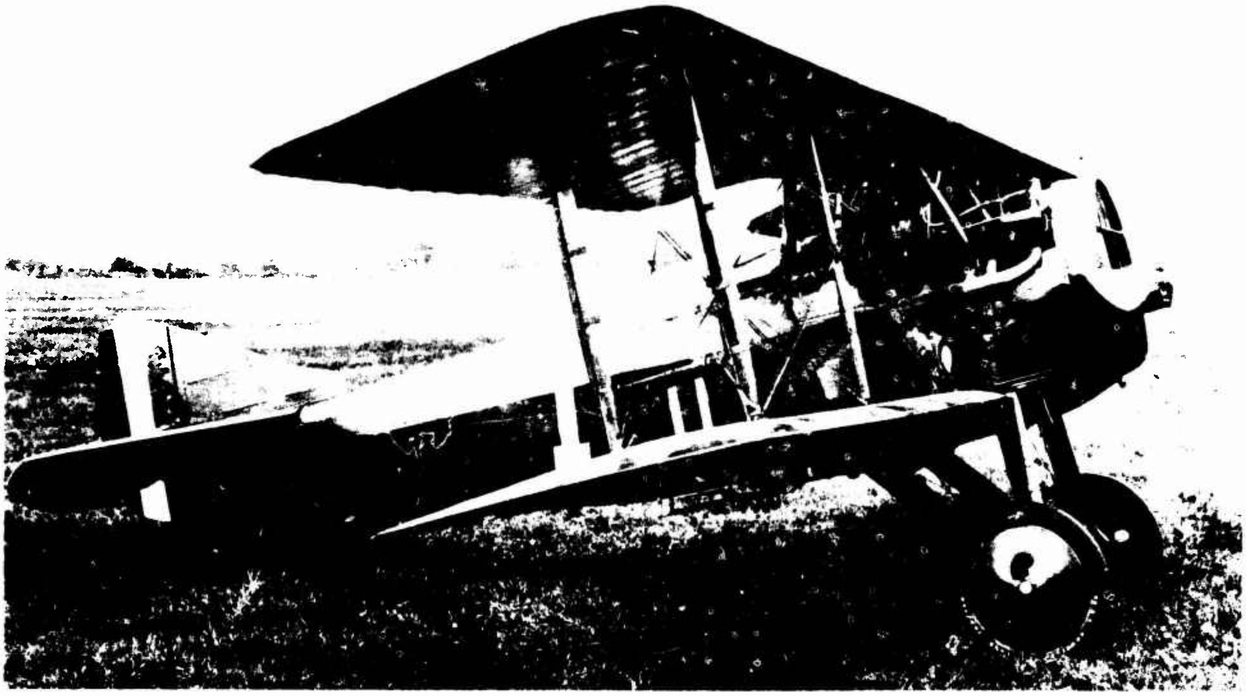

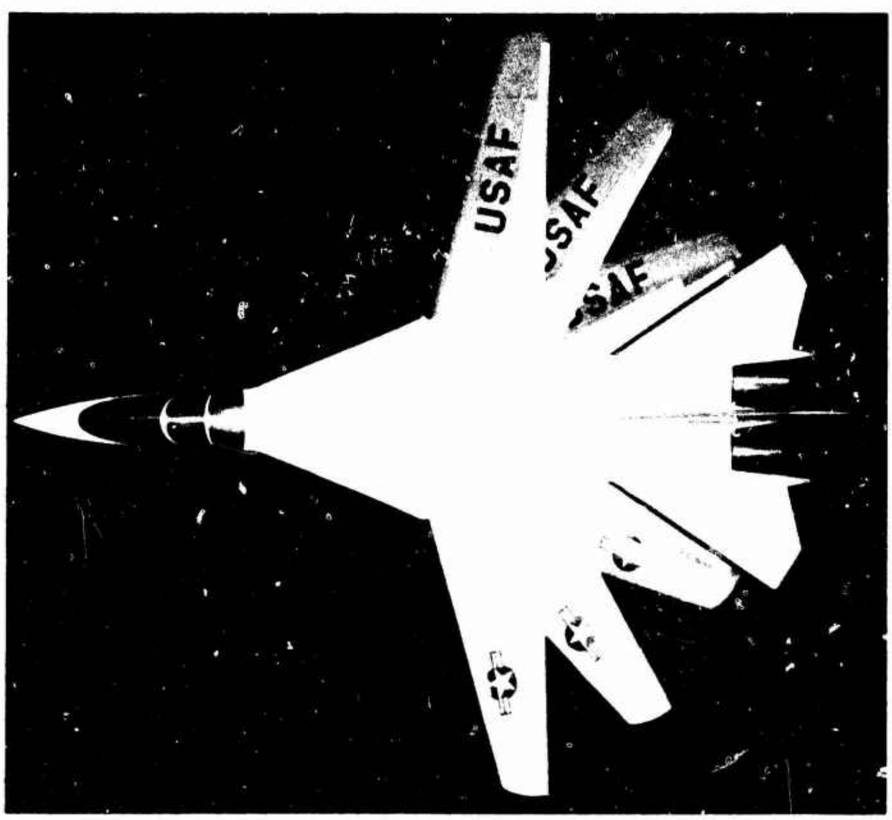
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CHAPTER 1

INTRODUCTION

1.1.0 This book is written as a general class room text for the theoretical approach to Stability and Control in the course curriculum of the USAF Aerospace Research Pilot School.

The theoretical discussion will, of necessity, incorporate certain simplifying assumptions. These simplifying assumptions are made in order to make the main elements of the subject more clear. The equations developed are by no means suitable for even preliminary design of modern aircraft, but the basic method of attacking the problem is valid. Now that analog and digital computers are available, the aircraft designers' more rigorous theoretical calculations, modified by data obtained from the wind tunnel, often give results which closely predict the flying qualities of new airplanes. Neither the theoretical nor the wind tunnel results are infallible. Therefore, there is still a valid requirement for the test pilot in the development cycle of new aircraft.

1.2.0 It is important at the beginning of the stability and control theory to redefine some of the terms to be used. The student will find it a great asset to be able to recall at a glance the definition represented by the symbols commonly used in this course.

c (M.A.C.) Mean Aerodynamic Chord: The chord of an imaginary airfoil which would have force vectors throughout the flight range identical with those of the actual wing.

c.p. Center of Pressure: The point on the chord of an airfoil, actual or prolonged, which is the intersection of the chord and the line of action of the resultant force.

C Chordwise Force: The component of the resultant aerodynamic force that is parallel to the aircraft reference axis.

N Normal Force: The component of the resultant aerodynamic force that is perpendicular to the aircraft reference axis.

L Lift: The component of the resultant aerodynamic force perpendicular to the relative wind. It must be specified whether this applies to a complete aircraft or to parts thereof.

D Drag: The component of the resultant aerodynamic force parallel to the relative wind. It too must be specified whether this applies to a complete aircraft or to parts thereof.

R Resultant Aerodynamic Force: The vector sum of the lift and drag forces on an airfoil or airplane.

a.c. Aerodynamic Center: A point located on the wing chord (approximately one quarter of the chord length back of the leading edge) about which the moment coefficient is practically constant for all angles of attack.

HM Hinge Moment: A moment which tends to restore or move a control surface to or from a condition of equilibrium.

Stability Derivatives: Nondimensional quantities expressing the variation of the force or moment coefficient with a disturbance from steady flight.

$$C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha} \; . \qquad (1.1)$$

$$C_{n_\beta} = \frac{\partial C_n}{\partial \beta} \; . \qquad (1.2)$$

Stability Parameters: A quantity that expresses the variation of force or moment on aircraft caused by flight as well as a disturbance from steady flight.

$$M_u = \frac{\rho U S c}{I_y} \left[C_m + \frac{U}{2} \frac{\partial C_m}{\partial u}\right] \; . \qquad (1.3)$$

$$L_q = \frac{\rho S c}{4m} C_{L_q} \; . \qquad (1.4)$$

I Moment of Inertia: With respect to any given axis, the "Moment of Inertia" is the sum of the products of the mass of each elementary particle by the square of its distance from the axis. It is a measure of the angular acceleration characteristics of a body or section about a given axis.

Static Stability: That characteristic of an aircraft which causes it, when its condition of steady flight is disturbed, to develop moments and forces which tend to restore the aircraft to its original condition of steady flight.

Dynamic Stability: That characteristic of an aircraft which causes it, when disturbed from a state of steady flight, to return to its original flight attitude because as the airplane diverges from its original steady flight, pressures are set up in a direction which tend to return the airplane (sometimes after a series of oscillations) to its original flight attitude.

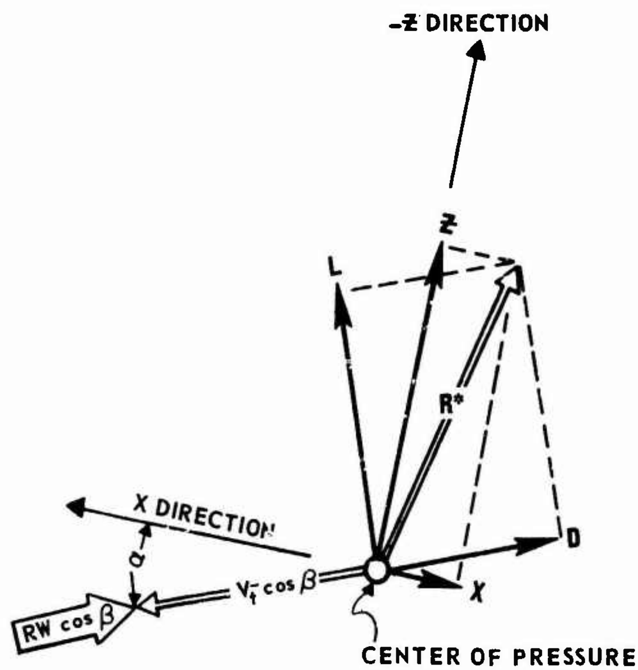
Neutral Stability: A neutrally stable airplane is one which, if once disturbed from a state of steady flight, will not return to its original flight attitude but may seek any new flight attitude and state of steady flight. Neutral statically, the aircraft would have no tendency to move from its disturbed condition. Neutral dynamically with static stability, the aircraft would sustain oscillation started by a disturbance.

Dynamically Unstable: An aircraft oscillation whose amplitude increases continuously until an attitude is reached from which there is no tendency to return toward the original attitude, the motion becoming a steady divergence.

Statically Unstable: A characteristic of an aircraft such that when disturbed from steady flight its tendency is to depart further or diverge from the original condition of steady flight.

Flight Control Sign Convention: Any control movement or deflection that causes a positive movement or moment on the airplane shall be considered a positive control movement. This sign convention does not conform to the convention used by NASA and some reference text books. This convention is the easiest to remember and is used at the Flight Test Center, therefore, it will be used in the School.

Degrees of Freedom: The number of paths that a physical system is free to follow.



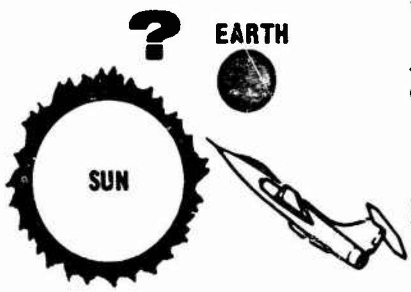
VIEW IN THE VEHICLE PLANE OF SYMMETRY

Figure 1.1

1.3.0 COORDINATE SYSTEM RELATIONSHIPS

1.3.1 Coordinate Systems There are many coordinate systems that are useful in the analysis of vehicle motion. In accordance with general practice, we will always define ours to be right hand and orthogonal.

1.3.1.1 True Inertial Coordinate System



Location of origin: unknown
Approximation for space dynamics: the center of the sun.

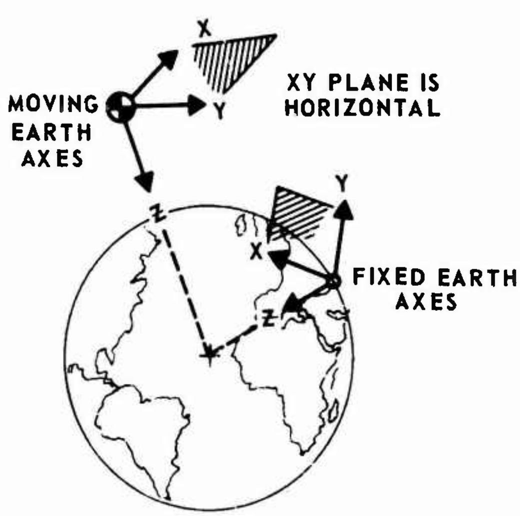
Approximation for aircraft: the center of the earth.

Unit Vectors $\vec{e}_x$, $\vec{e}_y$, $\vec{e}_z$

Figure 1.2

1.3.1.2 Earth Axes

There are two Earth Axis Systems, the fixed and the moving.



Location of Origin

Fixed System; arbitrary location
Moving System; at the vehicle cg

The Z axis ($\vec{e}_z$) points toward center of the earth.

The XY Plane (containing $\vec{e}_x$, $\vec{e}_y$) is parallel to local horizontal.

The Orientation of the X axis is arbitrary; may be North or on the initial vehicle heading.

Figure 1.3

1.3.1.3 BODY AXIS SYSTEMS These coordinate systems are fixed to the vehicle. There are many different types, e.g.,

General Body Axis System.
Stability Axis System.
Principal Axis System.
Wind Axis System.

The General Body Axis System frequently inherits the name "body Axes".

"BODY AXES"

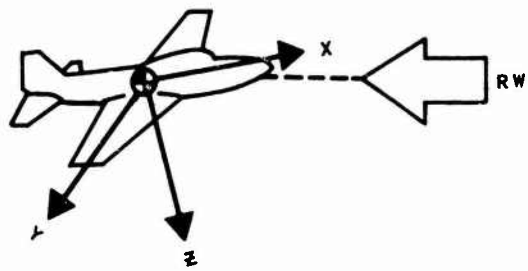


Figure 1.4

The Unit Vectors are $\vec{i}$ $\vec{j}$ $\vec{k}$.

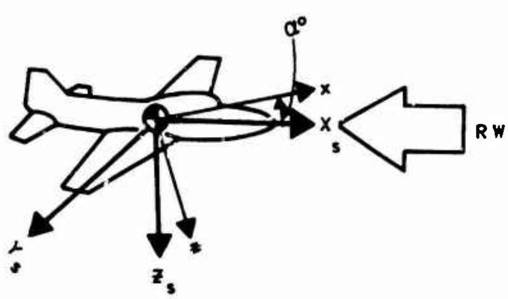
The Origin is at the cg.

The x z plane is in the vehicle plane of symmetry.

The positive x axis points forward along the vehicle horizontal reference line.

The positive z axis points downward toward the bottom of the vehicle.

"STABILITY AXES"



Y BODY = Y STAB
i.e., THE STABILITY XZ PLANE REMAINS IN THE VEHICLE PLANE OF SYMMETRY

Figure 1.5

The unit vectors are $\vec{i}_s$, $\vec{j}_s$, $\vec{k}_s$.

The origin is at the cg.

The positive x axis points forward coincident with the initial position of the relative wind.

The x z plane must remain in the vehicle plane of symmetry, hence this stability axis system is restricted to symmetrical initial flight conditions.

The positive z axis points downward toward the bottom of the vehicle.

1.3.2 VECTOR DEFINITIONS

The Equations of Motion describe the vehicle motion in terms of four vectors. The components of these vectors resolved along any body axis system are shown below.

1.3.2.1 $\vec{F}$ - Total Linear Force (Applied)

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k} .$$

1.3.2.2 G - Total Moment (Applied)

$$\vec{G} = G_x\vec{i} + G_y\vec{j} + G_z\vec{k} .$$

$$\vec{G} = \vec{G}_{\text{aerodynamic}} + \vec{G}_{\text{other sources}} .$$

$$\vec{G}_{\text{aerodynamic}} = L_{\text{aero}}\vec{i} + M_{\text{aero}}\vec{j} + N_{\text{aero}}\vec{k} .$$

$$\vec{G}_{\text{aerodynamic}} = \mathcal{L}\vec{i} + \mathcal{M}\vec{j} + \mathcal{N}\vec{k} .$$

Note: Control deflections that tend to produce positive $\mathcal{L}$, $\mathcal{M}$, or $\mathcal{N}$, are defined at ARPS to be positive.

1.3.2.3 $\vec{V}_T$ - True Velocity

$$\vec{V}_T = u\vec{i} + v\vec{j} + w\vec{k} .$$

where u = forward velocity.
v = side velocity.
w = vertical velocity.

1.3.2.4 $\vec{\omega}$ - Angular Velocity

$$\vec{\omega} = p\vec{i} + q\vec{j} + r\vec{k} .$$

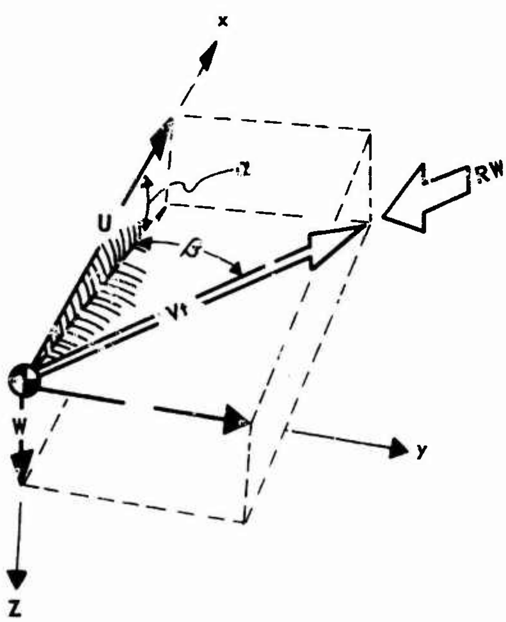
where p = roll rate.
q = pitch rate.
r = yaw rate.

1.8

1.3.3 POSITION OF THE RELATIVE WIND

We have already seen that the position of the relative wind in relation to the Stability Axis System is fixed by definition. The position in relation to two other axis systems leads to some necessary definitions.

In Relation to the "Body Axes"



α - Angle of Attack

$$\alpha = \tan^{-1}\left(\frac{w}{u}\right)$$

For small α and β

$$\alpha \approx \tan^{-1}\left(\frac{w}{V_T}\right) \approx \frac{w}{V_T}$$

β - Sideslip Angle

$$\beta = \sin^{-1}\left(\frac{v}{V_T}\right)$$

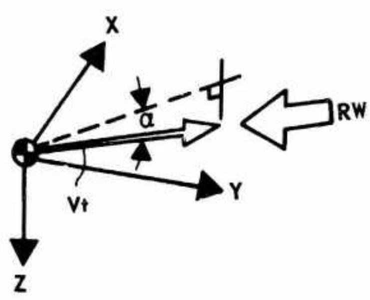
For small β

$$\beta \approx \frac{v}{V_T}$$

CAUTION - OTHER DEFINITIONS OF ARE POSSIBLE

Figure 1.6

In Relation to the Moving Earth Axes



γ = Flight Path Angle.

The angle between the velocity vector (V_T) and local horizontal. (Holds for space flight also).

Figure 1.7

1.3.4 Euler Angles - Transformation from the Moving Earth Axis System to the Body Axis System. The orientation of the body axis system (and hence the vehicle) with respect to the moving earth axis system is described by a sequenced multiple rotation involving Euler Angles. The sequence (YAW, PITCH, and ROLL) must be maintained.

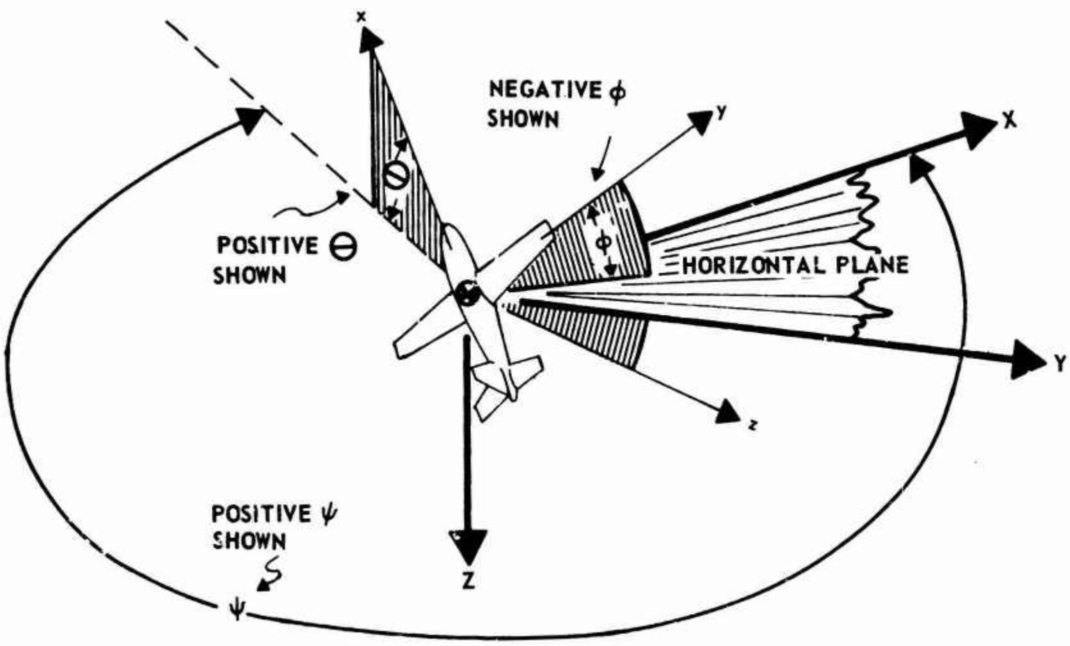


Figure 1.8

1.3.4.1 Yaw Angle - ψ The angle between the projection of x body axis onto the horizontal plane and the initial reference position of the X earth axis. (Yaw angle is the vehicle heading only if the initial reference is North).

1.3.4.2 Pitch Angle - θ The angle measured in a vertical plane between the x body axis and the horizontal plane (pitch angle is not necessarily the flight path angle α).

1.3.4.3 Bank Angle - ϕ The angle, measured in the y z plane of the body system, between the y body axis and the horizontal plane.

1.3.4.4 Angular Velocity Transformation The following relationships, derived by vector resolution, will be useful later in the study of dynamics.

$$p = \dot{\phi} - \dot{\psi} \sin \theta. \quad (1.5)$$

$$q = \dot{\theta} \cos \phi + \dot{\psi} \sin \phi \cos \theta. \quad (1.6)$$

$$r = \dot{\psi} \cos \phi \cos \theta - \dot{\theta} \sin \phi. \quad (1.7)$$

So that equations (1.5) through (1.7) might be better understood, consider an airplane with axes in the instantaneous position of the x_3, y_3, z_3, (Figure 1.9c) axes displaced from the steady flight axes by the Eulerian angles ψ, θ and ϕ. The instantaneous angular velocities are p, q and r, and the vectors representing these angular velocities are directed along the x_3, y_3 and z_3 axes respectively. The Eulerian angle ψ was a rotation about the z axis.

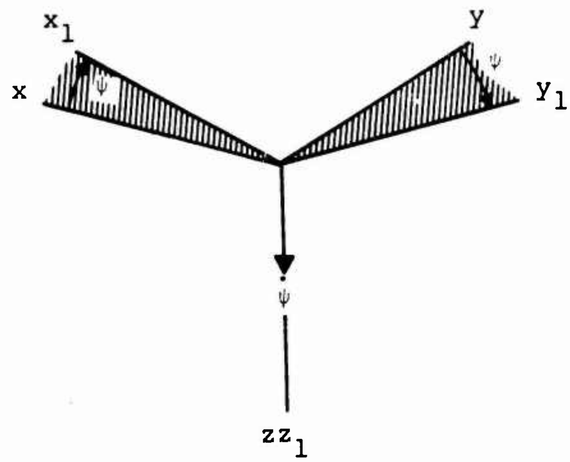


Figure 1.9a

Note that axes xyz are Orthogonal and $\dot{\psi}$ is a vector parallel to the z, z_1 axis. Axes x and y are displaced by ψ to positions x_1 and y_1. Next we pitch the axes system x_1, y_1, z_1 through an angle θ to a new position x_2, y_2, z_2. (See Figure 1.9b). This rotation is about the y_1 axis; therefore θ is a vector parallel to this axis.

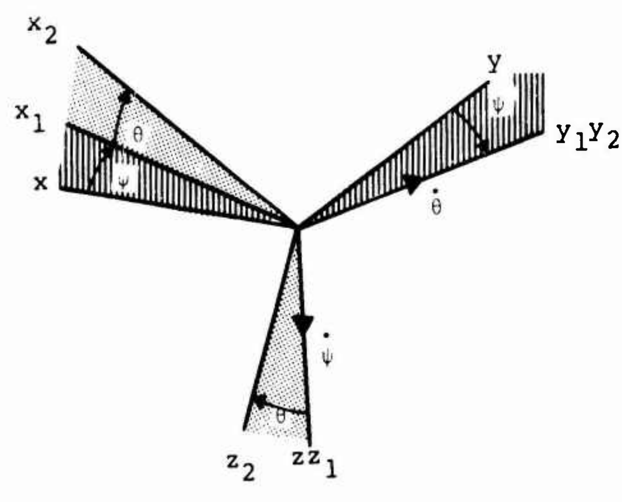


Figure 1.9b

The Euler angle ϕ is a rotation about the x body axis. (x_2 axis for our problem).

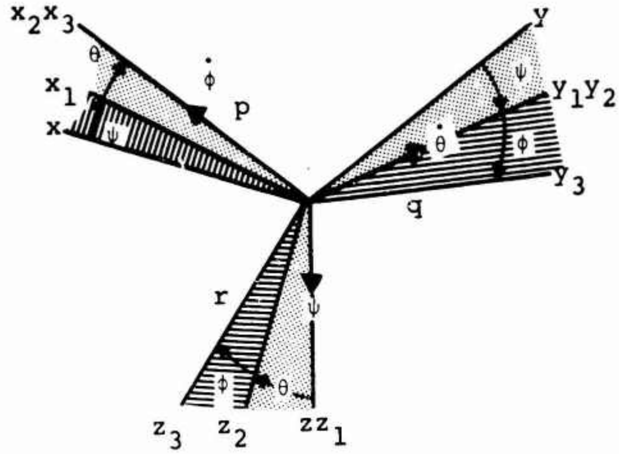
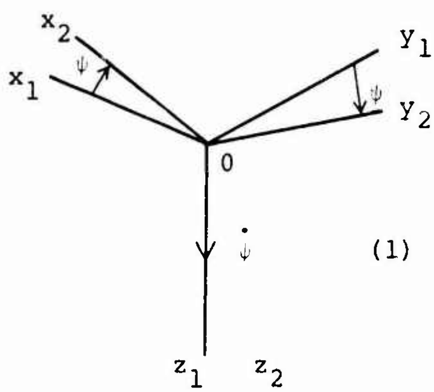


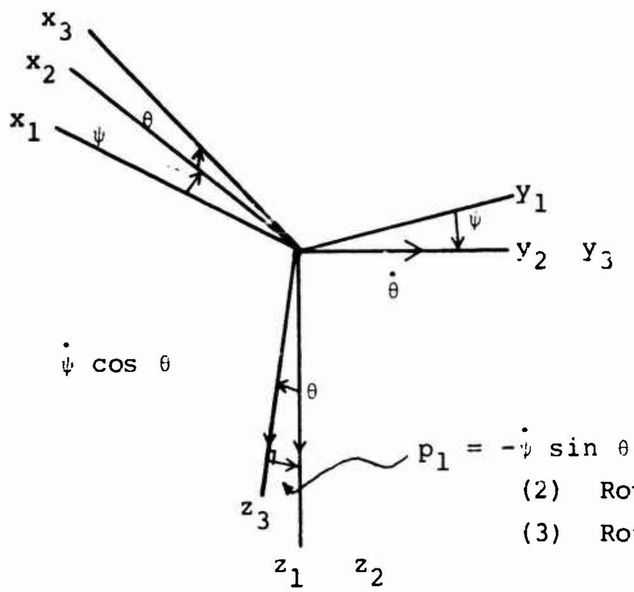
Figure 1.9c

Since the x_2 and x_3 axis are the same, $\dot\phi$ is a rotation about the x_3 axis. (Figure 1.9). Figure 1.9c shows all three angular rates ($\dot\psi$, $\dot\theta$, and $\dot\phi$) and can be used to write equations (1.5) through (1.7).

In summary, we have the following results:

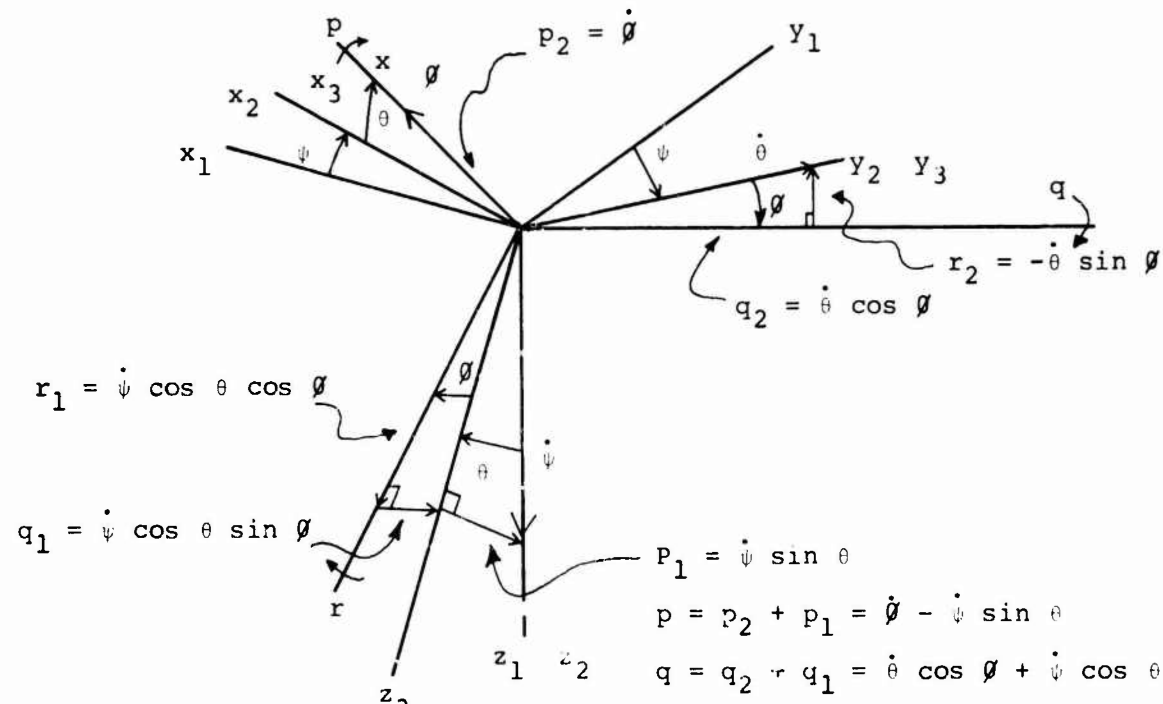


(1) Rotate Thru Yaw Angle - ψ



(2) Rotate Thru Pitch Angle - θ

(3) Rotate Thru Roll Angle - ϕ



$p = p_2 + p_1 = \dot{\phi} - \dot{\psi}\sin\theta$

$q = q_2 + q_1 = \dot{\theta}\cos\phi + \dot{\psi}\cos\theta\sin\phi$

$r = r_1 + r_2 = \dot{\psi}\cos\theta\cos\phi - \dot{\theta}\sin\phi$

1.4.0 DERIVATION OF THE AIRCRAFT EQUATIONS OF MOTION

1.4.1 Method of Derivation:

START WITH NEWTON'S SECOND LAW

$$\underbrace{\vec{F}}_{\text{externally applied force}} = \frac{d}{dt}\underbrace{(M\vec{V})}_{\text{linear momentum}} .$$

(3 linear degrees of freedom).

$$\underbrace{\vec{G}}_{\text{externally applied moment}} = \frac{d}{dt}\underbrace{(\vec{H})}_{\text{angular momentum}}$$

(3 rotational degrees of freedom).

Six Equations for the Six Degrees of Freedom of a Rigid Body.

↓

Algebraic Manipulation Using Two "Tools" :

1st Tool - Expression for a vector derivative in a moving reference system.

2nd Tool - Expression for the angular momentum of a rigid body.

↓

OBTAIN THE 6 AIRCRAFT EQUATIONS OF MOTION :

longitudinal

$$F_x = m\,(\dot{u} + q\,w - r\,v) . \tag{1.8}$$

$$F_z = m\,(\dot{w} + p\,v - q\,u) . \tag{1.9}$$

$$G_y = \dot{q}\,I_y - pr\,(I_z - I_x) + (p^2 - r^2)\,I_{xz} . \tag{1.10}$$

---

lateral-Directional

$$F_y = m\,(\dot{v} + r\,u - p\,w) . \tag{1.11}$$

$$G_x = \dot{p}\,I_x + qr\,(I_z - I_y) - (\dot{r} + pq)\,I_{xz} . \tag{1.12}$$

$$G_z = \dot{r}\,I_z + pq\,(I_y - I_x) + (qr - \dot{p})\,I_{xz} . \tag{1.13}$$

1.4.2 <u>Development of the First Tool - Vector Derivative in a Moving Reference.</u>

1.4.2.1 <u>Some Properties of Vectors:</u>

<u>PROPERTY 1</u> A vector is invarient with changes in reference.

$$\vec{A} = X\vec{e}_x + Y\vec{e}_y + Z\vec{e}_z = x\vec{i} + y\vec{j} + z\vec{k} . \qquad (1.14)$$

Hence we are free to change coordinates at any time, even in the middle of a discussion.

<u>PROPERTY 2</u> The rotation of a vector is conveniently expressed by the use of another vector, using the right-hand rule.

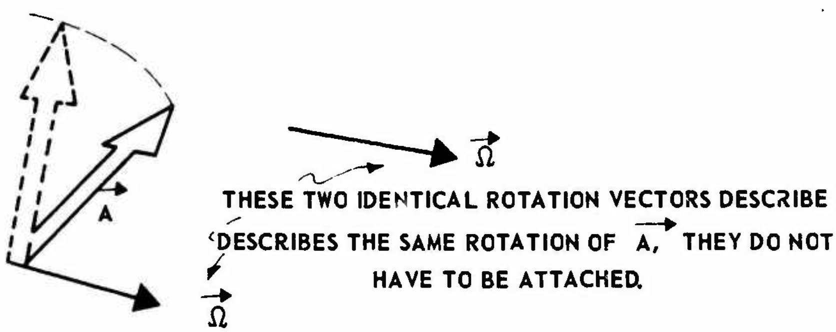


Figure 1.10

<u>PROPERTY 3</u> A vector is changed only by <u>growth</u> and <u>rotation</u>, not by <u>translation</u>. Several different forms of Vector Derivatives that express these changes are shown on the next page.

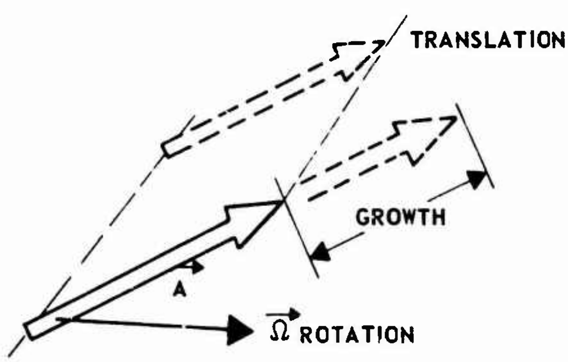


Figure 1.11

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1.4.2.2 General Expression for Vector Derivatives

"EASY" FORM:

$$\left.\frac{d\vec{A}}{dt}\right|_{XYZ} = \underbrace{\dot{X}\vec{e}_x + \dot{Y}\vec{e}_y + \dot{Z}\vec{e}_z}_{\substack{\text{describes both growth}\\ \text{and rotation changes}}} . \tag{1.15}$$

$$\left.\frac{d\vec{A}}{dt}\right|_{xyz} = \overbrace{\dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}} . \tag{1.16}$$

ORDINARY FORM:

rotation of $\vec{A}$

in XYZ

$$\left.\frac{d\vec{A}}{dt}\right|_{XYZ} = \underbrace{\frac{d}{dt}|\vec{A}|\vec{a}}_{\substack{\text{growth}\\ \text{changes}}} + \underbrace{\vec{\Omega} \times \vec{A}}_{\substack{\text{rotation}\\ \text{changes}}} . \tag{1.17}$$

$$\left.\frac{d\vec{A}}{dt}\right|_{xyz} = \overbrace{\frac{d}{dt}|\vec{A}|\vec{a}} + \overbrace{\vec{\phi} \times \vec{A}} . \tag{1.18}$$

rotation of $\vec{A}$

in xyz

1.4.2.3 <u>Vector Derivative in a Moving Reference:</u>

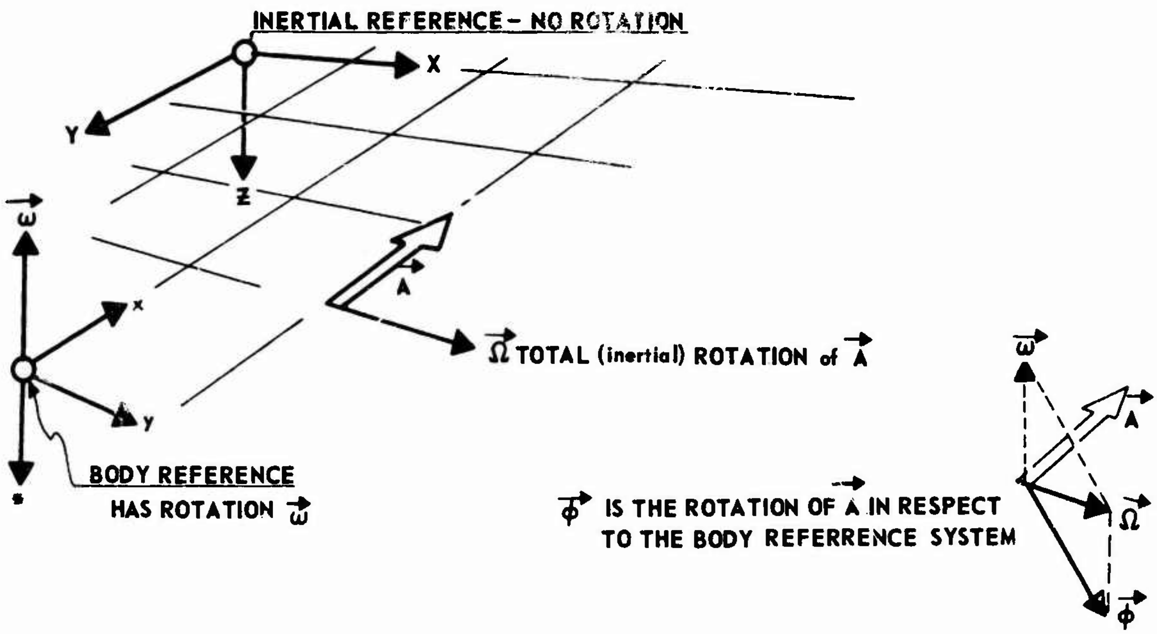


Figure 1.12

By PROPERTY 1 we choose to express all vectors in terms of the moving reference system

$$\vec{A} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{\Omega} = \vec{\phi} + \vec{\omega} \,. \tag{1.19}$$

Substituting in Equation (1.17) gives

$$\left.\frac{d\vec{A}}{dt}\right|_{XYZ} = \frac{d}{dt}\left|\vec{A}\right|\vec{a} + (\vec{\phi} + \vec{\omega}) \times \vec{A},$$

$$= \frac{d}{dt}\left|\vec{A}\right|\vec{a} + \vec{\phi} \times \vec{A} + \vec{\omega} \times \vec{A} \,.$$

By Equation (11) this is $\left.\frac{d\vec{A}}{dt}\right|_{xyz}$

$$\boxed{\left.\frac{d\vec{A}}{dt}\right|_{XYZ} = \left.\frac{d\vec{A}}{dt}\right|_{xyz} + \vec{\omega} \times \vec{A}} \qquad \text{THE "FIRST TOOL"} \tag{1.20}$$

This is an extremely convenient form - it expresses the INERTIAL TIME DERIVATIVE in terms of the moving reference system, i.e., small x, y, and z's.

1.4.2.4 Alternate Method of Derivation:

$$\vec{A} = x\vec{i} + y\vec{j} + z\vec{k} \quad .$$

By the product rule of calculus:

$$\left.\frac{d\vec{A}}{dt}\right|_{XYZ} = x\frac{d\vec{i}}{dt} + \dot{x}\vec{i} + y\frac{d\vec{j}}{dt} + \dot{y}\vec{j} + z\frac{d\vec{k}}{dt} + \dot{z}\vec{k} \quad . \qquad (1.21)$$

The total rotation of the unit vectors $\vec{i}$, $\vec{j}$, $\vec{k}$ is by definition the rotation of the small axis system, hence $\vec{\omega}$ may be substituted for $\vec{\Omega}$ in Equation 1.17 to get

$$\left.\frac{d\vec{i}}{dt}\right|_{XYZ} = \underbrace{\cancel{\frac{d}{dt}|\vec{i}|}}_{\text{Zero, unit vectors do not grow}}\vec{i} + \vec{\omega} \times \vec{i} \quad ,$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ p & q & r \\ 1 & 0 & 0 \end{vmatrix} = (r\vec{j} - q\vec{k}) \quad .$$

Similarly:

$$\left.\frac{d\vec{j}}{dt}\right|_{XYZ} = (p\vec{k} - r\vec{i}) \qquad \left.\frac{d\vec{k}}{dt}\right|_{XYZ} = (q\vec{i} - p\vec{j}) \quad .$$

Rearranging Equation (1.21) gives

$$\left.\frac{d\vec{A}}{dt}\right|_{XYZ} = \underbrace{\dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}}_{\text{From Eqn (9) this is } \left.\frac{d\vec{A}}{dt}\right|_{xyz}} + \underbrace{(zq - yr)\vec{i} + (xr - zp)\vec{j} + (yp - xq)\vec{k}}_{\text{By inspection this is } \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ p & q & r \\ x & y & z \end{vmatrix} = \vec{\omega} \times A}$$

$$\left.\frac{d\vec{A}}{dt}\right|_{XYZ} = \left.\frac{d\vec{A}}{dt}\right|_{xyz} + \vec{\omega} \times \vec{A} \quad . \qquad (1.22)$$

Hence the FIRST TOOL is confirmed.

1.4.2.5 Velocities are a Special Case

Note that thus far velocity has not been mentioned - it is a special case application of vector derivatives. The velocity of a particle relative to a reference is the derivative, as seen from that reference, of a <u>position vector</u> of the particle.

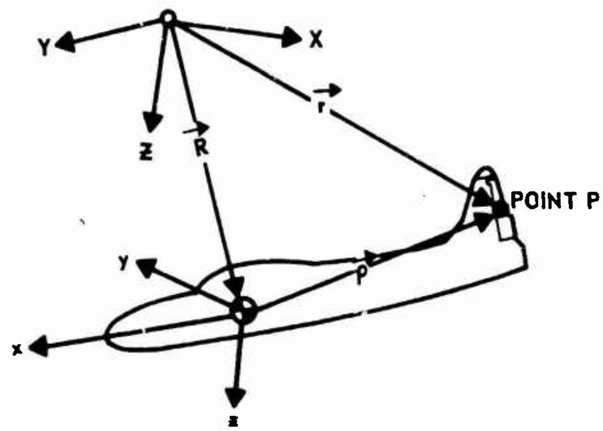


Figure 1.13

The local (or relative) velocity of P is $\left.\frac{d\vec{\rho}}{dt}\right|_{xyz}$.

The absolute (or inertial) velocity of P is $\left.\frac{d\vec{r}}{dt}\right|_{XYZ}$.

It is not convenient to use $\vec{r}$ (where is the center of inertial space?)
But we note that

$$\vec{r} = \vec{R} + \vec{\rho} \quad . \tag{1.23}$$

Hence

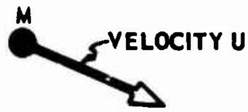
$$\vec{V}_{abs} = \left.\frac{d\vec{r}}{dt}\right|_{XYZ} = \left.\frac{d\vec{R}}{dt}\right|_{XYZ} + \left.\frac{d\vec{\rho}}{dt}\right|_{XYZ} \quad . \tag{1.24}$$

We then apply the FIRST TOOL

$$\vec{V}_{abs} = \underbrace{\left.\frac{d\vec{R}}{dt}\right|_{XYZ}}_{\substack{\text{velocity}\\ \text{of cg}\\ \text{(Translation)}}} + \underbrace{\left.\frac{d\vec{\rho}}{dt}\right|_{xyz}}_{\substack{\text{local}\\ \text{velocity}}} \underbrace{+ \vec{\omega} \times \vec{\rho}}_{\substack{\text{velocity due to}\\ \text{rotation of aircraft}}} \quad . \tag{1.25}$$

1.4.3 Angular Momentum of a Rigid Body

Linear Momentum



LINEAR MOMENTUM=mu

Figure 1.14

ANGULAR MOMENTUM (or Moment-of-Momentum)

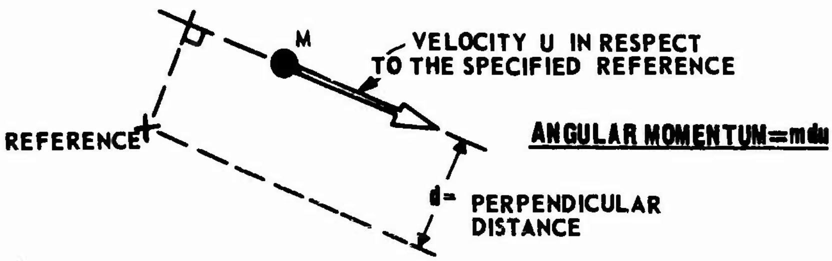


Figure 1.15

Three types of references may be used:

1. A point fixed in inertial space.
2. A point at the cg of a system of particles.
3. A point accelerating toward the cg.

The use of vector cross products greatly facilitates the expression of angular momentum.

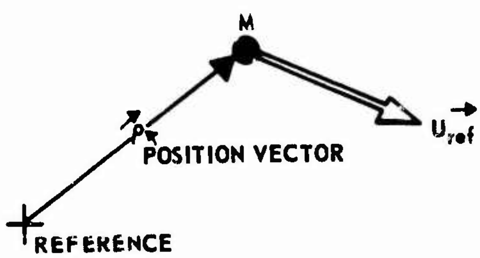


Figure 1.16

$$\vec{H} = m\,[\vec{\rho} \times \vec{u}] \qquad (1.26)$$

We will now extend this vector definition to a rigid body which can be thought of as a system of particles.

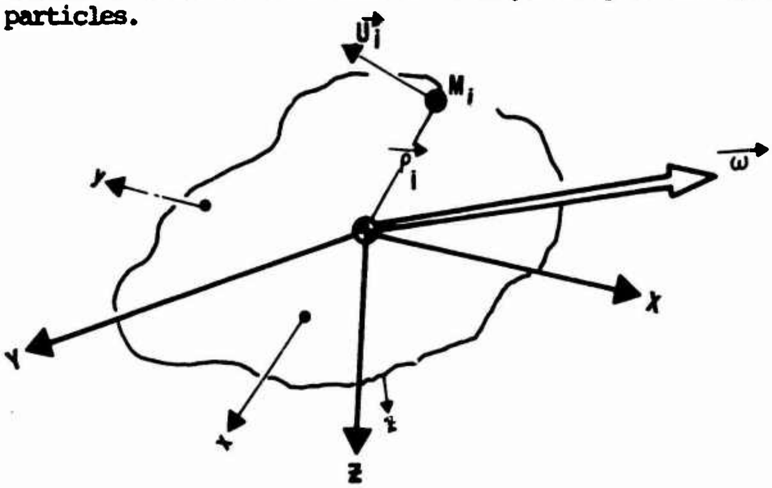


Figure 1.17

Choosing the cg as the reference, Angular Momentum becomes:

$$\vec{H} = \sum_{o}^{i} m_i \, [\vec{R}_i \times \vec{u}_i] \quad , \tag{1.27}$$

where u_i is the velocity with respect to the cg.

$$\vec{u}_i = \left.\frac{d\vec{\rho}}{dt}\right|_{XYZ} \quad . \quad \text{(i.e., with respect to a moving Earth Axis System at the cg)} \tag{1.28}$$

We are free, however, to EXPRESS ALL VECTORS in terms of the small Body Axis System, i.e.,

$$\vec{H} = H_x\vec{i} + H_y\vec{j} + H_z\vec{k} \quad .$$

$$\vec{\rho}_i = x_i\vec{i} + y_i\vec{j} + z_i\vec{k} \quad .$$

The First Tool is perfect for the job of converting Equation (1.28) to the Body Axis System, although another method is shown for comparison.

USING THE FIRST TOOL	USING VECTOR DERIVATIVE (EQUATION 1.17)
$\vec{u}_i = \underbrace{\cancel{\left.\frac{d\vec{\rho}}{dt}\right\|_{xyz}}}_{\text{Zero, rigid body}} + \vec{\omega} \times \vec{\rho}_i \quad .$	$\vec{u}_i = \underbrace{\cancel{\frac{d}{dt}\|\vec{\rho}\|}}_{\text{Zero, no growth, rigid body}} + \vec{\Omega} \times \vec{\rho} \quad .$ Also $\vec{\Omega} = \underbrace{\cancel{\vec{\phi}}}_{\text{Zero, no internal rotation}} + \vec{\omega} \quad ,$ $\vec{u}_i = \vec{\omega} \times \vec{\rho}_i \quad . \qquad (1.29)$

Substituting Equation (1.29) into Equation (1.27) gives

$$\vec{H} = \sum_{o}^{i} m_i \, [\vec{\rho}_i \times \{\vec{\omega} \times \vec{\rho}_i\}] \quad , \tag{1.30}$$

where m_i = a scalar

$$\vec{\rho}_i = x_i\vec{i} + y_i\vec{j} + z_i\vec{k}$$

$$\vec{\omega} = p\vec{i} + q\vec{j} + r\vec{k} \quad ,$$

and

$$\vec{\omega} \times \vec{\rho}_i = \begin{vmatrix} i & j & k \\ p & q & r \\ x_i & y_i & z_i \end{vmatrix} \quad .$$

DROPPING THE SUBSCRIPTS and expanding:

$$\vec{\omega} \times \vec{\rho} = \{(qz - ry)\vec{i} + (rx - pz)\vec{j} + (py - qx)\vec{k}\}$$

Equation (1.30 becomes:

$$\vec{H} = \sum_{o}^{i} m \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ (qz-ry) & (rx-pz) & (py-qx) \end{vmatrix}$$

Hence the components of H are:

$$H_x = \Sigma\, my(py-qx) - \Sigma\, mz(rx-pz) \quad , \tag{1.31}$$

$$H_y = \Sigma\, mz(qz-ry) - \Sigma\, mx(py-qx) \quad , \tag{1.32}$$

$$H_z = \Sigma\, mx(rx-pz) - \Sigma\, my(qz-ry) \quad . \tag{1.33}$$

Rearranging Equations (1.31), (1.32) and (1.33) gives

$$H_x = p\,\Sigma\, m(y^2 + z^2) - q\,\Sigma\, mxy - r\,\Sigma\, mxz \quad , \tag{1.34}$$

$$H_y = q\,\Sigma\, m(z^2 + x^2) - r\,\Sigma\, myz - p\,\Sigma\, mxy \quad , \tag{1.35}$$

$$H_z = r\,\Sigma\, m(x^2 + y^2) - p\,\Sigma\, mxz - q\,\Sigma\, myz \quad . \tag{1.36}$$

Define MOMENTS OF INERTIA:

$$I_x = \Sigma\, m(y^2 + z^2) \ .$$

$$I_y = \Sigma\, m(x^2 + z^2) \ .$$

$$I_z = \Sigma\, m(x^2 + y^2) \ .$$

These are a measure of resistance to rotation - they are never zero.

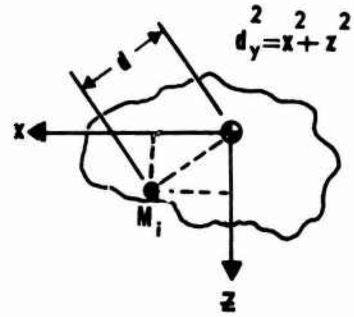


Figure 1.18

Define PRODUCTS OF INERTIA:

$$I_{xy} = \Sigma\, mxy \ .$$

$$I_{yz} = \Sigma\, myz \ .$$

$$I_{xz} = \Sigma\, mxz \ .$$

These are a measure of symmetry. They are zero for views having a line of symmetry.

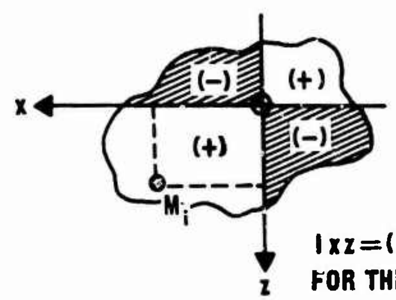


Figure 1.19

The Angular Momentum of a rigid body is therefore:

$$\vec{H} = H_x\vec{i} + H_y\vec{j} + H_z\vec{k} \quad , \tag{1.37}$$

So that

$$H_x = pI_x - qI_{xy} - rI_{xz} \quad , \tag{1.38}$$

$$H_y = qI_y - rI_{yz} - pI_{xy} \quad , \tag{1.39}$$

$$H_z = rI_z - pI_{xz} - qI_{yz} \quad . \tag{1.40}$$

1.4.4 Simplification for a Symmetric Aircraft - the Second Tool .

A symmetric aircraft has two views that contain a line of symmetry and hence two products of inertia that are zero.

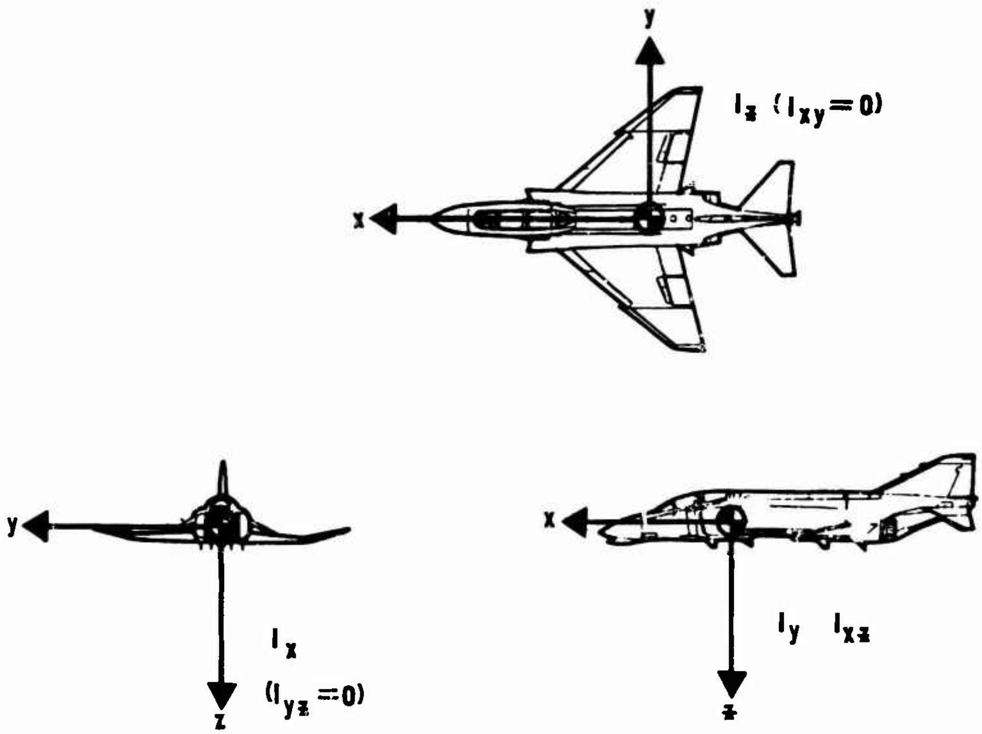


Figure 1.20

The angular momentum of a symmetric aircraft therefore simplifies to:

THE SECOND TOOL

$$\vec{H} = (pI_x - rI_{xz})\vec{i} + qI_y\vec{j} + (rI_z - pI_{xz})\vec{k} \tag{1.41}$$

We have now obtained the two tools necessary to derive the equations of motion from Newton's second law.

1.4.5 Derivation of the Three Rotational Equations:

Starting with Newton's Second Law

$$\vec{G} = \left.\frac{d\vec{H}}{dt}\right|_{XYZ} \quad , \tag{1.42}$$

we apply the FIRST TOOL

$$\vec{G} = \left.\frac{d\vec{H}}{dt}\right|_{xyz} + \vec{\omega} \times \vec{H} \tag{1.43}$$

and utilize the "easy form" of vector derivatives, Equation (1.20) to get

$$\vec{G} = \dot{H}_x\vec{i} + \dot{H}_y\vec{j} + \dot{H}_z\vec{k} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ p & q & r \\ H_x & H_y & H_z \end{vmatrix} \quad . \tag{1.44}$$

We next utilize the SECOND TOOL:

$$\vec{H} = (pI_x - rI_{xz})\vec{i} + (qI_y)\vec{j} + (rI_z - pI_{xz})\vec{k} \quad , \tag{1.45}$$

and note that the Moments and Products of Inertia are constant with respect to the body axis system.

$$\vec{G} = (\dot{p}I_x - \dot{r}I_{xz})\vec{i} + (\dot{q}I_y)\vec{j} + (\dot{r}I_z - \dot{p}I_{xz})\vec{k} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ p & q & r \\ (pI_x - rI_{xz}) & (qI_y) & (rI_z - pI_{xz}) \end{vmatrix} \tag{1.46}$$

Finally we equate vector components, obtaining the the three rotational equations:

$$G_x = \dot{p}I_x + qr(I_z - I_y) - (\dot{r} + pq)I_{xz} \quad , \tag{1.47}$$

$$G_y = \dot{q}I_y - pr(I_z - I_x) + (p^2 - r^2)I_{xz} \quad , \tag{1.48}$$

$$G_z = \dot{r}I_z + pq(I_y - I_x) + (qr - \dot{p})I_{xz} \quad . \tag{1.49}$$

1.4.6 Derivation of the Three Linear Equations

Starting with Newton's Second Law

$$\vec{F} = \left.\frac{d(m\vec{V}_T)}{dt}\right|_{XYZ} \tag{1.50}$$

We assume the mass is constant

$$\vec{F} = m \left.\frac{d\vec{V}_T}{dt}\right|_{XYZ} \tag{1.51}$$

and apply the FIRST TOOL

$$\vec{F} = m \left[\left.\frac{d\vec{V}_T}{dt}\right|_{xyz} + \vec{\omega} \times \vec{V}_T \right] . \tag{1.52}$$

Next we utilize the "easy form" of vector derivatives, Equation (1.20) to get

$$\vec{F} = m \left[\dot{u}\vec{i} + \dot{v}\vec{j} + \dot{w}\vec{k} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ p & q & r \\ u & v & w \end{vmatrix} \right] . \tag{1.53}$$

Finally we equate vector components to obtain the three linear equations:

$$F_x = m(\dot{u} + qw - rv) , \tag{1.54}$$

$$F_y = m(\dot{v} + ru - pw) , \tag{1.55}$$

and

$$F_z = m(\dot{w} + pv - qu) . \tag{1.56}$$

1.5.0 RESTRICTIONS AND SIMPLIFICATIONS

1.5.1 RESTRICTIONS

The aircraft equations of motion (Equations 1.8 through 1.13) are quite general; only four restrictions apply.

1.5.1.1 Rigid Body - Aeroelastic effects must be considered separately.

1.5.1.2 Earth and atmosphere must be assumed fixed - Allows use of Moving Earth Axis System as an "inertial reference", and true airspeed as absolute velocity. (Changes in V_{abs} are of more interest than the actual magnitude of V_{abs}).

1.5.1.3 Constant Mass - Most motion of interest in stability and control takes place in a relatively short time.

1.5.1.4 The x z plane must be a plane of symmetry - This restriction applies to the RHS of the equations only (shape and mass distribution); it does not preclude unbalanced forcing functions (F or G on the LHS) such as asymmetric thrust. If necessary, this restriction can be easily removed by deriving the equations using the complete expression for angular momentum (Equations 1.38, 1.39 and 1.40) instead of the "Second Tool." An indication of the extent of simplification that results from this fourth restriction is given below for the pitch equation:

$$G_{Y_{\substack{\text{symmetric}\\ \text{aircraft}}}} = \dot{q}\ I_y - pr\,(I_z - I_x) + (p^2 - r^2)\ I_{xz} \quad .$$

$$G_{Y_{\substack{\text{general}\\ \text{rigid}\\ \text{body}}}} = G_{Y_{\substack{\text{symmetric}\\ \text{aircraft}}}} + (pq - \dot{r})\,I_{yz} - (\dot{p} + qr)\ I_{xy} \quad .$$

1.5.2 TERMINOLOGY

1.5.2.1 Steady Flight: Flight in which the existing motion remains steady with time.

1.5.2.2 Symmetric Flight:

Flight in which the vehicle plane of symmetry remains fixed in space.

$$v = 0 \qquad p = r = 0 \quad .$$

$$(\beta = 0) \qquad (\phi \text{ and } \dot{\psi} = 0) \quad .$$

1.5.2.3 Asymmetric Flight:

Flight in which the vehicle plane of symmetry does not remain fixed in space.

$$v \neq 0 \qquad p \text{ and/or } r \neq 0 \quad .$$

$$(\beta \neq 0) \qquad (\phi \text{ and/or } \dot{\psi} \neq 0) \quad .$$

1.5.2 SOME SPECIAL-CASE VEHICLE MOTIONS

UNACCELERATED FLIGHT:

(Also called straight flight or equilibrium flight).

$$F_x = 0 \qquad F_y = 0 \qquad F_z = 0$$

Hence the cg travels a straight path at constant speed.

Note that equilibrium does not mean steady state. For example,

$$F_x = m(\dot{u} + qw - rv) = 0$$

could be maintained zero by fluctuation of the three terms on the right in an unsteady manner. In practice, however, it is difficult to predict that non-steady motion will remain unaccelerated and hence the straight motions most often discussed are also steady state.

STEADY STRAIGHT FLIGHT:	STEADY ROLLS OR SPINS:
$F_x = 0 \quad F_y = 0$	$F_x = 0 \quad F_y = 0$ by custom this is not called straight flight even though the cg may be travelling a straight path
$F_z = 0 \quad G_x = 0$	$F_z = 0 \quad G_x = 0$
$\underbrace{G_y = 0 \quad G_z = 0}_{\text{On the average}} \qquad \underbrace{p = q = r = 0}_{\text{Excluded by Custom}}$	$\underbrace{G_y = 0 \quad G_z = 0}_{\text{On the Average}}$
Trim Points, stabilized points.	Steady developed spins.

ACCELERATED FLIGHT:

(Non-equilibrium flight)

One or more of the linear equations is not zero, hence the cg is not travelling a straight path. Again the steady cases are of most interest.

STEADY TURNS:

An unbalanced horizontal force results in the cg being constantly deflected inward toward the center of a curved path. This results in a constantly changing yaw angle . By the Euler angle transform,

$$p = -\dot{\psi}\theta \text{ (assumes small } \theta)$$

$$q = \dot{\psi} \sin\phi \cos\theta = \dot{\psi} \sin\phi$$

$$r = \dot{\psi} \cos\phi \cos\theta = \dot{\psi} \cos\phi$$

and hence

$$F_y = m(\dot{\psi} \cos\phi)u \quad ,$$

$$F_z = -m(\dot{\psi} \sin\phi)u \quad . \quad \text{(Assumes } \dot{\psi}\theta \text{ is very very small)}$$

Includes moderate climbs and descents.

SYMMETRICAL PULL UP:

Here an unbalanced z force is constantly deflecting the cg upward.

$$q = \dot{\theta} \quad ,$$

$$F_x \approx mqw \quad ,$$

and

$$F_z \approx -mqu \quad .$$

This is a quasi-steady motion since $\dot{u}$ and $\dot{w}$ cannot long remain zero.

1.6.0 PREPARATION FOR EXPANSION OF THE LEFT HAND SIDE

We have seen that the Equations of Motion relate the vehicle motion to the applied forces and moments.

LHS		RHS
Applied Forces and Moments	=	Observed Vehicle Motion

$$F_x = m\dot{u} + - - - -$$

$$G_x = \dot{p}I_x + - - - -$$

etc.

The right hand side (RHS) of each of these six equations has been completely expanded in terms of easily measured quantities. We must now likewise expand the left hand side (LHS) in terms of convenient variables, to include Stability Parameters and Derivatives. Before this can be accomplished, however, the following topics must be discussed and understood. (1.61 through 1.63)

1.6.1 Initial Breakdown of the LHS

In general the applied forces and moments can be broken up according to source as shown below.

		SOURCE						
		Aero-dynamic	Direct Thrust	Gravity	Gyro-scopic	Other		
LONGITUDINAL	F_x	X	X_T	X_g	0	X_{oth}	$= m\dot{u} + - - -$	(1.57)
	F_z	Z	Z_T	Z_g	0	Z_{oth}	$= m\dot{w} + - - -$	(1.58)
	G_y	$\mathcal{M}$	M_T	0	M_{gyro}	M_{oth}	$= \dot{q}I_y + - - -$	(1.59)
LAT-DIRECTIONAL	F_y	Y	Y_T	Y_g	0	Y_{oth}	$= m\dot{v} + - - -$	(1.60)
	G_x	$\mathcal{L}$	L_T	0	L_{gyro}	L_{oth}	$= \dot{p}I_x + - - -$	(1.61)
	G_z	$\mathcal{N}$	N_T	0	N_{gyro}	N_{oth}	$= \dot{r}I_z + - - -$	(1.62)

Gravity Forces: These vary with orientation of the weight vector.

$X_g = -mg \sin \theta \qquad Y_g = mg \cos\theta \sin \phi \qquad Z_g = mg \cos\theta \cos \phi$.

Gyroscopic Moments: These occur as a result of large rotating masses such as engines and props.

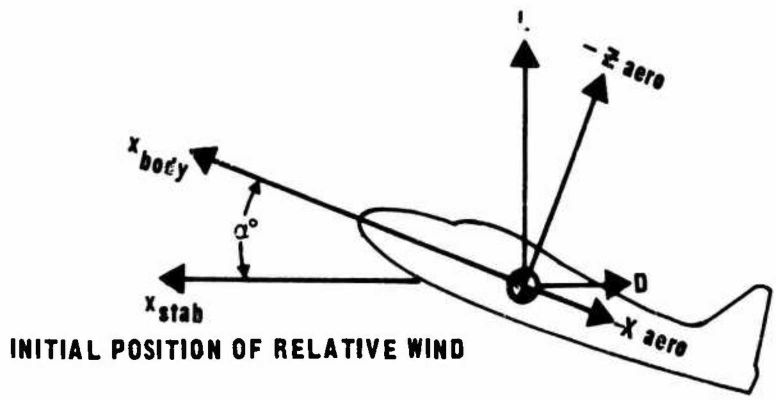
Direct Thrust Forces and Moments: These terms include the effect of the thrust vector itself - they usually do not include the indirect or induced effects of jet flow or running propellers.

Aerodynamic Forces and Moments: These will be further expanded into Stability Parameters and Derivatives.

Other Sources: These include spin chutes, reaction controls, etc.

1.6.2 Choice of Axis System

The derivation of the Equations of Motion made use of a body axis system. As discussed earlier, there are several different types, e.g., the General Body Axis System ("Body Axes") and the Stability Axis System. It will be convenient for us to continue to think of the RHS in terms of the "Body Axes" but to expand the aerodynamic forces and moments on the LHS in terms of the Stability Axis System.



BOTH AXES ARE RIGIDLY ATTACHED TO THE AIRCRAFT

USE STABILITY AXES ON LHS	USE BODY AXES ON RHS
FACILITATES EXPANSION OF THE AERODYNAMIC FORCES AND MOMENTS	Ix, Iy, etc REMAIN CONSTANT p, q, r CAN BE MEASURED BY RIGIDLY MOUNTED GYROS

Figure 1.21

Fortunately this seemingly irresponsible procedure can be utilized without loss or sacrifice, for the following reasons:

1. Any quantity in one system can be easily expressed in the other by means of a trigonometric transform involving α_o.

   For example:

   $$- X = D\cos \alpha_o - L \sin \alpha_o .$$

   Numerical analyses can be made completely accurate in this manner, particularly if a computer is used.

2. The actual errors resulting from use of the different systems are usually small enough to allow them to be disregarded in discussions of stability and control. In other words, we can frequently accept the following approximation .

$$G_{x_{stab}} = \dot{p}_{stab} I_{y_{stab}} + - - - -$$

$$\approx \dot{p}_{body} I_{y_{body}} + - - - -$$

At this point, and for the remainder of the discussion, we will utilize the stability axis system on the LHS and assume proper allowance has been made on the RHS.

The equations, which of course retain the same form, are now as shown below.

"DRAG"	$-D + X_T + X_g + X_{oth}$	$= m\dot{u} + - - - -$	(1.63)
"LIFT"	$-L + Z_T + Z_g + Z_{oth}$	$= m\dot{w} + - - - -$	(1.64)
"PITCH"	$\mathcal{M} + M_T + M_{gyro} + M_{oth}$	$= \dot{q}I_y + - - - -$	(1.65)
"SIDE"	$Y + Y_T + Y_g + Y_{oth}$	$= m\dot{v} + - - - -$	(1.66)
"ROLL"	$\mathcal{L} + L_T + L_{gyro} + L_{oth}$	$= \dot{p}I_x + - - - -$	(1.67)
"YAW"	$\mathcal{N} + N_T + N_{gyro} + N_{oth}$	$= \dot{r}I_z + - - - -$	(1.68)

1.6.3 The Small Disturbance Assumption:

The aerodynamic forces and moments are primarily a function of the following variables:

1. Temperature and altitude

   Accounted for by ρ, M, R_e .

2. Angular Velocities

   Accounted for by p, q, r .

3. Control Deflections

   Accounted for by δ_e, δ_a, δ_r .

4. Position and Magnitude of the Relative Wind

Accounted for by the components u, v, w of true velocity, or alternately, by:

$$u, \qquad \alpha \equiv \frac{w}{V_T} , \qquad \beta \equiv \frac{v}{V_t}$$

In general, the time derivatives of these variables could also be significant. In other words:

D, L, $\mathcal{M}$, Y, $\mathcal{L}$, $\mathcal{N}$ Are a Function of

VARIABLE	1st DERIVATIVE	HIGHER ORDER TERMS
$u \ \alpha \ \beta$	$\dot{u} \ \dot{\alpha} \ \dot{\beta}$	$\ddot{u}$ - - - -
$p \ q \ r$	$\dot{p}$ - -	$\ddot{p}$ - - - -
$\delta_e \ \delta_a \ \delta_r$	$\dot{\delta}_e$ - -	- - -

$\rho \ M \ R_e$ → assumed constant

This rather formidible list can be reduced to workable proportions by making the assumption that the vehicle motion will consist only of small deviations from some initial reference condition. Fortunately, this small-disturbance assumption applies to many cases of practical interest, and, as a bonus, stability parameters and derivatives derived under this assumption continue to give good results for motions somewhat larger.

The variables are considered to consist of some initial value plus an incremental change, called the "perturbated value." The notation for these perturbated values is sometimes lower case and sometimes lower case with a bar.

$$P = P_o + p \ . \qquad p = p_o + \bar{p} \ .$$

$$U = U_o + u \ . \qquad u = u_o + \bar{u} \ .$$

It has been found from experience that when operating under the small disturbance assumption the vehicle motion can be thought of as two independent motions each of which is a function only of the variables shown below.

LONGITUDINAL MOTION:

$$(D, L, \mathcal{M}) = f\,(U, \alpha, \dot{\alpha}, Q, \delta_e) \ . \qquad (1.69)$$

LATERAL-DIRECTIONAL MOTION:

$$(Y, \mathcal{L}, \mathcal{N}) = g\ (\beta,\ \dot{\beta},\ P,\ R,\ \delta_a,\ \delta_r) \quad . \tag{1.70}$$

1.7.0 EXPANSION OF THE AERODYNAMIC FORCES AND MOMENTS:

1.7.1 Initial Conditions.

It will be assumed that the motion consists of small perturbations about some initial condition of steady straight symmetrical flight. From this and the definition of stability axes, the following can be stated:

$$V_t = U_o = \text{constant}$$

$$V_o = 0 \qquad \beta_o = 0$$

$$W_o = 0 \qquad \alpha_o = \text{constant}$$

$$P_o = Q_o = R_o = 0 \ .$$

$$(\rho,\ M,\ R_e,\ \text{aircraft configuration}) = \text{constant}$$

1.7.2 Normalization of the Equations.

To obtain the stability derivatives and parameters most commonly used throughout the industry, the equations are "normalized" to arrange the first term on the RHS into a convenient form.

EQUATION	NORMALIZING FACTOR		FIRST TERM IS NOW PURE ACCEL OR $\dot{\alpha}$ $\dot{\beta}$	UNITS	
"DRAG"	$\frac{1}{m}$	$-\frac{D}{m} + \frac{X_T}{m} + \text{- - - -} =$	$\dot{u} + \text{---}$	$[\frac{ft}{sec^2}]$	(1.71)
"LIFT"	$\frac{1}{mU_o}$	$-\frac{L}{mU_o} + \frac{Z_T}{mU_o} + \text{- - - -} =$	$\frac{\dot{w}}{U_o} + \text{---}$	$[\frac{rad}{sec}]$	(1.72)
"PITCH"	$\frac{1}{I_y}$	$\frac{\mathcal{M}}{I_y} + \frac{M_T}{I_y} + \text{- - - -} =$	$\dot{q} + \text{---}$	$[\frac{rad}{sec^2}]$	(1.73)
"SIDE"	$\frac{1}{mU_o}$	$\frac{Y}{mU_o} + \frac{Y_T}{mU_o} + \text{- - - -} =$	$\frac{\dot{v}}{U_o} + \text{---}$	$[\frac{rad}{sec}]$	(1.74)
"ROLL"	$\frac{1}{I_x}$	$\frac{\mathcal{L}}{I_x} + \frac{L_T}{I_x} + \text{- - - -} =$	$\dot{p} + \text{---}$	$[\frac{rad}{sec^2}]$	(1.75)
"YAW"	$\frac{1}{I_z}$	$\frac{\mathcal{N}}{I_z} + \frac{N_T}{I_z} + \text{- - - -} =$	$\dot{r} + \text{---}$	$[\frac{rad}{sec^2}]$	(1.76)

1.7.3 STABILITY PARAMETERS FROM TAYLOR'S EXPANSION:

As previously noted, the longitudinal motion can be assumed to be a function of five variables, U, α, $\dot{\alpha}$, Q, δ_e. The normalized aerodynamic forces and moments can therefore be expressed by a Taylor's expansion.

For example:

$$\frac{L}{mU_o} = \frac{1}{mU_o}\left[\begin{array}{llll} L_o + \frac{\partial L}{\partial U}\,\Delta U & + \frac{1}{2}\,\frac{\partial^2 L}{\partial U^2}\,\Delta U^2 & + - - - \\ \quad + \frac{\partial L}{\partial \alpha}\,\Delta\alpha & + \frac{1}{2}\,\frac{\partial^2 L}{\partial \alpha^2}\,\Delta\alpha^2 & + - - - \\ \quad + \frac{\partial L}{\partial \dot{\alpha}}\,\Delta\dot{\alpha} & + - - - - & \\ \quad + \frac{\partial L}{\partial Q}\,\Delta Q & + - - - - & \\ \quad + \frac{\partial L}{\partial \delta_e}\,\Delta\delta_e & + - - - - & \end{array}\right] \tag{1.77}$$

But we have decided to express the variables as the sum of an initial value plus a small perturbated value

$$U = U_o + u \qquad \text{where} \qquad u = U - U_o = \Delta U. \tag{1.78}$$

Therefore

$$\frac{\partial L}{\partial U} = \frac{\partial L}{\partial U_o}\cdot\cancelto{\text{Zero}}{\frac{\partial U_o}{\partial U}} + \frac{\partial L}{\partial u}\,\cancelto{1.0}{\frac{\partial u}{\partial U}} = \frac{\partial L}{\partial u}\,, \tag{1.79}$$

and the first term of the expansion becomes

$$\frac{\partial L}{\partial U}\,\Delta U = \frac{\partial L}{\partial u}\,u\ . \tag{1.80}$$

Similarly

$$\frac{\partial L}{\partial Q}\,\Delta Q = \frac{\partial L}{\partial q}\,q\ . \tag{1.81}$$

We also elect to let $\alpha = \Delta\alpha$, $\dot{\alpha} = \Delta\dot{\alpha}$ and $\delta_e = \Delta\delta_e$

Dropping higher order terms involving u^2, q^2, etc., Equation (1.77) now becomes:

$$\frac{L-L_o}{mU_o} = \underbrace{\frac{1}{mU_o}\frac{\partial L}{\partial u}}_{L_u}u + \underbrace{\frac{1}{mU_o}\frac{\partial L}{\partial \alpha}}_{L_\alpha}\alpha + \underbrace{\frac{1}{mU_o}\frac{\partial L}{\partial \dot{\alpha}}}_{L_{\dot{\alpha}}}\dot{\alpha} + \underbrace{\frac{1}{mU_o}\frac{\partial L}{\partial q}}_{L_q}q + \underbrace{\frac{1}{mU_o}\frac{\partial L}{\partial \delta_e}}_{L_{\delta_e}}\delta_e \; [\frac{rad}{sec}] \qquad (1.82)$$

The indicated quantities are defined as STABILITY PARAMETERS

$$\frac{\Delta L}{mU_o} = L_u u + L_\alpha \alpha + L_{\dot{\alpha}}\dot{\alpha} + L_q q + L_{\delta_e}\delta_e \qquad [\frac{rad}{sec}] \quad . \qquad (1.83)$$

Stability parameters have various dimensions depending on whether they are multiplied by a linear velocity, an angle, or an angular rate.

$$L_u[\frac{1}{ft}]\; u\; [\frac{ft}{sec}] = [\frac{rad}{sec}] \quad , \quad L_\alpha\; [\frac{1}{sec}]\; \alpha\; [rad] = [\frac{rad}{sec}] \quad ,$$

$$L_{\dot{\alpha}}\; [none]\; \dot{\alpha}\; [\frac{rad}{sec}] = [\frac{rad}{sec}] \quad .$$

The lateral-directional motion is a function of β, $\dot{\beta}$, P, R, δ_a, δ_r, and can be handled in a similar manner. For example, the normalized aerodynamic rolling moment becomes:

$$\frac{\Delta \mathcal{L}}{I_x} = \mathcal{L}_\beta \beta + \mathcal{L}_{\dot{\beta}}\dot{\beta} + \mathcal{L}_p p + \mathcal{L}_r r + \mathcal{L}_{\delta_a}\delta_a + \mathcal{L}_{\delta_r}\delta_r \quad [\frac{1}{sec^2}] \quad , \qquad (1.84)$$

where

$$\mathcal{L}_\beta = \frac{1}{I_x}\frac{\partial \mathcal{L}}{\partial \beta}\; [\frac{1}{sec^2}] \qquad \text{etc.}$$

These stability parameters are sometimes called "Dimensional derivatives" but we will reserve the word "derivative" to indicate the non-dimensional form which can be obtained by rearrangement, as shown on the following pages.

1.7.4 Stability Derivatives

The Stability Parameters can be rearranged into a form that includes the non-dimensional stability Derivatives. The process is best illustrated by example.

EXAMPLE 1. Rearrange the parameter M_q to include the derivative C_{m_q}.

PROCEDURE. Mentally start with the normalized pitch equation:

$$\frac{\Delta\mathcal{M}}{I_y} = M_u u + M_\alpha \alpha + M_{\dot\alpha}\dot\alpha + M_q q + M_{\delta_e}\delta_e \quad , \quad [\frac{\text{rad}}{\text{sec}^2}] \tag{1.85}$$

then write the desired parameter in general form

$$M_q = \frac{1}{I_y}\frac{\partial M}{\partial q} \quad . \tag{1.86}$$

Substitute the coefficient form, $M = \frac{1}{2}\rho U_o^2 S c C_m$ to get

$$M_q = \frac{1}{I_y}\frac{\partial}{\partial q}\left(\frac{1}{2}\rho U_o^2 S c C_m\right) \quad . \tag{1.87}$$

Extract the constants, i.e., those variables that are not dependent on q:

$$M_q = \frac{\rho U_o^2 S c}{2 I_y}\frac{\partial C_m}{\partial q} \; . \quad \text{this has dimensions} \quad \frac{\partial C_m [\text{dim'less}]}{\partial q [\text{rad/sec}]} = [\text{sec}]. \tag{1.88}$$

There exist certain predetermined "compensating factors" which are shown later.

For this case the compensating factor is

$$\frac{c\ [\text{ft}]}{2U_o\ [\text{ft/sec}]} \quad .$$

Applying the compensating factor

$$M_q = \frac{\rho U_o^{2}\, S\, c}{2 I_y} \cdot \frac{C/2U_o}{C/2U_o} \left(\frac{\partial C_m}{\partial q} \right). \tag{1.89}$$

The circled quantity is rearranged and defined as the Stability Derivative C_{m_q}.

$$C_{m_q} = \frac{\partial C_m \ [\text{dimensionless}]}{\partial \left(\frac{qc}{2U_o}\right) \left[\frac{\text{rad/sec - ft}}{\text{ft/sec}}\right]} = [\text{dimensionless}]. \tag{1.90}$$

Hence

$$M_q = \frac{\rho U_o\, S\, c^2}{4\, I_y}\, C_{m_q} \quad . \tag{1.91}$$

C_{m_q} → Derivative is dimensionless

$\frac{\rho U_o S c^2}{4 I_y}$ → Constants

M_q → Parameter has dimensions, $[\frac{1}{\text{sec}}]$

Both the Derivative and the Parameter describe the variation of the aerodynamic pitching moment with pitch rate, i.e., "pitch damping."

Additional stability notation has now appeared:

ANGULAR RATES	NON-DIMENSIONAL ANGULAR RATES	
$p = \frac{\text{rad}}{\text{sec}}$	$\hat{p} = \frac{pb}{2U_o}$ dimensionless	
$q = \frac{\text{rad}}{\text{sec}}$	$\hat{q} = \frac{qc}{2U_o}$ "	
$r = \frac{\text{rad}}{\text{sec}}$	$\hat{r} = \frac{rb}{2U_o}$ "	(1.92)
$\dot{\beta} = \frac{\text{rad}}{\text{sec}}$	$\hat{\dot{\beta}} = \frac{\dot{\beta} b}{2U_o}$ "	
$\dot{\alpha} = \frac{\text{rad}}{\text{sec}}$	$\hat{\dot{\alpha}} = \frac{\dot{\alpha} c}{2U_o}$ "	

These non-dimensional angular rates are used in conjunction with the Stability Derivatives

$$C_{\ell} = C_{\ell_o} + C_{\ell_\beta}\beta + C_{\ell_{\dot\beta}}\hat{\dot\beta} + C_{\ell_p}\hat{p} + C_{\ell_r}\hat{r} + C_{\ell_{\delta_a}}\delta_a + C_{\ell_{\delta_r}}\delta_r \quad . \tag{1.93}$$

<u>EXAMPLE 2.</u> Rearrange the parameter M_u, to include the derivative C_{m_u}.

<u>PROCEDURE</u>. Mentally visualize the normalized pitch equation and write the desired parameter in general form

$$M_u = \frac{1}{I_y}\frac{\partial M}{\partial u} \quad . \tag{1.94}$$

Substitute the coefficient form

$$M_u = \frac{1}{I_y}\frac{\partial}{\partial u}\left(\frac{1}{2}\rho U^2 S c C_m\right) \quad ; \tag{1.95}$$

extract the constants

$$M_u = \frac{\rho S c}{2 I_y}\frac{\partial}{\partial u}(U^2 C_m) = \frac{\rho S c}{2 I_y}\left(2U_o\frac{\partial U}{\partial u}C_{m_o} + U_o^{\,2}\frac{\partial C_m}{\partial u}\right) \quad . \tag{1.96}$$

Rearrange the last term to isolate the proper compensating factor, in this case $U_o/2$

$$M_u = \frac{\rho S c}{2 I_y}\left(2U_o C_{m_o} + 2U_o \underbrace{\frac{U_o}{2}\frac{\partial C_m}{\partial u}}_{\text{this is } C_{m_u}}\right) \quad . \tag{1.97}$$

Hence

$$M_u + \frac{\rho U_o S c}{I_y}\left(C_{m_o} + C_{m_u}\right) \quad . \tag{1.98}$$

1.7.5 Basic Factors: The stability student must be able to expand the parameters indicated on this page. Memorization of the basic relationships shown will allow this to be accomplished by the procedure illustrated above.

LONGITUDINAL MOTION

EQUATION	COEFFICIENT	NORMALIZING FACTOR	PARAMETERS
Drag	$D = \frac{1}{2}\rho U^2 S C_D$	$\frac{1}{m}$	D_u D_{δ_e} (D_α requires special derivation) ($D_{\dot\alpha}$ and D_q are insignificant)
Lift	$L = \frac{1}{2}\rho U^2 S C_L$	$\frac{1}{mU_o}$	L_u $L_{\dot\alpha}$ L_q L_{δ_e} (L_α requires special derivation)
Pitch	$\mathcal{M} = \frac{1}{2}\rho U^2 S c C_m$	$\frac{1}{I_y}$	M_u M_α $M_{\dot\alpha}$ M_q M_{δ_e}

	LINEAR VELOCITY	ANGLES	ANGULAR RATES
Independent Variables	u	α δ_e	$\dot\alpha$ q
Compensating Factors	$\frac{U_o}{2}$	None	$\frac{2U_o}{c}$

LATERAL-DIRECTIONAL MOTION

EQUATION	COEFFICIENT	NORMALIZING FACTOR	PARAMETERS
Side	$Y = \frac{1}{2}\rho U^2 S C_Y$	$\frac{1}{mU_o}$	Y_β $Y_{\dot\beta}$ Y_p Y_r Y_{δ_a} Y_{δ_r}
Roll	$\mathcal{L} = \frac{1}{2}\rho U^2 S b C_\ell$	$\frac{1}{I_x}$	$\mathcal{L}_\beta$ $\mathcal{L}_{\dot\beta}$ $\mathcal{L}_p$ $\mathcal{L}_r$ $\mathcal{L}_{\delta_a}$ $\mathcal{L}_{\delta_r}$
Yaw	$\mathcal{N} = \frac{1}{2}\rho U^2 S b C_n$	$\frac{1}{I_z}$	$\mathcal{N}_\beta$ $\mathcal{N}_{\dot\beta}$ $\mathcal{N}_p$ $\mathcal{N}_r$ $\mathcal{N}_{\delta_a}$ $\mathcal{N}_{\delta_r}$

	ANGLES	ANGULAR RATES
Independent Variables	β δ_a δ_r	$\dot\beta$ p r
Compensating Factors	None	$\frac{2U_o}{b}$

CHAPTER 2

LONGITUDINAL STATIC STABILITY

2.1 DEFINITION OF LONGITUDINAL STATIC STABILITY

Static stability is the reaction of a body to a disturbance from equilibrium. To determine the static stability of a body, the body must be initially disturbed from its equilibrium state. If when disturbed from equilibrium, the body returns to its original equilibrium position, the body displays positive static stability or is stable. If the body remains in the disturbed position, the body is said to be neutrally stable. However, should the body, when disturbed, continue to displace from equilibrium, the body has negative static stability or is unstable.

Longitudinal static stability or "gust stability" of an aircraft is determined similarly. If an aircraft in equilibrium is momentarily disturbed by a vertical gust, the resulting change in angle of attack causes changes in lift coefficients on the aircraft. (Velocity is constant for this time period.) The changes in lift coefficients produce additional aerodynamic forces and moments in this disturbed position. If the aerodynamic forces and moments created tend to return the aircraft to its original undisturbed condition, the aircraft possessses positive static stability or is stable. Should the aircraft remain in the disturbed position, it possesses neutral stability. If the forces and moments cause the aircraft to diverge further from equilibrium, the aircraft possesses negative longitudinal static stability or is unstable.

2.2 ANALYSIS OF LONGITUDINAL STATIC STABILITY

Longitudinal static stability is only a special case for the total equations of motion of an aircraft. Of the six equations of motion, longitudinal static stability is concerned with only one, the pitch equation, that equation describing the aircraft's motion about the y - axis.

$$G_y = \dot{q}I_y - pr(I_z - I_x) + (p^2 - r^2)I_{xz} \qquad (2.1)$$

The fact that theory pertains to an aircraft in straight, steady, symmetrical flight with no unbalance of forces or moments permits longitudinal static stability motion to be independent of the lateral and directional equations of motion. This is not an oversimplification since most aircraft spend much of the flight under symmetric equilibrium conditions. Furthermore the disturbance required for stability determination and the measure of the aircraft's response takes place about the y - axis or in the longitudinal plane.

Since longitudinal static stability is concerned with resultant aircraft pitching moments caused by momentary changes in angle of attack and lift coefficients, the primary stability derivatives become C_{m_α} and $C_{m_{C_L}}$. The value of either derivative is a direct indication of the longitudinal static stability of the particular aircraft.

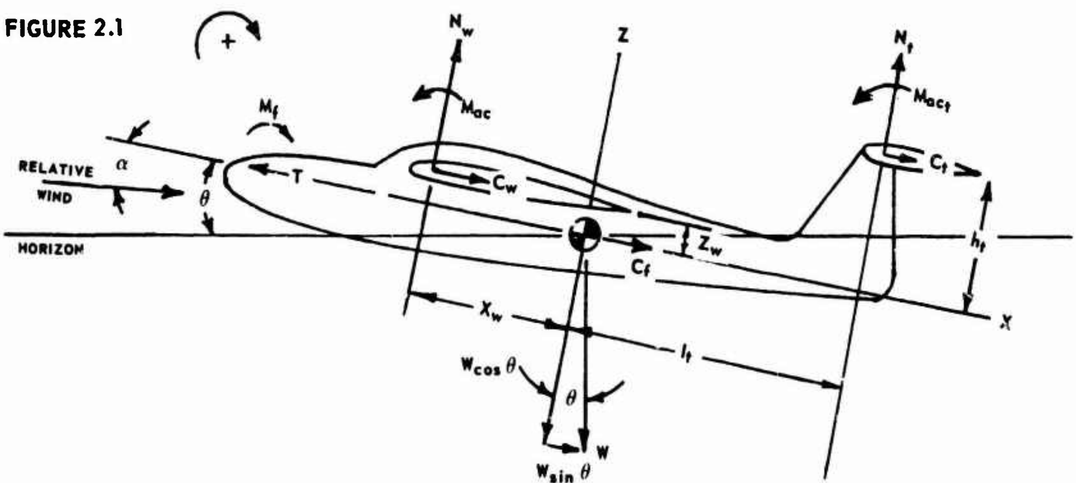
To determine an expression for the derivative, $C_{m_{C_L}}$, an air-

craft in stabilized, equilibrium flight with horizontal stabilizer control surface fixed will be analyzed. A moment equation will be determined from the forces and moments acting on the aircraft. Once this equation is nondimensionalized, in moment coefficient form, the derivative with respect to C_L will be taken. This differential equation will be an expression for $C_{m_{C_L}}$ and will relate directly to the aircraft's stability. Individual term contribution to stability will in turn be analyzed. A flight test relationship for determining the stability of an aircraft will be developed followed by analysis of the aircraft with a free control surface.

2.3 THE STABILITY EQUATION

To derive the longitudinal pitching moment equation, refer to the aircraft in figure 2.1. Writing the moment equation using the sign convention of pitchup being a positive moment and assuming a small angle of attack α so that $\cos\alpha \approx 1$ and $\sin\alpha \approx \alpha$;

FIGURE 2.1



$$\overset{\curvearrowright}{+}\quad M_{CG} = N_w X_w + C_w Z_w - M_{ac} + M_f - N_T \ell_T + C_T h_T - M_{ac_T} \qquad (2.2)$$

If an order of magnitude check is made, some of the terms can be logically eliminated because of their relative size. C_T can be omitted since

$$C_T = \frac{C_w}{10} = \frac{L_w}{100}$$

Mac_T is zero for a symmetrical airfoil horizontal stabilizer section. Rewriting the simplified equation:

$$\overset{\curvearrowright}{+}\quad M_{CG} = N_w X_w + C_w Z_w - M_{ac} + M_f - N_T \ell_T \qquad (2.3)$$

It is convenient to express equation 2.3 in nondimensional coefficient form by dividing both sides of the equation by $q_w S_w c_w$

$$\frac{M_{CG}}{q_w S_w c_w} = \frac{N_w X_w}{q_w S_w c_w} + \frac{C_w Z_w}{q_w S_w c_w} - \frac{M_{ac}}{q_w S_w c_w} + \frac{M_f}{q_w S_w c_w} - \frac{N_T \ell_T}{q_w S_w c_w} \qquad (2.4)$$

Substituting the following coefficients in equation 2.4:

$$C_{m_{CG}} = \frac{M_{CG}}{q_w S_w c_w}$$ total pitching moment coefficient

$$C_{m_{ac}} = \frac{M_{ac}}{q_w S_w c_w}$$ wing aerodynamic pitching moment coefficient

$$C_{m_f} = \frac{M_f}{q_w S_w c_w}$$ fuselage aerodynamic pitching moment coefficient

$$C_N = \frac{N_w}{q_w S_w}$$ wing aerodynamic normal force coefficient

$$C_{N_T} = \frac{N_T}{q_T S_T}$$ tail aerodynamic normal force coefficient

$$C_C = \frac{C_W}{q_w S_w}$$ wing aerodynamic chordwise force coefficient

Equation 2.4 may now be written:

$$C_{m_{CG}} = C_N \frac{X_w}{c} + C_c \frac{Z_w}{c} - C_{m_{ac}} + C_{m_f} - \frac{N_T \ell_T}{q_w S_w c_w} \qquad (2.5)$$

where C and c_W are used interchangeably to represent the mean aerodynamic chord of the wing. To have the tail term in terms of a coefficient, multiply and divide the term by $q_T S_T$

$$\frac{N_T \ell_T}{q_w S_w c_w} \cdot \frac{q_T S_T}{q_T S_T}$$

Substituting tail efficiency factor $\eta_T = q_T/q_w$ and designating tail volume coefficient $V_H = \ell_T S_T / c_W S_W$, Equation (2.5) becomes:

$$C_{m_{CG}} = C_N \frac{X_w}{c} + C_C \frac{Z_w}{c} - C_{m_{ac}} + C_{m_f} - C_{N_T} V_H \eta_T \qquad (2.6)$$

Equation 2.6 is referred to as the equilibrium equation in pitch. If the magnitudes of the individual terms in the above equation are adjusted to the proper value, the aircraft may be placed in equilibrium flight where $C_{m_{CG}} = 0$.

Taking the derivative of equation 2.6 with respect to C_L and assuming that X_w, Z_w, V_H, and η_T do not vary with C_L,

$$\frac{dC_{m_{CG}}}{dC_L} = \underbrace{\frac{dC_N}{dC_L}\frac{X_w}{c} + \frac{dC_C}{dC_L}\frac{Z_w}{c} - \frac{dC_{m_{ac}}}{dC_L}}_{\text{WING}} + \underbrace{\frac{dC_{m_f}}{dC_L}}_{\text{FUSELAGE}} - \underbrace{\frac{dC_{N_T}}{dC_L} V_H \eta_T}_{\text{TAIL}} \qquad (2.7)$$

Equation 2.7 is the stability equation and is related to the stability derivative C_{m_α} by the slope of the lift curve, a.

$$\frac{dC_m}{d\alpha} = a \frac{dC_m}{dC_L} \qquad (2.8)$$

Equation 2.6 and equation 2.7 determine the two criteria necessary for longitudinal stability:

1. The aircraft is balanced.

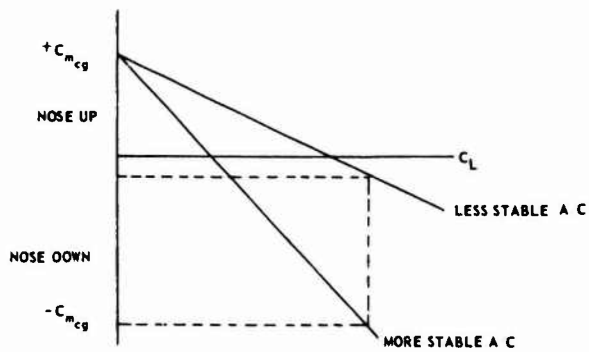
2. The aircraft is stable.

The final condition is satisfied if the pitching moment equation may be forced to $C_{m_{CG}} = 0$ for all useful positive values of C_L. This condition is achieved by

trimming the aircraft so that moments about the center of gravity are zero (i.e., $M_{CG} = 0$).

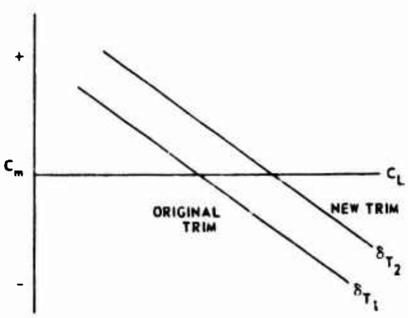
The second condition is satisfied if equation 2.7 or $dC_{m_{CG}}/dC_L$ has a negative value. From figure 2.2 a negative value for equation 2.7 is necessary if the aircraft is to be stable. Should a gust cause an angle of attack increase and a corresponding increase in C_L, a negative $C_{m_{CG}}$ should be produced to return the aircraft to equilibrium, or $C_{M_{CG}} = 0$. The greater the slope or the negative value, the more restoring moment is generated for an increase in C_L. The slope or dC_m/dC_L is a direct measure of the "gust stability" of the aircraft.

FIGURE 2.2



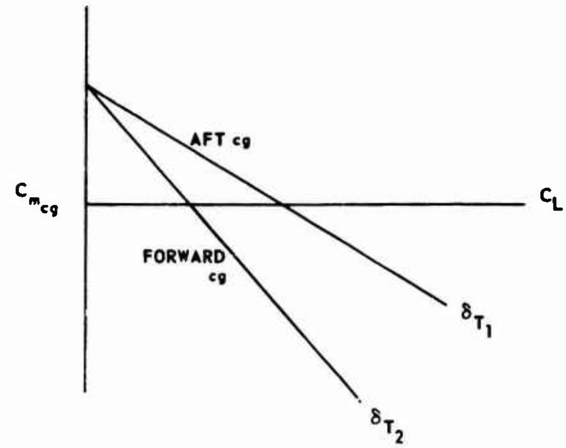
If the aircraft is retrimmed from one angle of attack to another, the basic stability of the aircraft or slope dC_m/dC_L does not change. Note figure 2.3.

FIGURE 2.3



However, if the cg is changed or values of X_w, Z_w, and V_H are changed, the slope or stability of the aircraft is changed. See equation 2.7. For no change in trim tab setting, the stability curve may shift as in figure 2.4.

FIGURE 2.4

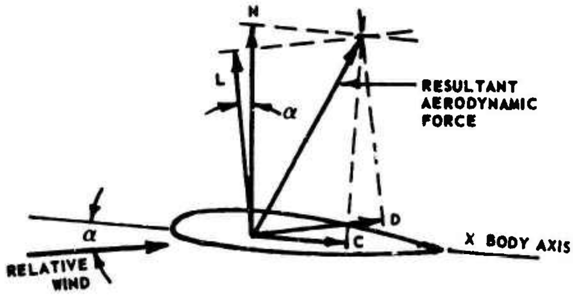


2.4 EXAMINATION OF THE WING, FUSELAGE, AND TAIL CONTRIBUTION TO THE STABILITY EQUATION

<u>The Wing Contribution to Stability:</u>

The lift and drag are by definition always perpendicular and parallel to the relative wind. It is therefore inconvenient to use these forces to obtain moments, for their arms to the center of gravity vary with angle of attack. For this reason, all forces are resolved into normal and chordwise forces whose axes remain fixed with the aircraft and whose arms are therefore constant:

FIGURE 2.5



Assuming the wing lift to be the airplane lift and the angle of attack of the wing to be the airplane's angle of attack, the following relationship exists between the normal and lift forces (figure 2.5):

$$N = L \cos \alpha + D \sin \alpha \qquad (2.9)$$

$$C = D \cos \alpha - L \sin \alpha \qquad (2.10)$$

Therefore, the coefficients are similarly related:

$$C_N = C_L \cos \alpha + C_D \sin \alpha \qquad (2.11)$$

$$C_C = C_D \cos \alpha - C_L \sin \alpha \qquad (2.12)$$

The stability contributions, dC_N/dC_L and dC_C/dC_L, are obtained:

$$\frac{dC_N}{dC_L} = \frac{dC_L}{dC_L} \cos \alpha - C_L \frac{d\alpha}{dC_L} \sin \alpha + \frac{dC_D}{dC_L} \sin \alpha + C_D \frac{d\alpha}{dC_L} \cos \alpha \qquad (2.13)$$

$$\frac{dC_C}{dC_L} = \frac{dC_D}{dC_L} \cos \alpha - C_D \frac{d\alpha}{dC_L} \sin \alpha - \frac{dC_L}{dC_L} \sin \alpha - C_L \frac{d\alpha}{dC_L} \cos \alpha \qquad (2.14)$$

Making an additional assumption that:

$$C_D = C_{D_{parasite}} + \frac{C_L^2}{\pi AR e} \quad \text{and if}$$

$C_{D_{parasite}}$ is constant with change in C_L:

Then $$\frac{dC_D}{dC_L} = \frac{2C_L}{\pi AR e}$$

If the angles of attack are small such that $\cos \alpha = 1.0$ and $\sin \alpha \approx \alpha$, equations 2.13 and 2.14 become:

$$\frac{dC_N}{dC_L} = 1 + C_L \alpha \left(\frac{2}{\pi AR e} - \frac{d\alpha}{dC_L} \right) + C_D \frac{d\alpha}{dC_L} \qquad (2.15)$$

$$\frac{dC_C}{dC_L} = \frac{2}{\pi AR e} C_L - C_D \frac{d\alpha}{dC_L} \alpha - \alpha - C_L \frac{d\alpha}{dC_L} \qquad (2.16)$$

Examining the above equations for relative magnitude,

C_D	is on the order of 0.3
C_L	usually ranges from .2 to 2.0
α	is small, $\approx$ .2 radians
$\frac{d\alpha}{dC_L}$	is nearly constant at .2 radians
$\frac{2}{\pi AR e}$	is on the order of .1

Making these substitutions, equations 2.15 and 2.16 become

$$\frac{dC_N}{dC_L} = 1 - .04 + .06$$

$$= 1.02 \approx 1.0 \qquad (2.17)$$

$$\frac{dC_C}{dC_L} = .1\,C_L - .012 - .2 - .2C_L$$

$$= -(.2 + .1\,C_L) \qquad (2.18)$$

By definition the coefficient of moment about the aerodynamic center is invariant with respect to angle of attack. Therefore,

$$\frac{dC_{m_{ac}}}{dC_L} = 0$$

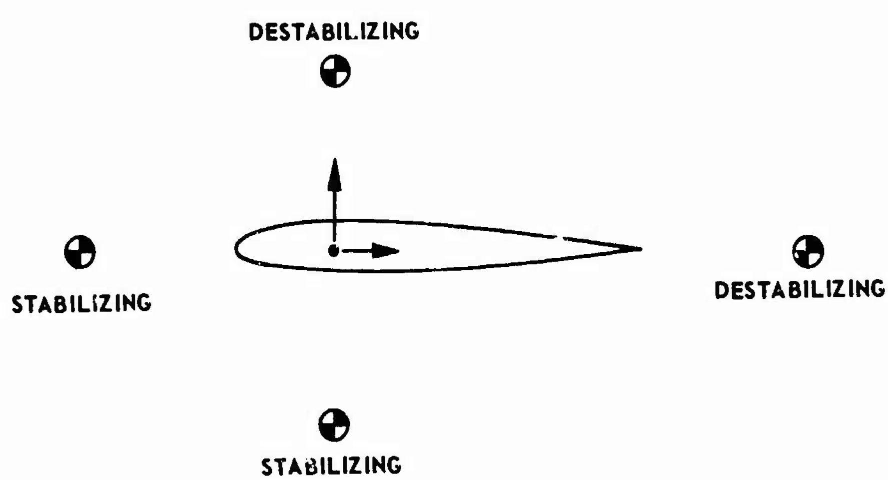
Rewriting the wing contribution of the stability equation 2.7,

$$\frac{dC_m}{dC_{L_{WING}}} = \frac{X_w}{c} - (.2 + .1\,C_L)\,\frac{Z_w}{c} \qquad (2.19)$$

From figure 2.5 when α increases, the normal force increases and the chordwise force decreases. Equation 2.19 shows the relative magnitude of these changes. The position of the cg above or below the aircraft chord (a.c.) has a much smaller effect on stability than does the position of the cg ahead or behind the a.c. With the cg ahead of the a.c., the normal force is stabilizing. From equation 2.19, the more forward the cg location, the more stable the aircraft. With the cg below the a.c., the chordwise force is stabilizing since this force decreases as the angle of attack increases. The further the cg is located below the a.c., the more stable the aircraft or the more negative the value of dC_m/dC_L. The wing contribution to stability depends on the cg and a.c. relationship shown in figure 2.6.

FIGURE 2.6

WING CONSTRUCTION TO STABILITY



For a stable wing contribution to stability, the aircraft should be designed with a high wing aft of the center of gravity.

Symmetrical Wing Contribution.

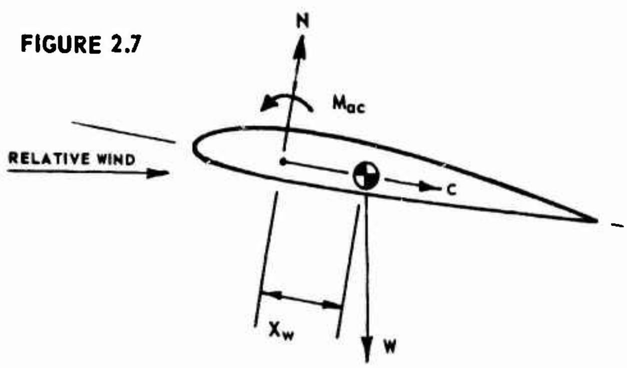
Fighter type aircraft and most low wing, large aircraft have cg's very close to the top of the mean aerodynamic chord. Z_W is on the order of .03. For these aircraft, the chordwise force contribution to stability can be neglected. The wing contribution then becomes:

$$\frac{dC_m}{dC_{L_{WING}}} = \frac{X_w}{c} \qquad (2.20)$$

The Flying Wing.

In order for a flying wing to be a usable aircraft, it must be balanced (fly in equilibrium at a useful positive C_L) and be stable. The problem may be analyzed as follows:

FIGURE 2.7



For the wing in figure 2.7, assuming that the chordwise force acts through the cg, the equilibrium equation in pitch may be written:

$$M_{CG} = NX_w - M_{ac} \qquad (2.21)$$

Writing the equation in coefficient form,

$$C_{m_{CG}} = C_N \frac{X_w}{c} - C_{m_{ac}} \qquad (2.22)$$

For controls fixed, the stability equation becomes,

$$\frac{dC_{m_{CG}}}{dC_L} = \frac{dC_N}{dC_L}\frac{X_w}{c} \qquad (2.23)$$

Equations 2.22 and 2.23 show that the wing in figure 2.7, is balanced and unstable. To make the wing stable, or dC_m/dC_L negative, the center of gravity must be ahead of the wing aerodynamic center. Making this cg change, however, now changes the signs in equation 2.21. The equilibrium and stability equations become:

$$C_{m_{CG}} = -C_N \frac{X_w}{c} - C_{m_{ac}} \qquad (2.24)$$

$$\frac{dC_{m_{CG}}}{dC_L} = -\frac{dC_N}{dC_L}\frac{X_w}{c} \qquad (2.25)$$

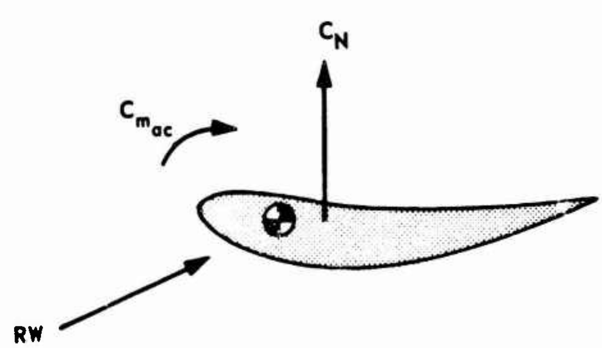
The wing is now stable but unbalanced. The balanced condition is possible with a positive $C_{m_{ac}}$.

Three methods of obtaining a positive $C_{m_{ac}}$ are:

1. Use a negative camber airfoil section. The positive $C_{m_{ac}}$ will give a flying wing that is stable and balanced (figure 2.8).

FIGURE 2.8

NEGATIVE CAMBERED FLYING WING



This type of wing is not realistic because of unsatisfactory dynamic characteristics, small cg range, and extremely low C_L capability.

2. A reflexed airfoil section reduces the effect of camber by creating a download near the trailing edge. Similar results are possible with an upward deflected flap on a symmetrical airfoil.

3. A symmetrical airfoil section in combination with sweep and wing tip washout (reduction in angle of incidence at the tip) will produce a positive Cm_{ac} by virtue of the aerodynamic couple produced between the down loaded tips and the normal lifting force.

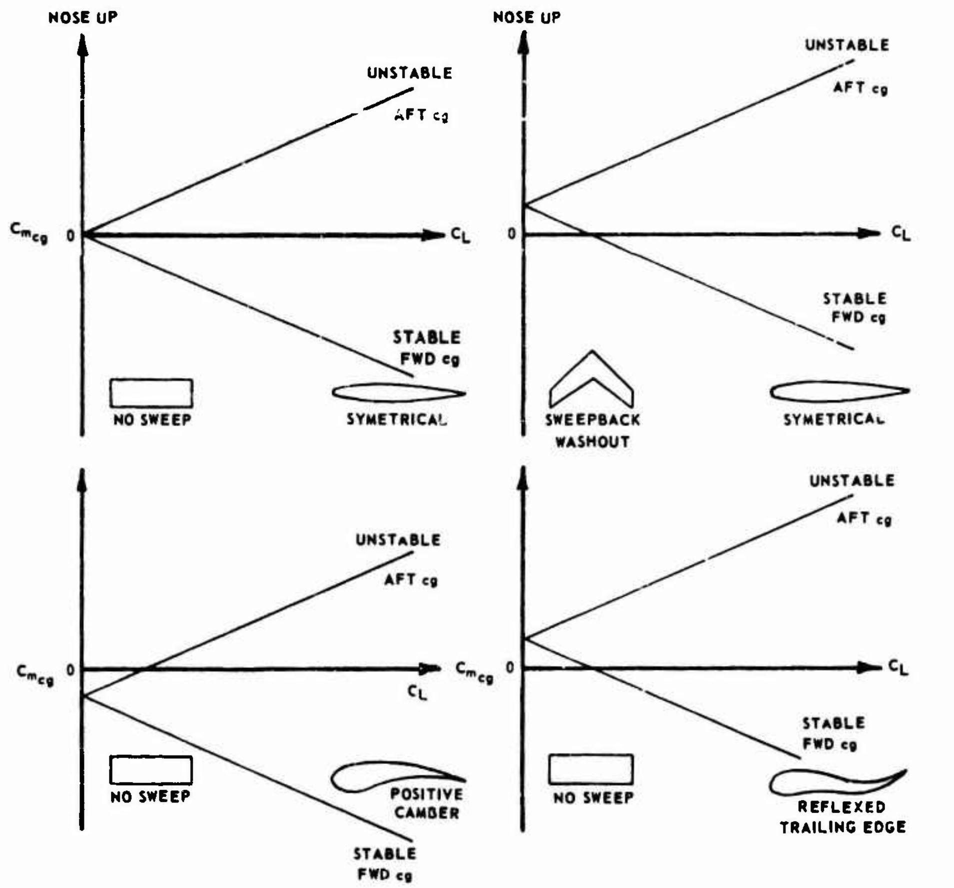
Figure 2.9 shows idealized Cm_{cg} versus C_L for various wings in a control fixed position. Only two of the wings are capable of sustained flight.

<u>The Fuselage Contribution to Stability:</u>

The fuselage contribution is difficult to separate from the wing terms because it is strongly influenced by interference from the wing flow field. Wind tunnel tests of the wing body combination are used by airplane designers to obtain information about the fuselage influence on stability.

A fuselage by itself is almost always destabilizing because the center of pressure is usually

FIGURE 2.9



ahead of the center of gravity. The magnitude of the destabilizing effects of the fuselage requires their consideration in the equilibrium and stability equations.

$$\frac{dC_m}{dC_{L_{Fus}}} = \text{Positive quantity}$$

<u>The Tail Contribution to Stability:</u>

From equation 2.7, the tail contribution to stability was found to be:

$$\frac{dC_m}{dC_{L_{Tail}}} = -\frac{dC_{N_T}}{dC_L} V_H \eta_T \qquad (2.26)$$

For small angles of attack, the lift curve slope of the tail is very nearly the same as the slope of the normal force curve (i.e., $C_{N_T} \approx C_{L_T}$).

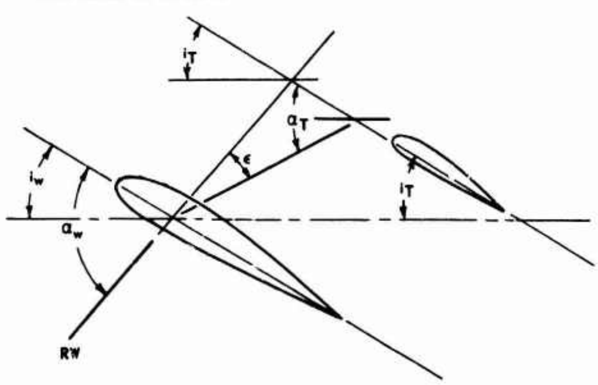
$$a_T = \frac{dC_L}{d\alpha_{Tail}} \approx \frac{dC_N}{d\alpha_{Tail}} \qquad (2.27)$$

Therefore:

$$C_{N_T} = a_T \alpha_T \qquad (2.28)$$

An expression for α_T in terms of C_L is required before solving for dC_{N_T}/dC_L.

FIGURE 2.10



From figure 2.10,

$$\alpha_T = \alpha_w - i_w + i_T - \epsilon \qquad (2.29)$$

Substituting equation 2.29 into 2.28 and taking the derivative with respect to C_L, where $a_w = dC_L/d\alpha$

$$\frac{dC_{N_T}}{dC_L} = a_T\left(\frac{d\alpha_w}{dC_L} - \frac{d\epsilon}{dC_L}\right)$$

$$= a_T\left(\frac{1}{a_w} - \frac{d\epsilon}{d\alpha}\frac{1}{a_w}\right) \qquad (2.30)$$

of factoring out $1/a_w$,

$$\frac{dC_{N_T}}{dC_L} = \frac{a_T}{a_w}\left(1 - \frac{d\epsilon}{d\alpha}\right) \qquad (2.31)$$

Substituting equation 2.31 into 2.26, the expression for the tail contribution becomes,

$$\frac{dC_m}{dC_{L_{Tail}}} = -\frac{a_T}{a_w}\left(1 - \frac{d\epsilon}{d\alpha}\right) V_H \eta_T \qquad (2.32)$$

The value of a_T/a_w is very nearly constant. These values are usually obtained from experimental data.

The tail volume coefficient, V_H, is a term determined by the geometry of the aircraft. To vary this term is to redesign the aircraft.

$$V_H = \frac{\ell_T S_T}{cS} \qquad (2.33)$$

The further the tail is located aft of the cg (increase ℓ_T) or the greater the tail surface area (S_T), the greater the tail volume coefficient V_H which increases the tail contribution to stability.

The expression, η_T, is the ratio of the tail dynamic pressure to the wing dynamic pressure and η_T is greater than unity for a prop aircraft and less than unity for a jet. For power-off considerations, $\eta_T \approx 1.0$.

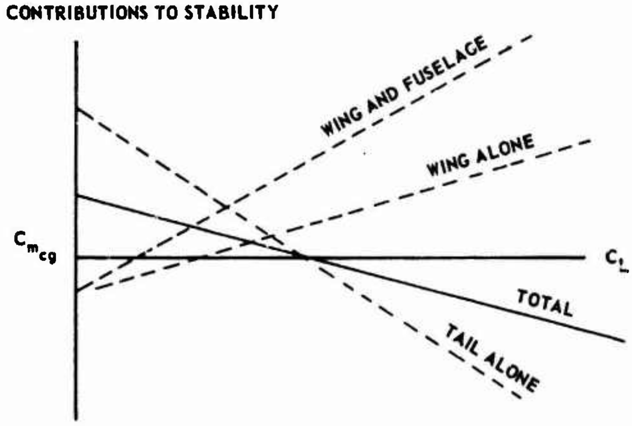
The term $(1 - d\varepsilon/d\alpha)$ is an important factor in the stability contribution of the tail. Large positive values of $d\varepsilon/d\alpha$ produce destabilizing effects by reversing the sign of the term $(1 - d\varepsilon/d\alpha)$ and consequently, the sign of $dC_m/dC_{L_{Tail}}$.

For example at high angles of attack the F-104 experiences a sudden increase in $d\varepsilon/d\alpha$. The term $(1 - d\varepsilon/d\alpha)$ goes negative causing the entire tail contribution to be positive or destabilizing, causing aircraft pitchup. The stability of an aircraft is definitely influenced by the wing vortex system. For this reason the downwash variation with angle of attack should be evaluated in the wind tunnel.

The horizontal stabilizer provides the necessary positive stability contribution (negative dC_m/dC_L) to offset the negative stability of the wing and fuselage combination and to make the entire aircraft stable and balanced (figure 2.11).

FIGURE 2.11

CONTRIBUTIONS TO STABILITY



The stability equation 2.7 may now be written as,

$$\frac{dC_m}{dC_L} = \frac{X_w}{c} + \frac{dC_m}{dC_{L_{Fus}}} - \frac{a_T}{a_w} V_H \eta_T \left(1 - \frac{d\varepsilon}{d\alpha}\right) \tag{2.33a}$$

The Power Contribution to Stability:

The addition of a power plant to the aircraft may have a decided effect on the equilibrium as well as the stability equations. The overall effect may be quite complicated. This section will be a qualitative discussion of the power effects. The actual end result as to the power effects on trim and stability should come from large scale wind tunnel models or actual flight test.

The power effects on a propeller-driven aircraft which influence the static longitudinal stability of the aircraft are:

1. Thrust force effect - effect on stability from the thrust force acting along the propeller axis.

2. Normal force effect - effect on stability from a force normal to the thrust line and in the plane of the propeller.

3. Indirect effects - power plant effects on the stability contribution of other parts of the aircraft.

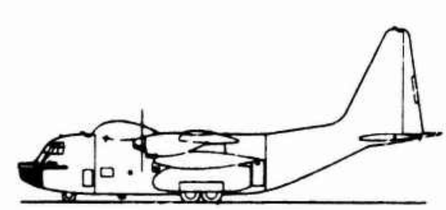
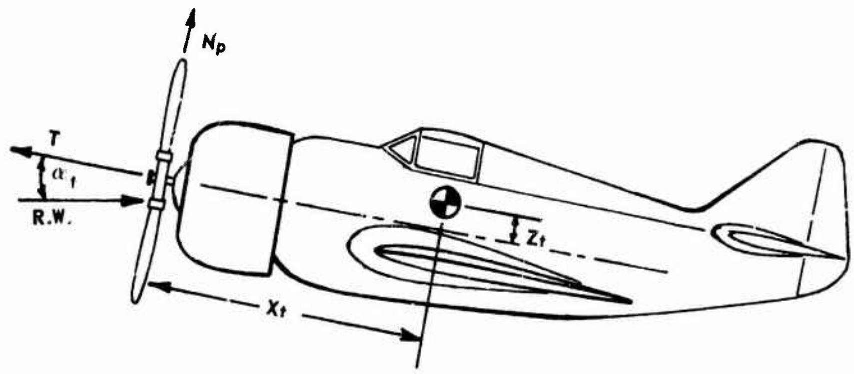


FIGURE 2.12

PROLLER THRUST AND NORMAL FORCE



Writing the moment equation for the power terms as:

$$\overset{+}{\curvearrowright} \quad M_{CG} = TZ_T + N_pX_T \qquad (2.34)$$

In coefficient form,

$$C_{m_{CG}} = C_T \frac{Z_T}{c} + C_{N_p} \frac{X_T}{c} \qquad (2.35)$$

The direct power effect on the aircraft's stability equation is then:

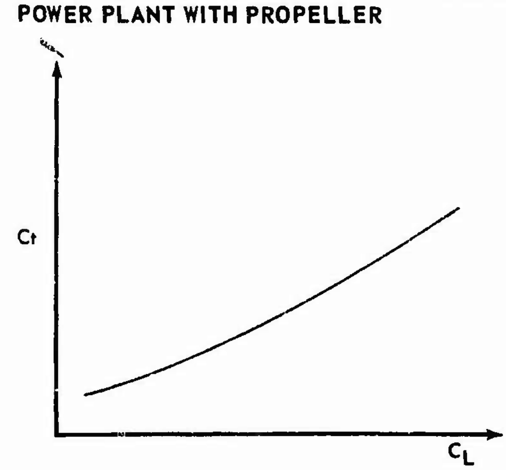
$$\frac{dC_m}{dC_L}_{power} = \frac{dC_T}{dC_L}\frac{Z_T}{c} + \frac{dC_{N_p}}{dC_L}\frac{X_T}{c} \qquad (2.36)$$

The sign of $dC_m/dC_{L_{power}}$ then depends on the sign of the derivatives dC_{N_p}/dC_L and dC_T/dC_L.

We shall first consider the dC_T/dC_L derivative. As speed varies at different flight conditions, throttle position is held constant. Consequently, C_T varies in a manner that can be represented by dC_T/dC_L. The coefficient of thrust for a reciprocating power plant varies with C_L and propeller efficiency. Propeller efficiency which is available from propeller performance estimates in manufacturer's data, decreases with increase in velocity. Coefficient of thrust variation with C_L is nonlinear with the derivative large at low speeds. The combination of these two variations approximately linearize C_T versus C_L (figure 2.13). The sign of dC_m/dC_L is positive.

FIGURE 2.13

COEFFICIENT OF THRUST CURVE RECIPROCATING POWER PLANT WITH PROPELLER



The derivative, dC_{N_p}/dC_L, is positive since the normal propeller force increases linearly with the local angle of attack of the propeller axis, α_T.

The direct power effects are then destabilizing if the cg is as

shown in figure 2.12, or where the power plant is ahead and below the cg.

$$\frac{dC_m}{dC_{L_{power}}} = \frac{dC_T}{dC_L}\frac{Z_T}{c} + \frac{dC_{N_p}}{dC_L}\frac{X_T}{c} \quad (2.37)$$

The indirect power effects must also be considered in evaluating the overall stability contribution of the propeller power plant. No attempt will be made to determine their quantitative magnitudes; however, their general influence on the aircraft's stability and trim condition can be great.

1. Increase of Angle of Downwash, ε:

Since the normal force on the propeller increases with angle of attack under powered flight, the slipstream is deflected downward netting an increase downwash at the tail. The downwash in the slipstream will increase more rapidly with angle of attack than the downwash outside the slipstream. The derivative $d\varepsilon/d\alpha$ has a positive increase with power. The term $(1 - d\varepsilon/d\alpha)$ in equation 2.32 is reduced causing the tail trim contribution to be less negative or less stable than the power-off situation.

2. Increase of $\eta_T = (q_T/q_W)$:

The dynamic pressure, q_T, of the tail is increased by the slipstream and η_T is greater than unity. From equation 2.32, the increase of η_T with addition of a power plant increases the tail contribution to stability. However, if the tail is carrying a download at trim and if it should move into a high velocity region of the slipstream at higher C_L, more of a noseup moment would be present as C_L increased, causing an obvious destablizing effect.

Both slipstream effects mentioned above may be reduced by locating the horizontal stabilizer high on the tail and out of the slipstream at operating angles of attack.

<u>Power Effects on Jet Aircraft.</u>

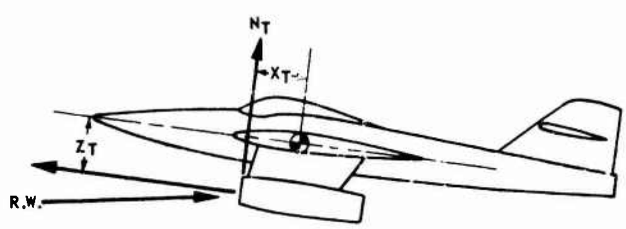
The magnitude of the power effects on jet-powered aircraft are generally smaller than on propeller-driven aircraft. By assuming that jet engine thrust does not change with velocity or angle of attack, and by assuming constant power settings, smaller power effects would be expected than with a similar reciprocating engine aircraft.

There are three major contributions of a jet engine to the equilibrium static longitudinal stability of the aircraft. These are the direct thrust effects, the normal force effects at the air duct inlet, and the indirect effect of the induced flow at the tail.

The thrust and normal force contribution may be determined from figure 2.13a.

FIGURE 2.13a

TEST THRUST AND NORMAL FORCE



Writing the equation,

$$\overset{+}{M_{CG}} = TZ_T + N_TX_T \quad (2.38)$$

or

$$C_{m_{CG}} = \frac{T}{qSc} Z_T + C_{N_T}\frac{X_T}{c} \quad (2.39)$$

With the aircraft in unaccelerated flight, the dynamic pressure is a function of lift coefficient.

$$q = \frac{W}{C_L S} \tag{2.40}$$

Therefore,

$$C_{m_{CG}} = \frac{T}{W}\frac{Z_T}{c}C_L + C_{N_T}\frac{X_T}{c} \tag{2.41}$$

If thrust is considered independent of speed,* then

$$\frac{dC_m}{dC_L} = \frac{T}{W}\frac{Z_T}{c} + \frac{dC_{N_T}}{dC_L}\frac{X_T}{c} \tag{2.42}$$

The thrust contribution to stability then depends on whether the thrust line is above or below the cg. Locating the engine below the cg causes a destablizing influence, and above the cg a stabilizing influence.

The normal force contribution depends on the sign of the derivative dC_{N_T}/dC_L. The normal force N_T is created at the air-duct inlet to the turbojet unit. This force is created as a result of the momentum change of the free stream which bends to flow along the duct axis. The magnitude of the force is a function of the mass airflow rate, W_a, and the angle α_T between the local flow at the duct entrance and the duct axis.

$$N_T = \frac{Wa}{g} V \alpha_T \tag{2.43}$$

With an increase in α_T, N_T will increase, causing dC_{N_T}/dC_L to be positive. The normal force contribution will be destabilizing if the inlet duct is ahead of the center of gravity. The magnitude of the destabilizing moment will depend on the distance the inlet duct is ahead of the center of gravity.

For a jet engine to definitely contribute to positive longitudinal stability, (dC_m/dC_L negative), the jet engine would be located above and behind the center of gravity.

The indirect contribution of the jet unit to longitudinal stability is the effect of the jet induced downwash at the horizontal tail. This applies to the situation where the jet exhaust passes under or over the horizontal tail surface. The jet exhaust as it discharges from the tail pipe spreads outward. Turbulent mixing causes outer air to be drawn in towards the exhaust area. Downwash at the tail is directly affected. With the exhaust below the tail surface, the downwash is increased, causing the tail term to be less stabilizing.

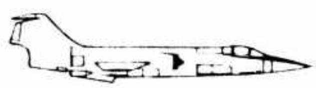
From the above discussion it can be seen that several factors are important in deciding the power effect on stability. Each aircraft must be examined individually. This is the reason that aircraft are tested for stability in several configurations and at different power settings.

2.5 THE NEUTRAL POINT

The stick-fixed neutral point is defined as the center of gravity position at which the aircraft displays neutral stability or where $dC_m/dC_L = 0$.

The symbol h is used for center of gravity position where,

$$h = \frac{X_{CG}}{c} \tag{2.44}$$



* For aircraft which have large thrust variation with airspeed, the pitching moment coefficient must be calculated for different values of the aircraft's lift coefficient.

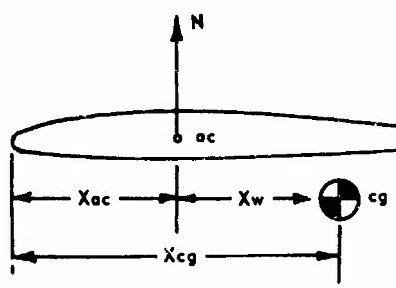
The stability equation for the powerless aircraft is:

$$\frac{dC_m}{dC_L} = \frac{X_w}{c} + \frac{dC_m}{dC_{L_{Fus}}} - \frac{a_T}{a_w} V_H \eta_T \left(1 - \frac{d\varepsilon}{d\alpha}\right) \tag{2.45}$$

Looking at the relationship between cg and a.c. in figure 2.14,

FIGURE 2.14

cg AND A.C. RELATIONSHIP



$$\frac{X_w}{c} = h - \frac{X_{ac}}{c} \tag{2.46}$$

Substituting equation 2.46 into equation 2.45 and setting dC_m/dC_L equal to zero,

$$\frac{dC_m}{dC_L} = 0 = h - \frac{X_{ac}}{c} + \frac{dC_m}{dC_{L_{Fus}}} - \frac{a_T}{a_w} V_H \eta_T \left(1 - \frac{d\varepsilon}{d\alpha}\right) \tag{2.47}$$

Solving for h which is h_n,

$$h_h = \frac{X_{ac}}{c} - \frac{dC_m}{dC_{L_{Fus}}} + \frac{a_T}{a_w} V_H \eta_T \left(1 - \frac{d\varepsilon}{d\alpha}\right) \tag{2.48}$$

Substituting equation 2.48 back into equation 2.47, the stick-fixed stability derivative in terms of cg positions becomes,

$$\frac{dC_m}{dC_L} = h - h_n \tag{2.49}$$

The stick-fixed static stability is equal to the distance between the cg position and the neutral point in percent of the mean aerodynamic chord. "Static Margin" refers to the same distance but is positive in sign for a stable aircraft.

$$\text{"Static Margin"} = h_n - h \tag{2.50}$$

It is the test pilot's responsibility to evaluate the aircraft's handling qualities and to determine the acceptable static margin for the aircraft.

● 2.6 ELEVATOR POWER

As previously mentioned, for an aircraft to be a usable flying machine, it must possess stability and must be capable of being placed in equilibrium ($C_{m_{cg}} = 0$) throughout the useful C_L range (balanced).

For trimmed or equilibrium flight, $C_{m_{cg}}$ must be zero. Some means must be available for balancing the various terms in the moment coefficient of equation 2.51,

$$C_{m_{CG}} = C_N \frac{X_w}{c} + C_C \frac{Z_w}{c} + C_{m_{ac}} + C_{m_f} - a_T \alpha_T V_H \eta_T \tag{2.51}$$

Several possibilities are available. The center of gravity could be moved fore and aft or up and down thus changing X_w/C or Z_w/C. However, this would not only affect the equilibrium lift coefficient but would also change the stability dC_m/dC_L in the stability equation 2.52. This is undesirable.

$$\frac{dC_m}{dC_L} = \frac{dC_N}{dC_L}\frac{X_w}{c} + \frac{dC_C}{dC_L}\frac{Z_w}{c} + \frac{dC_m}{dC_{L_{Fus}}}$$

$$- \frac{a_T}{a_w} V_H \eta_T \left(1 - \frac{d\varepsilon}{d\alpha}\right) \qquad (2.52)$$

The pitching moment coefficient about the aerodynamic center could be changed by effectively changing the camber of the wing by using trailing edge flaps as is done in flying wing vehicles. On the conventional tail-to-the-rear aircraft, trailing edge wing flaps are ineffective in trimming the pitching moment coefficient to zero.

The remaining solution is to change the angle of attack of the horizontal stabilizer to achieve a $C_{m_{cg}} = 0$ without a change to the basic aircraft stability. The control means is either an elevator on the stabilizer or an all moving stabilizer (called a slab). The slab is used in most high speed aircraft and is the most powerful means of longitudinal control.

Movement of the slab or elevator changes the effective angle of attack of the horizontal stabilizer and, consequently, the lift on the horizontal tail. This in turn changes the moment about the center of gravity due to the horizontal tail. It is of interest to know the amount of pitching moment change associated with a degree of elevator deflection. This may be determined by differentiating equation 2.51 with respect to δ_e.

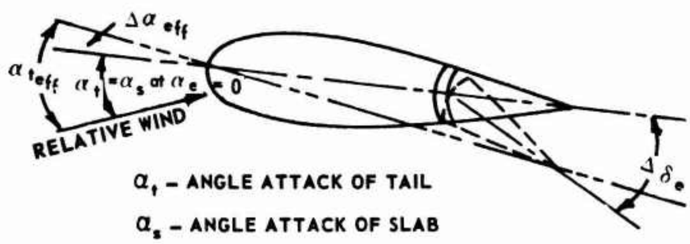
$$\frac{dC_m}{d\delta_e} = C_{m_{\delta_e}} = - a_T V_H \eta_T \frac{d\alpha_T}{d\delta_e} \qquad (2.53)$$

This change in pitching moment coefficient with respect to elevator deflection $C_{m_{\delta_e}}$ is referred to as "elevator power." It indicates the capability of the elevator in producing moments about the center of gravity.

The term $d\alpha_T/d\delta_e$ in equation 2.53 is termed elevator effectiveness and is given the shorthand notation τ. The elevator effectiveness may be considered as the equivalent change in effective tail plane angle of attack per unit change in elevator deflection. The relationship between elevator effectiveness τ and the effective angle of attack of the stabilizer is seen in figure 2.15.

FIGURE 2.15

CHANGE IN EFFECTIVE ANGLE OF ATTACK WITH ELEVATOR DEFLECTION



As seen, elevator effectiveness is a design parameter and is determined from wind tunnel tests. Elevator effectiveness is a negative number for all tail to the rear aircraft. The values range from zero to the limiting case of the all moving stabilizer (slab) where τ equals (-1). The tail angle of attack would change plus one degree for every minus degree the slab moves. For the elevator stabilizer combination, the elevator effectiveness is a function of the ratio of overall elevator area to the entire horizontal tail area.

2.7 STABILITY CURVES

Figure 2.16 is a wind tunnel plot of C_m versus C_L for an aircraft tested under two cg positions and two elevator positions.

FIGURE 2.16

cg AND δ_e VARIATION ON STABILITY

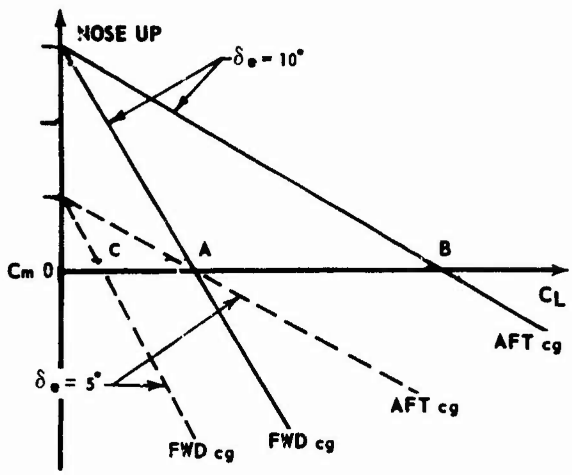
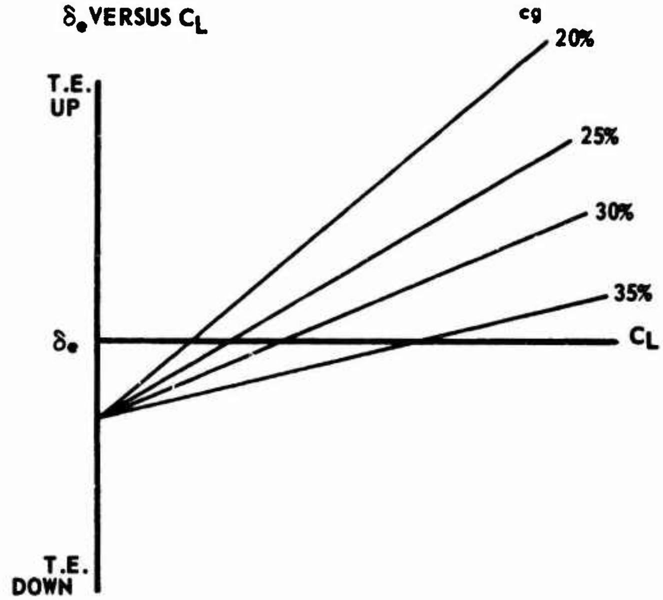


FIGURE 2.17

δ_e VERSUS C_L



Assuming the elevator effectiveness and the elevator power to be constant, then equal elevator deflections produce equal moments about the cg. Points A and B represent the same elevator deflection corresponding to the $C_{m_{cg}}$ needed to maintain equilibrium. The pilot selects elevator deflection of 10 degrees. In the aft cg condition, the aircraft will fly in equilibrium at point B. If the cg is moved forward with no change to the elevator deflection, the equilibrium point is now at A or at a new C_L. Note the increase in the stability of the aircraft (greater negative slope dC_m/dC_L).

If the pilot desires to fly at a lower C_L or at A and not change the cg, he does so by deflecting the elevator to 5 degrees. The stability level of the aircraft has not changed (same slope).

A cross plot of figure 2.3 is elevator deflection versus C_L for $C_m = 0$. This is shown in figure 2.17. The slopes of the cg curves are indicative of the aircraft's stability.

● 2.8 FLIGHT TEST RELATIONSHIP

The stability equation 2.52 derived previously pertains to theoretical applications and text book solutions. The equation has no use in flight testing. There is no aircraft instrumentation which will measure the change in pitching moment coefficient with change in lift coefficient or angle of attack. Therefore an expression involving parameters easily measurable in flight is required. This expression should relate directly to the stick-fixed longitudinal static stability dC_m/dC_L of the aircraft.

The external moment acting longitudinally on an aircraft is:

$$M = f(\alpha, \dot{\alpha}, q, V, \delta_e) \qquad (2.54)$$

Assuming further that the aircraft is in equilibrium and in unaccelerated flight, then

$$M = f(\alpha, \delta_e) \qquad (2.55)$$

Therefore,

$$\Delta M = \frac{\partial M}{\partial \alpha}\Delta\alpha + \frac{\partial M}{\partial \delta_e}\Delta\delta_e \qquad (2.56)$$

and

$$C_m = C_{m_\alpha}\alpha + C_{m_{\delta_e}}\delta_e = 0 \qquad (2.57)$$

where $\Delta\alpha = \alpha - \alpha_o = \alpha$

$\Delta\delta_e = \alpha_e - \delta_{e_o} = \delta_e$

assuming $\alpha_o = 0$

$\delta_{e_o} = 0$

The elevator deflection required to maintain equilibrium is,

$$\delta_e = -\frac{C_{m_\alpha}\alpha}{C_{m_{\delta_e}}} \qquad (2.58)$$

Taking the derivative of δ_e with respect to C_L,

$$\frac{d\delta_e}{dC_L} = -\frac{\frac{dC_m}{d\alpha}\frac{d\alpha}{dC_L}}{C_{m_{\delta_e}}} = -\frac{\frac{dC_m}{dC_L}}{C_{m_{\delta_e}}} \qquad (2.59)$$

In terms of the static margin, the flight test relationship is,

$$\frac{d\delta_e}{dC_L} = \frac{h_n - h}{C_{m_{\delta_e}}} \qquad (2.60)$$

The amount of elevator required to fly at equilibrium varies directly as the amount of static stick-fixed stability and inversely as the amount of elevator power.

2.9 LIMITATION TO DEGREE OF STABILITY

The degree of stability tolerable in an aircraft is determined by the physical limits of the longitudinal control. The elevator power and amount of elevator deflection is fixed once the aircraft has been designed. If the relationship between δ_e required to maintain the aircraft in equilibrium flight and C_L is linear, then the elevator deflection required to reach any C_L is,

$$\delta_e = \delta_{e_{\text{Zero Lift}}} + \frac{d\delta_e}{dC_L}C_L \qquad (2.61)$$

The elevator stop determines the absolute limit of the elevator deflection available. Similarly, the elevator must be capable of bringing the aircraft into equilibrium at $C_{L_{Max}}$.

Recalling the flight test relationship,

$$\frac{d\delta_e}{dC_L} = \frac{-\frac{dC_m}{dC_L}}{C_{m_{\delta_e}}} \qquad (2.62)$$

Substituting equation 2.62 into 2.61 and solving for $dC_m/dC_{L_{Max}}$ corresponding to $C_{L_{Max}}$

$$\frac{dC_m}{dC_{L_{Max}}} = \frac{\left(\delta_{e_{\text{Zero Lift}}} - \delta_{e_{\text{Limit}}}\right)}{C_{L_{Max}}} C_{m_{\delta_e}} \qquad (2.63)$$

Given a maximum C_L required for landing approach, equation 2.63 represents the maximum stability possible, or defines the most forward cg movement. A cg forward of this point prevents obtaining maximum C_L with limit elevator.

If a pilot were to maintain the $C_{L_{Max}}$ for the approach, the value of dC_m/dC_L corresponding to this $C_{L_{Max}}$ would be satisfactory. However, as is the case, the pilot desires additional C_L to maneuver as in flaring the aircraft. Additional elevator is required. This requirement then dictates a $dC_m/dC_{L_{Max}}$ less than the value required for $C_{L_{Max}}$ only.

In addition to maneuvering the aircraft in the landing flare, the pilot must adjust for ground effect. The ground imposes a boundary condition which affects the downwash associated with the lifting action of the wing. This ground interference places the horizontal stabilizer at a reduced angle of attack. The equilibrium condition at the desired C_L is disturbed. To maintain the desired C_L, the pilot must increase δ_e to obtain the original tail angle of attack. The maximum stability dC_m/dC_L must be further reduced to obtain additional δ_e to counteract the reduction in downwash.

The three conditions that limit the amount of static longitudinal stability or most forward cg position are:

a. The ability to land at high C_L in ground effect.

b. The ability to maneuver at landing C_L (flare capability).

c. The total elevator deflection available.

Figure 2.17A illustrates the limitations in $dC_m/dC_{L_{Max}}$.

2.10 STICK-FREE STABILITY

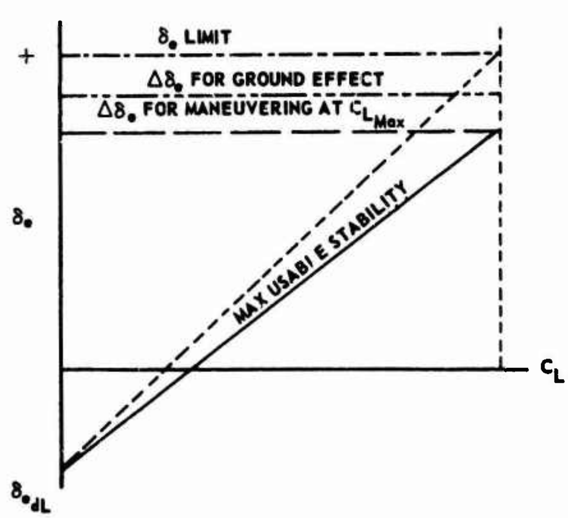
The name stick-free stability comes from the era of reversible control systems and is that variation related to the longitudinal stability which an aircraft would possess if the longitudinal control surface were left free to float in the slip stream. The control force variation with a change in airspeed is a measure of this stability.

If an airplane had an elevator that would float in the slip stream when the controls were free, then the change in the dynamic pressure pattern of the stabilizer would cause a change in the stability level of the airplane. The change in the stability contribution of the tail would be manifested by the floating characteristics of the elevator. Thus, the stick-free stability would depend upon the elevator hinge moments, control friction, or any device that would affect the moment of the elevator.

An airplane with an irreversible control system has very little tendency for its elevator to float. Yet the control forces presented to the pilot during flight, even though artificially produced, appear to be the effects of having a free elevator. If the control feel system can be altered artifically, then the pilot will see only good handling qualities and be able to fly

FIGURE 2.17a

LIMITATIONS ON $dC_m/dC_{L_{Max}}$



what would normally be an unsatisfactory flying machine.

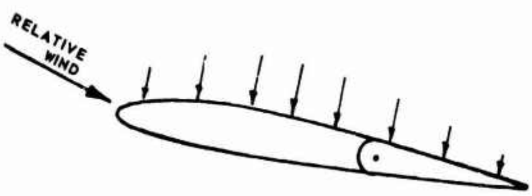
Stick-free stability can be analyzed by considering the effect of freeing the elevator of a tail-to-the-rear aircraft with a reversible control system. In this case the stick free stability would be indicated by the stick forces required to maintain the airplane in equilibrium at some speed other than trim.

The change in stability due to freezing the elevator, is a function of the floating characteristics of the elevator. The floating characteristics depend upon the elevator hinge moments which depend upon the change in pressure distribution over the elevator associated with changes in elevator deflection and tail angle of attack.

The analysis will look at the effect that pressure distribution has on the elevator hinge moments, the floating characteristics of the elevator, and then the effects of freeing the elevator.

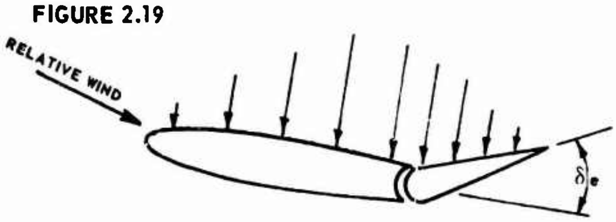
For a standard stable tail to the rear airplane, the pressure distribution would produce a downward load on the tail.

FIGURE 2.18



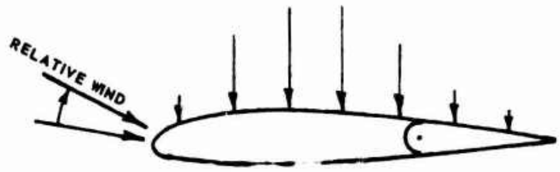
When the elevator is deflected the pressure distribution is changed.

FIGURE 2.19



When the stabilizer angle of attack is changed the pressure distribution is also changed.

FIGURE 2.20

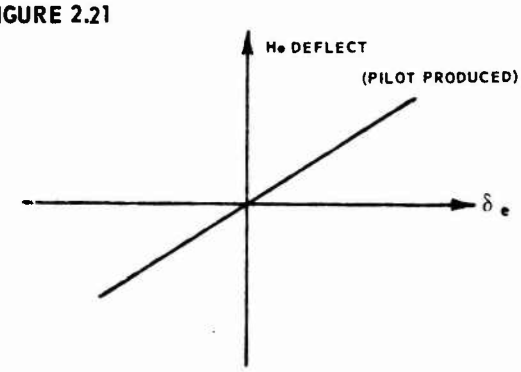


When the pressure distribution is changed, the hinge moments are changed. In order to deflect the elevator, the pilot had to apply a force to the stick and create a moment on the elevator hinge. The elevator hinge moment the pilot applied is now balanced by a moment caused by the pressure distribution on the control surface, and the elevator remains in the deflected position.

The pilot normally pulls back on the stick in order to produce a pitchup moment on the airplane. The hinge moment produced tends to move the control such that a positive moment on the airplane results. Therefore, the hinge moment is called positive. The pilot applies a positive moment to move the elevator. The pressure distribution produces a negative moment that opposes that of the pilot.

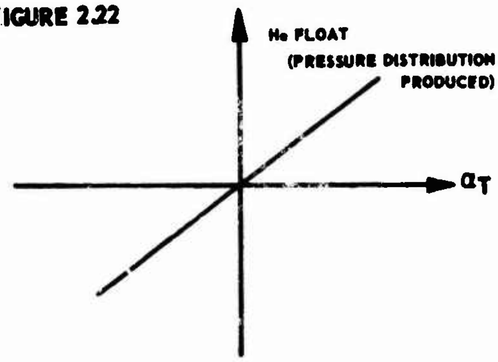
A plot of the pilot's hinge moment to deflect the elevator would be:

FIGURE 2.21



The hinge moment produced by the pressure distribution would be as shown in figure 2.22.

FIGURE 2.22

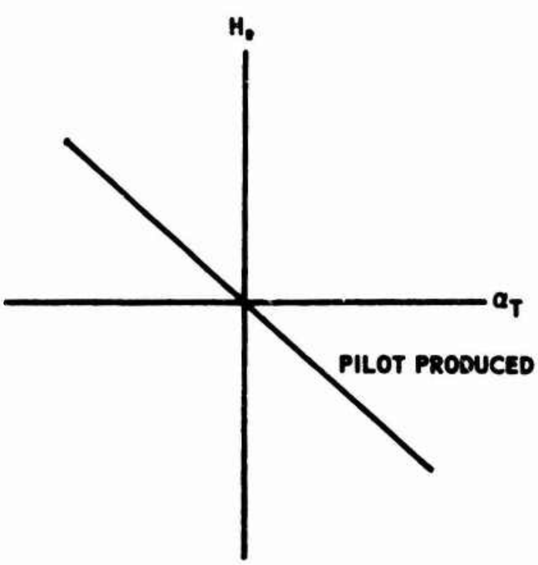


When the stabilizer angle of attack (α_T) is changed, the pilot must produce a control force in order to keep the elevator from floating in the slip stream.

Normally as the angle of attack is increased the elevator would tend to float up and the pilot would have to apply a negative push force in order to keep the stick from moving.

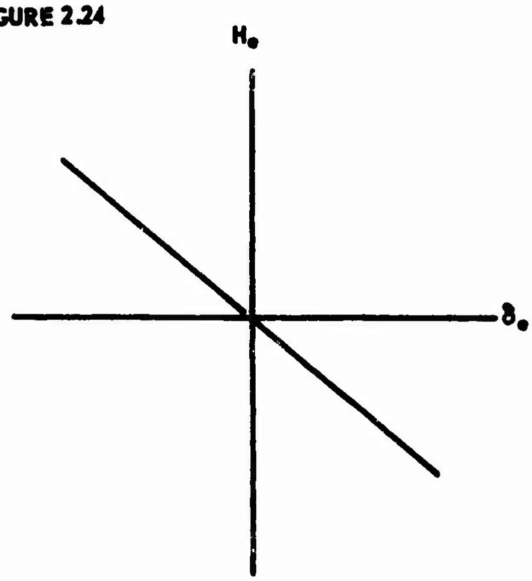
The hinge moment produced by the pilot to maintain trim deflection would be:

FIGURE 2.23



The hinge moment produced by the pressure distribution to float the elevator would be as shown in figure 2.24.

FIGURE 2.24



If we consider the moments produced by the pressure distribution on the elevator only, then we could analyze the floating characteristics of the elevator.

The hinge moments can be put in coefficient form in much the same manner as the airplane's aerodynamic moments. The H_e Restore Slope due to elevator deflection in coefficient form would be:

$$\frac{\delta C_h}{\delta \delta_e} \text{ Restore} = C_{h_\delta} \qquad (2.64)$$

The He_{Float} slope due to angle of attack change in coefficient form would be:

$$\frac{\delta C_h}{\delta \alpha_e} \text{ Float} = C_{h_\alpha} \qquad (2.65)$$

Examining a floating elevator, it is seen that the total hinge moment is a function of elevator deflection, angle of attack, and mass distribution.

$$H_e = f(\delta_e, \alpha_T, W) \tag{2.66}$$

If the elevator is held at zero elevator deflection and zero angle of attack there may be some residual aerodynamic hinge moment Ch_o. The total hinge moment where W = weight of the elevator would be:

$$C_h = C_{h_o} + C_{h_\alpha}\alpha_T + C_{h_\delta}\delta_e + \frac{W}{qS}\frac{X}{c} \tag{2.67}$$

The weight effect is usually eliminated by mass balancing the elevator. Proper design of a symmetrical airfoil will cause Ch_o to be negligible.

When the elevator assumes its equilibrium position the total hinge moments will be zero and solving for the elevator deflection at this floating position.

$$\delta_{e_{Float}} = -\frac{C_{h_\alpha}}{C_{h_\delta}}\alpha_T \tag{2.68}$$

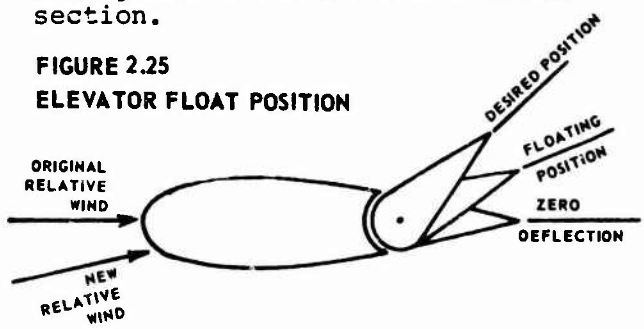
The stability of the aircraft with the elevator free is going to be affected by this floating position.

If the pilot desires to hold a new angle of attack from trim, he will have to deflect the elevator from this floating position to the position desired.

The floating position will greatly affect the forces the pilot is required to use. If the ratio Ch_α/Ch_δ can be adjusted, then the forces the pilot is required to use can be controlled.

If Ch_α/Ch_δ is small, then the elevator will not float very far and the stick-free stability characteristics will be much the same as those with the stick-fixed. But Ch_δ must be small or the stick forces required to hold deflection will be unreasonable. The values of Ch_α and Ch_δ can be controlled by aerodynamic balance. Types of aerodynamic balancing will be covered in a later section.

FIGURE 2.25
ELEVATOR FLOAT POSITION



2.11 STICK-FREE STABILITY EQUATIONS

Stick free stability may be considered the summation of the stick-fixed stability and the contribution to stability of freeing the elevator.

$$\frac{dC_m}{dC_{L_{Stick\text{-}Free}}} = \frac{dC_m}{dC_{L_{Stick\text{-}Fixed}}} + \frac{dC_m}{dC_{L_{Freeing\ Elevator}}} \tag{2.69}$$

Solving first for the effect to stability of freeing the elevator,

$$\frac{dC_m}{dC_{L_{Free\ Elev.}}} = \frac{dC_m}{d\delta_e}\frac{d\delta_e}{dC_L} = C_{m_{\delta_e}}\frac{d\delta_e}{dC_L} \tag{2.70}$$

The stability contribution of the free elevator depends upon the elevator floating position. Equation 2.68 relates this position.

$$\delta_{e_{Float}} = -\frac{C_{h_\alpha}}{C_{h_\delta}}\alpha_T \tag{2.71}$$

Substituting for α_T from figure 2.10,

$$\delta_{e_{Float}} = -\frac{C_{h_\alpha}}{C_{h_\delta}}(\alpha_w - i_w + i_T - \varepsilon) \quad (2.72)$$

Taking the derivative of equation 2.72 with respect to C_L,

$$\frac{d\delta_e}{dC_L} = -\frac{C_{h_\alpha}}{C_{h_\delta}} \frac{\left(1 - \frac{d\varepsilon}{d\alpha}\right)}{a_w} \quad (2.73)$$

Solving for $dC_m/dC_{L_{Free\ Elev.}}$ in equation 2.70 and substituting the expression for elevator power,

$$C_{m_{\delta_e}} = -a_T \tau V_H \eta_T \quad (2.74)$$

$$\frac{dC_m}{dC_{L_{Free\ Elevator}}} = -\frac{a_T}{a_w} V_H \eta_T \left(1 - \frac{d\varepsilon}{d\alpha}\right)\left(-\tau \frac{C_{h_\alpha}}{C_{h_\delta}}\right) \quad (2.75)$$

Substituting equation 2.75 and equation 2.33a ($dC_m/dC_{L_{Fixed}}$) into equation 2.69, the stick-free stability becomes

$$\frac{dC_m}{dC_{L_{Stick\ Free}}} = \frac{X_w}{c} + \frac{dC_m}{dC_{L_{Fus}}} - \frac{a_T}{a_w} V_H \eta_T \left(1 - \frac{d\varepsilon}{d\alpha}\right)\left(1 - \tau \frac{C_{h_\alpha}}{C_{h_\delta}}\right) \quad (2.76)$$

The difference between stick-fixed and stick-free stability is the multiplier in equation 2.76, $(1 - \tau\, C_{h_\alpha}/C_{h_\delta})$, called the "free elevator factor" and which is designated F. The magnitude and sign of F depends on the relative magnitudes of τ and the ratio of $C_{h_\alpha}/C_{h_\delta}$. An elevator with only slight floating tendency has a small $C_{h_\alpha}/C_{h_\delta}$ giving a value of F around unity. The stick fixed and stick free stability are practically the same. If the elevator has a large floating tendency (ratio of $C_{h_\alpha}/C_{h_\delta}$ large), the stability contribution of the horizontal tail is reduced materially ($dC_m/dC_{L_{Free}}$ is less negative).

For instance, a ratio of $C_{h_\alpha}/C_{h_\delta}$ = -2 and a τ of -.5, the floating elevator can obviate the whole tail contribution to stability. Generally, freeing the elevator causes a destablizing effect. With elevator free to float, the aircraft is less stable.

The stick-free neutral point, h_n', is that cg position at which $dC_m/dC_{L_{Free}}$ is zero. Continuing as in the stick-fixed case, the stick-free neutral point is,

$$h_n' = \frac{X_{ac}}{c} - \frac{dC_m}{dC_{L_{Fus}}} + \frac{a_T}{a_w} V_H \eta_T \left(1 - \frac{d\varepsilon}{d\alpha}\right) F \quad (2.77)$$

and

$$\frac{dC_m}{dC_{L_{Free}}} = h - h_n' \quad (2.78)$$

The stick-free static margin is defined as,

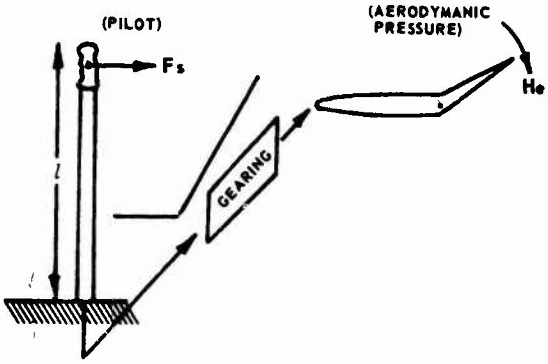
$$\text{Static Margin} = h_n' - h \quad (2.79)$$

2.12 STICK-FREE FLIGHT TEST RELATIONSHIP

As was done for stick-fixed stability, a flight test relationship is required that will relate measureable flight test parameters with the stick-free stability of

the aircraft $dC_m/dC_{L_{Free}}$. This relationship may be developed with reference to figure 2.26.

FIGURE 2.26
ELEVATOR-STICK GEARING



The pilot holds a stick deflected with a stick force F_s. The control system transmits the moment from the pilot through the gearing to the elevator. The elevator deflects and the aerodynamic pressure produces a hinge moment at the elevator that exactly balances the moment produced by the pilot with force F_s.

$$F_s \ell = - G'H_e \quad (2.80)$$

If the length ℓ is included with the gearing, the stick force becomes,

$$F_s = - GH_e \quad (2.81)$$

The hinge moment H_e may be written,

$$H_e = C_{h_e} q S_e c_e \quad (2.82)$$

Equation 2.81 then becomes,

$$F_s = - GC_{h_e} q S_e c_e \quad (2.83)$$

Substituting

$$C_{h_e} = C_{h_o} + C_{h_{\alpha_T}} \alpha_T + C_{h_{\delta_e}} \delta_e + C_{h_{\delta_T}} \delta_T \quad (2.84)$$

where $C_{h_{\delta_T}} \delta_T$ represents the tab contribution for an elevator with tab

where

$$\delta_e = \delta_{e_{zero\ lift}} + \frac{d\delta_e}{dC_L} C_L \quad (2.85)$$

$$\alpha_T = \alpha_w - i_w + i_T - \varepsilon \quad (2.86)$$

Equation 2.83 may be written,

$$F_s = \overbrace{- GS_e c_e q}^{A}$$

$$\overbrace{\left(C_{h_o} + C_{h_\alpha} (\alpha_{OL} - i_w + i_T) + C_{h_{\delta_e}} \delta_e\right.}^{B}$$

$$\left. + C_{h_{\delta_T}} \delta_T - \frac{C_L C_{h_{\delta_e}}}{C_{m_{\delta_e}}} \frac{dC_m}{dC_{L_{Free}}} \right) \quad (2.87)$$

Rewriting equation 2.87 with the above substitutions,

$$F_s = Aq \left(B + C_{h_{\delta_T}} \delta_T - \frac{C_L C_{h_{\delta_e}}}{C_{m_{\delta_e}}} \frac{dC_m}{dC_{L_{Free}}} \right) \quad (2.88)$$

Writing equation 2.88 as a function of airspeed and substituting for unaccelerated flight, $C_L q = W/S$ and using equivalent airspeed, V_e,

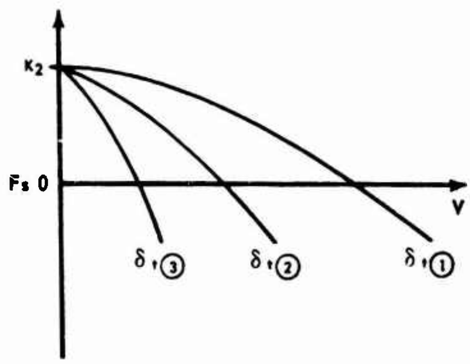
$$F_s = 1/2\, \rho_o V_e^2 A (B + C_{h_{\delta_T}} \delta_T) - A \frac{W}{S} \frac{C_{h_{\delta_e}}}{C_{m_{\delta_e}}} \frac{dC_m}{dC_{L_{Free}}} \quad (2.89)$$

Simplifying equation 2.89 by combining constant terms,

$$F_s = K_1 V_e^2 + K_2 \qquad (2.90)$$

K_1 contains δ_T which determines trim speed. K_2 contains $dC_m/dC_{L_{Free}}$. Equation 2.90 gives a relationship between an inflight measurement of stick force gradient and stick free stability. The equation is plotted in figure 2.27.

FIGURE 2.27
STICK FORCE VERSUS AIRSPEED



The plot is made up of a constant force springing from the stability term plus a variable force proportional to the velocity squared, introduced through some constants and the tab term $C_{h_{\delta_T}}\delta_T$. Equation 2.90 introduces an interesting fact that the stick force variation with airspeed is apparently dependent on the first term only and independent in general of the stability level. That is, the slope of the curve F_s versus V is not a direct function of $dC_m/dC_{L_{Free}}$. If the derivative of equation 2.89 is taken with respect to V, the second term containing the stability drops out.

$$\frac{dF_s}{dV} = \rho_o V_e A(B + C_{h_{\delta_T}} \delta_T) \qquad (2.91)$$

However, dF_s/dV may be made a function of the stability term using another approach. The tab setting δ_T in equation 2.89 should be adjusted to obtain $F_s = 0$. This is δ_T for trim velocity, i.e.,

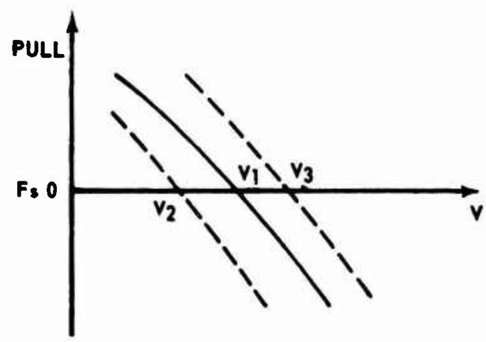
$$\delta_{T_{F_s}} = 0 = f(V_{Trim}, \frac{dC_m}{dC_{L_{Free}}}) \qquad (2.92)$$

This value of $\delta_{T_{F_s}} = 0$ is then substituted into equation 2.91 so that,

$$\frac{dF_s}{dV_{Trim}} = f\,(V_{Trim}, \frac{dC_m}{dC_{L_{Free}}}) \qquad (2.93)$$

Thus it appears that if an aircraft is flown at three cg locations and dF_s/dV_{Trim} through the same trim speed each time is determined, then one could extrapolate or interpolate to determine the stick-free neutral point h_n'. Unfortunately, if there is a significant amount of friction in the control system, it is impossible to precisely determine this trim speed. In order to investigate briefly the effects of friction on the longitudinal control system, suppose that the aircraft represented in figure 2.28 is perfectly trimmed at V_1 (i.e., $\delta_e = \delta_{e1}$ and $\delta_T = \delta_{T_1}$). If the airspeed is decreased or increased with no change to the trim setting, the friction in the control system will

FIGURE 2.28
CONTROL SYSTEM FRICTION



prevent the elevator from returning all the way back to δ_{e1} when the controls are released. The aircraft will return only to V_2 or V_3. With the trim tab at δ_{T1}, the aircraft is content to fly at any speed between V_2 and V_3. The more friction that exists in the system, the wider this speed range becomes.

Therefore, if there is a significant amount of friction in the control system, it becomes impossible to say that there is one exact speed for which the aircraft is trimmed. Equation 2.93 then, is something less than perfect for predicting the stick-free neutral point of an aircraft. To reduce the undesirable effect of friction in the control system, a different approach is made to equation 2.88.

If equation 2.88 is divided by the dynamic pressure q, then,

$$F_s/q = A(B + C_{h_{\delta_T}} \delta_T) - \frac{AC_L C_{h_{\delta_e}}}{C_{m_{\delta_e}}} \frac{dC_m}{dC_{L_{Free}}} \tag{2.94}$$

Differentiating with respect to C_L,

$$\frac{dF_s/q}{dC_L} = - \frac{AC_{h_{\delta_e}}}{C_{m_{\delta_e}}} \frac{dC_m}{dC_{L_{Free}}} \tag{2.95}$$

or

$$\frac{dF_s/q}{dC_L} = f\left(\frac{dC_m}{dC_{L_{Free}}}\right) \tag{2.96}$$

Trim velocity is now eliminated from consideration, and the prediction of stick-free neutral point h_n' is more exact. A plot of $dF_s/q/dC_L$ versus cg position may be extrapolated to obtain h_n'.

● 2.13 APPARENT STICK-FREE STABILITY

Speed stability or stick force gradient dF_s/dV in most cases does not reflect the actual stick-free stability $dC_m/dC_{L_{Free}}$ of an aircraft. In fact this apparent stability dF_s/dV may be quite different from the actual stability of the aircraft. Where the actual stability of the aircraft may be marginal ($dC_m/dC_{L_{Free}}$ small), or even unstable ($dC_m/dC_{L_{Free}}$ positive), the apparent stability dF_s/dV may be such as to make the aircraft quite acceptable. In flight, the test pilot feels and evaluates the apparent stability of the aircraft and not the actual stability $dC_m/dC_{L_{Free}}$.

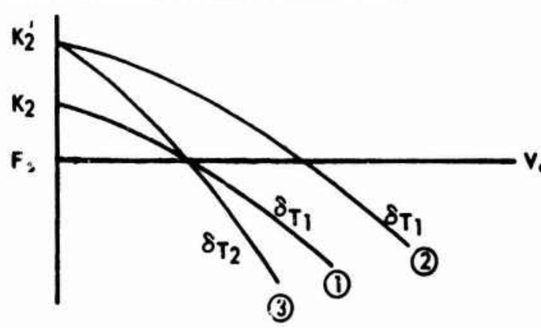
The apparent stability dF_s/dV is affected by:

1. Changes in $dC_m/dC_{L_{Free}}$
2. Aerodynamic balancing
3. Downsprings and/or bob weights.

The apparent stability or the stick force gradient through a given trim speed increases if $dC_m/dC_{L_{Free}}$ is made more negative. The constant K_2 of equation 2.90 is made more positive and in order for the stick force curve to continue to pass through the desired trim speed, a more positive tab selection is required. An aircraft operating at a certain cg with a tab setting δ_{T1} is shown on figure 2.29, line 1.

FIGURE 2.29

EFFECT ON APPARENT STABILIZER



If $dC_m/dC_{L_{Free}}$ is increased by moving the cg forward, K_2, which is a function of $dC_m/dC_{L_{Free}}$ in equation 2.89 becomes more positive or increases. The new equation becomes,

$$F_s = K_1 V_e^{2} + K_2' \qquad (2.97)$$

This equation plots as line 2 in figure 4.7. The aircraft with no change in tab setting δ_{T_1} operates on line 2 and is trimmed to V_2. Stick forces at all airspeeds have increased. At this juncture, although the actual stability $dC_m/dC_{L_{Free}}$ has increased, there has been no effect on the stick force gradient or apparent stability. (The slopes of line 1 and line 2 being the same.) So as to retrim to the original trim airspeed V_1, the pilot applies additional nose up tab to δ_{T_2}. The aircraft is now operating in line 3. The stick force gradient through V_1 has increased because of an increase in the K_1 term in equation 2.89. The apparent stability dF_s/dV has increased.

The same effect on apparent stability as cg movement may be obtained by means of aerodynamic balancing. This is a design means of controlling the hinge moment coefficients, C_{h_α} and C_{h_δ}. The primary reason for aerodynamic balancing is to increase or reduce the hinge moments and, in turn, the control stick forces. Changing C_{h_δ}, affects the stick forces as seen in equation 2.89. In addition to the influence on hinge moments, aerodynamic balancing may very well affect the real and apparent stability of the aircraft. Assuming that the restoring hinge moment coefficient C_{h_δ} is increased by an appropriate aerodynamic balanced control surface, the ratio of $C_{h_\alpha}/C_{h_\delta}$ in stability equation 2.98 is increased.

$$\frac{dC_m}{dC_{L_{Free}}} = \frac{X_w}{c} + \frac{dC_m}{dC_{L_{Fus}}} - \frac{a_T}{a_w} V_H \eta_T \left(1 - \frac{d\varepsilon}{d\alpha}\right)\left(1 - \tau \frac{C_{h_\alpha}}{C_{h_\delta}}\right) \qquad (2.98)$$

The combined increase in $dC_m/dC_{L_{Free}}$ and C_{h_δ}, increases the K_2 term in equation 2.90 since

$$K_2 = - A \frac{W}{S} \frac{C_{h_\delta}}{C_{m_{\delta_e}}} \frac{dC_m}{dC_{L_{Free}}} \qquad (2.99)$$

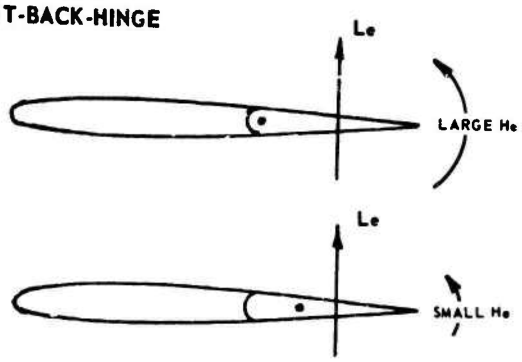
Figure 2.29 shows the effect of increased K_2. The apparent stability is not affected by the increase in K_2 while the aircraft retrims at V_2. However, once the aircraft is retrimmed to the original airspeed V_1 by increasing the tab setting to δ_{T_2}, the apparent stability is increased.

Types of aerodynamic balancing used to control the hinge moment coefficients are as follows:

Set-Back-Hinge:

Perhaps the simplest method of reducing the aerodynamic hinge moments is simply to move the hinge line rearward. Thus the hinge moment is reduced because of the moment arm between the elevator lift and the elevator hinge line is reduced. (One may arrive at the same conclusion by arguing that part of the elevator lift acting behind the hinge line has been reduced, while that in front of the hinge line has been increased.) The net result is a reduction in the absolute value of both C_{h_α} and C_{h_δ}. In fact if the hinge line is set back far enough, the sign of both derivatives can be changed.

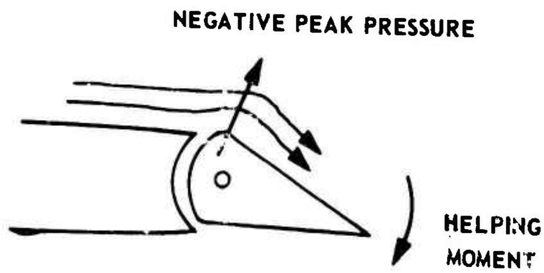
FIGURE 2.30
SET-BACK-HINGE



Overhang Balance:

This method is simply a special case of set-back hinge in which the elevator is designed so that when the leading edge protrudes into the airstream, the local velocity is increased significantly; causing an increase in negative pressure at that point. This negative pressure peak creates a hinge moment which opposes the normal restoring hinge moment, reducing Ch_{δ}. Figure 2.31 shows an elevator with an overhang balance.

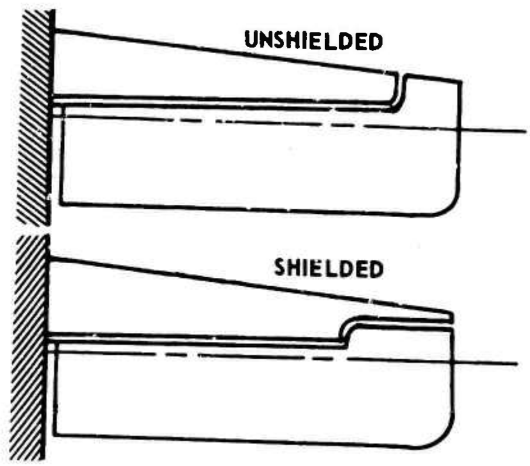
FIGURE 2.31
OVERHANG BALANCE



Horn Balance:

The horn balance works on the same principle as the set-back hinge, i.e., to reduce hinge moments by increasing the area forward of the hinge line. The horn balance, especially the unshielded horn, is very effective in reducing Ch_{α} and Ch_{δ}. This arrangement shown in figure 2.32, is also a handy way of improving the mass balance of the control surface.

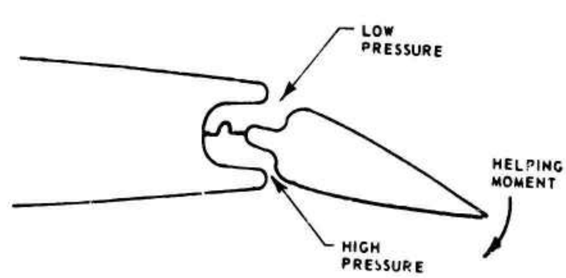
FIGURE 2.32
HORN BALANCE



Internal Balance or Internal Seal:

The internal seal allows the negative pressure on the upper surface and the positive pressure on the lower surface to act on an internal sealed surface forward of the hinge line in such a way that a helping moment is created, again opposing the normal hinge moments. As a result, the absolute values of Ch_{α} and Ch_{δ} are both reduced. This method is good at high indicated airspeeds but is sometimes troublesome at high Mach numbers.

FIGURE 2.33
INTERNAL SEAL



Elevator Modifications:

Bevel Angle on Top or on Bottom of the Stabilizer.

This device, which causes flow separation on one side but not on the other, reduced the absolute values of Ch_α and Ch_δ.

Trailing Edge Strips.

This device, found on the B-57, increases both Ch_α and Ch_δ. In combination with a balance tab, trailing edge strips produce a very high positive Ch_α, but still a low Ch_δ. Thus results in what is called a favorable "Response Effect," i.e., it takes a lower control force to hold a deflection than was originally required to produce it.

Convex Trailing Edge.

This type surface produces a more negative Ch_δ, but tends as well to produce a dangerous short-period oscillation.

Tabs:

A tab is simply a small flap which has been placed on the trailing edge of a larger one. The tab greatly modifies the flap hinge moments but has only a small effect on the lift of the elevator or the entire airfoil. Tabs in general are designed to modify stick-forces and therefore Ch_δ but will not affect Ch_α. A positive tab deflection is one which will tend to move the elevator in a positive direction.

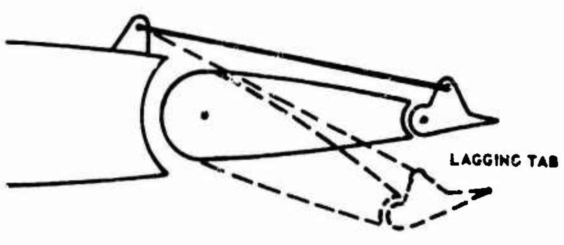
Trim Tab.

A tab which is controlled by a switch or control separate from the normal cockpit pilot control is called a trim tab. The purpose of the trim tab is to reduce the elevator hinge moment and, therefore, the stick force to zero for a given flight condition. A satisfactory trim tab should be able to accomplish this throughout the aircraft flight envelope. Ordinarily, a trim tab will not significantly vary Ch_α or Ch_δ. The functions of the spring and trim or balance and trim tabs may be combined in a single tab. Another method of trimming an aircraft is the use of an adjustable horizontal stabilizer. Normally the trim tab or horizontal stabilizer setting will have a small effect on stability.

Balance Tab.

A balance tab is a simple tab which is mechanically geared to the elevator so that a certain elevator deflection produces a given tab deflection. If the tab is geared to move in the same direction as the surface, it is called a leading tab. If it moves in the opposite direction, it is called a lagging tab. The purpose of the balance tab is usually to reduce the hinge moments and stick forces (lagging tab) at the price of a certain loss in control effectiveness. Sometimes, however, a leading tab is used to increase control effectiveness at the price of increased stick forces. The leading tab may also be used for the express purpose of increasing control forces. Thus Ch_δ may be increased or decreased, while Ch_α remains unaffected. If the linkage shown in figure 2.34 is made so that the length may be varied by the pilot, then the tab may also serve as a trimming device.

FIGURE 2.34
BALANCE TAB



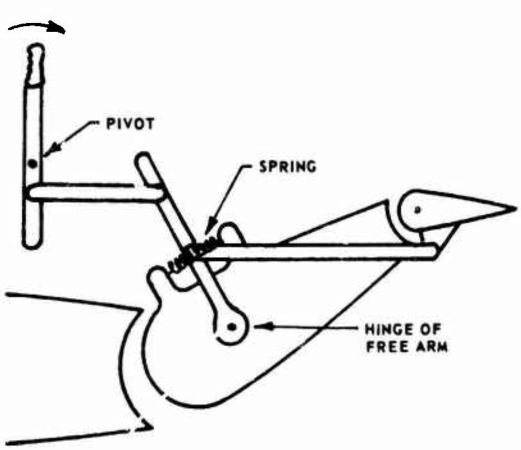
Servo or Control Tab.

The servo tab is linked directly to the aircraft control system in such a manner that the pilot moves the tab and the tab moves the elevator, which is free to float. The summation of elevator hinge moments, therefore, always equals zero since the elevator will float until the hinge moment due to elevator deflection just balances out the hinge moments due to α_s and δ_t. The stick forces are now a function of the tab hinge moment or Ch_{δ_T}. Again Ch_α is not affected.

Spring Tab.

A spring tab is a lagging balance tab which is geared in such a way that the pilot receives the most help from the tab at high speeds where he needs it the most, i.e., the gearing is a function of dynamic pressure. The basic principles of its operation are:

FIGURE 2.35
SPRING TAB



1. An increase in dynamic pressure causes an increase in hinge moment and stick force for a given control deflection.

2. The increased stick force causes an increased spring deflection and, therefore, an increased tab deflection.

3. The increased tab deflection causes a decrease in stick force. Thus an increased proportion of the hinge moment is taken by the tab.

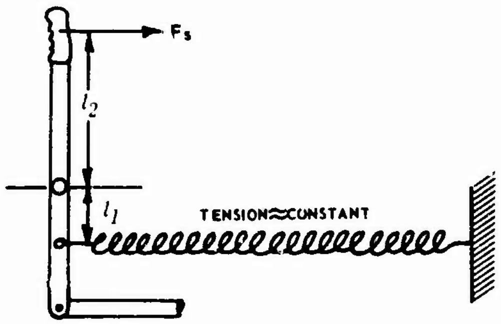
4. Therefore, the spring tab is a geared balance tab where the gearing is a function of dynamic pressure.

5. Thus the stick forces are more nearly constant over the speed range of the aircraft. That is, the stick force required to produce a given deflection at 300 knots is still greater than at 150 knots, but not by as much as before.

6. After full spring or tab deflection is reached the balancing feature is lost and the pilot must supply the full force necessary for further deflection. (This acts as a safety feature.)

Because of the very low force gradients in most modern aircraft at the aft center of gravity ($dC_m/dC_{L_{Free}}$ less negative), improvements in the stick-free longitudinal stability are obtained by devices which produce a constant pull force on the stick independent of airspeed which allows a more noseup tab setting and steeper stick force gradients. Two such gadgets for improving the stick force gradients are the downspring and bobweight. Both effectively increase the apparent stability of the aircraft.

FIGURE 2.36
DOWNSPRING



Downspring:

A virtually constant stick force may be demanded of the pilot by incorporating a downspring or bungee into the control system which tends to pull the top of the stick forward. From figure 2.36, the force required to counteract the spring is,

$$F_{s_{Downspring}} = T \frac{\ell_1}{\ell_2} = K_3 \quad (2.100)$$

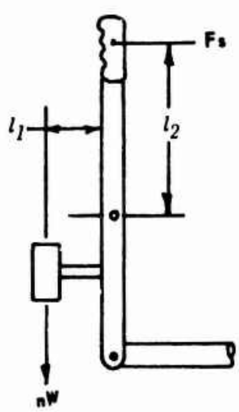
If the spring is a long one, the tension in it will be increased only slightly as the top moves rearward and can be considered to be constant.

The equation with the downspring in the control system becomes,

$$F_s = K_1 V_e^2 + K_2 + K_{3_{Downspring}} \quad (2.101)$$

As shown in figure 2.36, the apparent stability will increase when the aircraft is once again retrimmed to the original trim airspeed by increasing the tab setting. Note that the downspring increases apparent stability but does not affect the actual stability $dC_m/dC_{L_{Free}}$ (no change to K_2) of the aircraft.

FIGURE 2.37
BOBWEIGHT



Bobweight:

Another method of introducing a nearly constant stick force is by placing a bobweight somewhere in the control system which causes a constant moment which must be overcome by the pilot. The force which the pilot must apply to counteract the bobweight is,

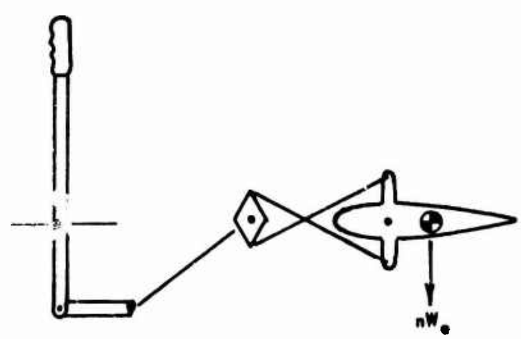
$$F_{s_{Bobweight}} = nW \frac{\ell_1}{\ell_2} = K_3 \quad (2.102)$$

Like the downspring the bobweight increases the stick force throughout the airspeed range and, at increased tab settings, the apparent stability or stick force gradient. The bobweight has no effect on the actual $dC_m/dC_{L_{Free}}$ of the aircraft.

Elevator Unbalance:

There are other devices which increase the stick force gradient through trim or apparent stability. The unbalance in the control system resulting from the center of gravity of the elevator falling aft of the hinge line is shown in figure 2.38.

FIGURE 2.38
ELEVATOR UNBALANCE



From the figure it can be seen that an elevator cg behind the hinge line will tend to rotate the top of the stick forward. This must be counteracted by a positive pull stick force.

As the elevator is moved from the horizontal, the hinge moment is reduced by the cosine of the deflection angle; this moment remains virtually constant. Thus a forward hinge line which usually produces a destabilizing (positive) Ch_α will also produce a "stabilizing" elevator unbalance.

Comment:

Since h_n' is usually found by equation 2.96, it would be worthwhile to examine the effect of the stick force gradient dF_s/dV on this equation. Rewriting equation 2.88, with a downspring used as the control system gadget,

$$F_s = Aq(B + C_{h_{\delta_T}} \delta_T) - AC_L q \frac{C_{h_\delta}}{C_{m_{\delta_e}}} \left.\frac{dC_m}{dC_L}\right|_{\substack{Free \\ No\ Gadget}} + K'_{3_{Gadget}} \qquad (2.103)$$

$$F_s/q = A(B + C_{h_{\delta_T}} \delta_T) - AC_L \frac{C_{h_\delta}}{C_{m_{\delta_e}}} \left.\frac{dC_m}{dC_L}\right|_{\substack{Free \\ No\ Gadget}} + \frac{K_3' C_L}{W/S} \qquad (2.104)$$

$$\frac{dF_s/q}{dC_L} = K_2' \left.\frac{dC_m}{dC_L}\right|_{\substack{Free \\ No\ Gadget}} + \frac{K_3'}{W/S} \qquad (2.105)$$

Obviously the cg location at which $dF_s/q/dC_L$ goes to zero will not be the true h_n'. However, the only reason that the term $dC_m/dC_{L_{Free}}$ was of interest in the first place was because it was proportional to the stick force gradient. The pilot is more interested in the apparent stability for the same reason. The fact that the addition to the stick-free stability caused by this gadgetry is "artificial" rather than genuine is only of academic interest.

2.14 HIGH SPEED LONGITUDINAL STATIC STABILITY

The effects of high speeds (transonic and supersonic) on longitudinal static stability can be analyzed in the same manner as that done for subsonic speeds. The assumptions that were made for the incompressible flow are no longer valid and, therefore, cannot be neglected.

Compressibility associated with the transonic and supersonic speed regime has noticeable effect upon both the gust stability (longitudinal static stability $C_{m_{C_L}}$) and speed stability (F_s/V). The gust stability depends mainly on the contributions to stability of the wing, fuselage, and tail in the stability equation below during transonic and supersonic flight.

$$\frac{dC_m}{dC_L} = \frac{X_w}{c} + \left.\frac{dC_m}{dC_L}\right|_{Fus} - \frac{a_T}{a_w} V_H \eta_T \left(1 - \frac{d\varepsilon}{d\alpha}\right) \qquad (2.106)$$

The terms in the stability equation will be examined in turn.

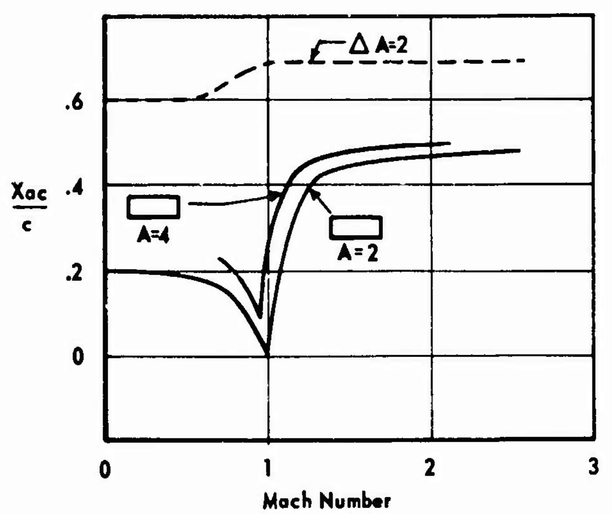
The Wing Contribution:

In subsonic flow or low Mach flight, the aerodynamic center is at the quarter chord. As subsonic Mach approaches unity or the transonic speed is approached, flow separation occurs behind the shock formations causing the aerodynamic center to move forward of the quarter chord position. The immediate effect is a reduction in stability since X_w/c increases. Following the flow separation behind the shocks at

positions of sonic speed, the flow pattern on the airfoil eventually transitions to supersonic flow. The shocks move off the surface and the wing recovers lift. The aerodynamic center now moves aft towards the 50-percent chord position. There is a sudden increase in the wing's contribution to stability since X_w/c is reduced (figure 2.1).

The extent of the aerodynamic center shift forward and rearward depends greatly on the aspect ratio of the aircraft. The shift is least for low aspect ratio aircraft. Among the plan forms, the rectangular wing has the largest shift for a given aspect ratio whereas the triangular wing as the least (figures 2.39 and 2.14).

FIGURE 2.39
A.C. VARIATION WITH MACH



The Fuselage Contribution:

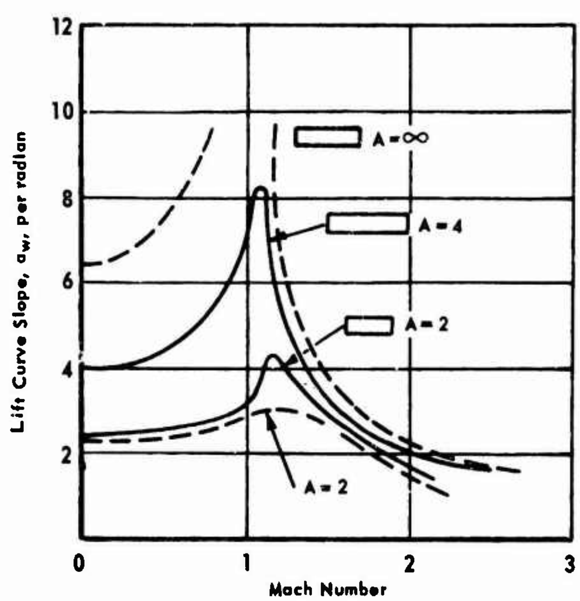
In supersonic flow the fuselage center of pressure moves forward causing a positive increase in the fuselage dC_m/dC_L or a destabilizing influence on the stability equation. The fuselage term variation with Mach number will be ignored.

The Tail Contribution:

The tail contribution to stability depends on the variation of lift curve slopes, a_W and a_T, plus downwash ε with Mach during transonic and supersonic flight. It is expressed as: $-a_T/a_W\ V_H\eta_T\ (1 - d\varepsilon/d\alpha)$

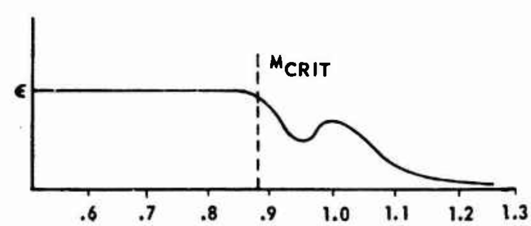
During subsonic flight a_T/a_W remains approximately constant. The slope of the lift curve, a_W varies as shown in figure 2.40. This variation of a_W in the transonic speed range is a function of geometry (i.e., aspect ratio, thickness, camber, and sweep). Limiting further discussion to aircraft designed for transonic flight or aircraft which employ airfoil shapes with small thickness to chord ratios, then a_W increases slightly in the transonic regime. For all airfoil shapes the value of a_W decreases as the airspeed increases supersonically. The a_T/a_W contribution is therefore destabilizing in the transonic regime and stabilizing in the supersonic regime.

FIGURE 2.40
LIFT CURVE SLOPE VARIATION WITH MACH



The tail contribution is further affected by the variation in downwash, ϵ, with Mach increase. The downwash at the tail is a result of the vortex system associated with the lifting wing. It is recognized that the tail location will have considerable influence as to the degree of variation of δ_e with $\Delta\epsilon$. An aircraft such as the F-100 has a great deal more variation of δ_e due to downwash effects than the F-104. Since downwash is a direct function of wing lift, a sudden loss of downwash occurs transonically with a resulting increase in tail angle of attack. The effect is to require the pilot to apply additional up elevator with increasing airspeed to maintain altitude. This additional up elevator contributes to speed instability. (Speed stability will be covered more thoroughly later.) Downwash variation with Mach is seen in figure 2.41.

FIGURE 2.41
DOWNWASH VARIATION WITH MACH

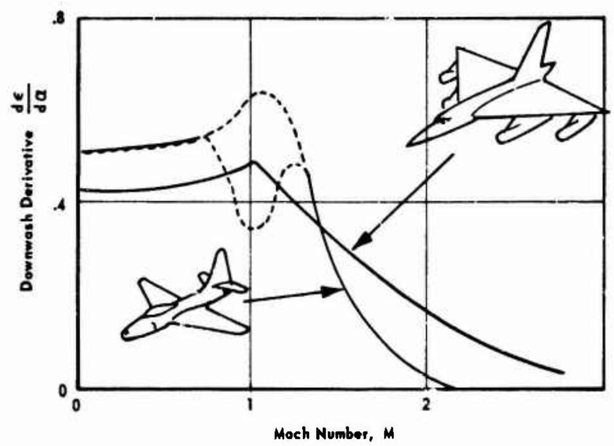


The variation of $d\epsilon/d\alpha$ with Mach number greatly influences the aircraft's gust stability dC_m/dC_L. Recalling,

$$\epsilon = \frac{114.6C_L}{\pi A\!\!R} \quad \text{where} \quad \frac{d\epsilon}{d\alpha} = \frac{114.6a_w}{\pi A\!\!R} \tag{2.107}$$

Since the downwash angle behind the wing is directly proportional to the lift coefficient of the wing, it is apparent that the value of the derivative $d\epsilon/d\alpha$ is a function of a_w. The general trend of $d\epsilon/d\alpha$ is an initial increase with Mach starting at subsonic speeds. This increase follows a trend similar to but at a lesser slope than the increase of the lift curve slope, a_w, of the wing. Above Mach 1.0, $d\epsilon/d\alpha$ decreases and approaches zero. This variation depends on the particular wing geometry of the aircraft. As shown in figure 2.42, $d\epsilon/d\alpha$ may dip for thicker wing sections where considerable flow separation occurs. Again, $d\epsilon/d\alpha$ is very much dependent on a_w.

FIGURE 2.42
DOWNWASH DERIVATIVE vs MACH



For an aircraft designed for high speed flight, the variation of $d\epsilon/d\alpha$ with increasing Mach number results in a slight destabilizing effect in the transonic regime and contributes to increased stability in the supersonic speed regime.

As the wing surface becomes a less efficient lifting surface, a loss of stabilator effectiveness is experienced in supersonic flight. The elevator power, $C_{m_{\delta_e}}$, increases as airspeed approaches Mach 1.0. Beyond Mach 1.0, elevator effectiveness decreases. Consequently, increase of elevator power causes a positive $\Delta\delta_e$ contribution or again an indication of speed instability

as Mach 1.0 is approached. With decrease in elevator power, a negative $\Delta\delta_e$ contribution once again produces speed stability. For the F-104 the relative order of magnitude of these values cause an initial increase in gust stability in the transonic regime followed by a steadily decreasing stability influence as $C_{m_{\delta_e}}$ approaches zero.

$$\frac{dC_m}{dC_{L_{Tail}}} = \frac{C_{m_{\delta_e}}}{a_w \tau}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \qquad (2.108)$$

The overall effect of transonic and supersonic flight on gust stability or dC_m/dC_L is shown in figure 2.43. Static longitudinal stability increases transonically and then decreases supersonically. The speed stability of the aircraft is affected as well. Recalling the pitching moment coefficient equation,

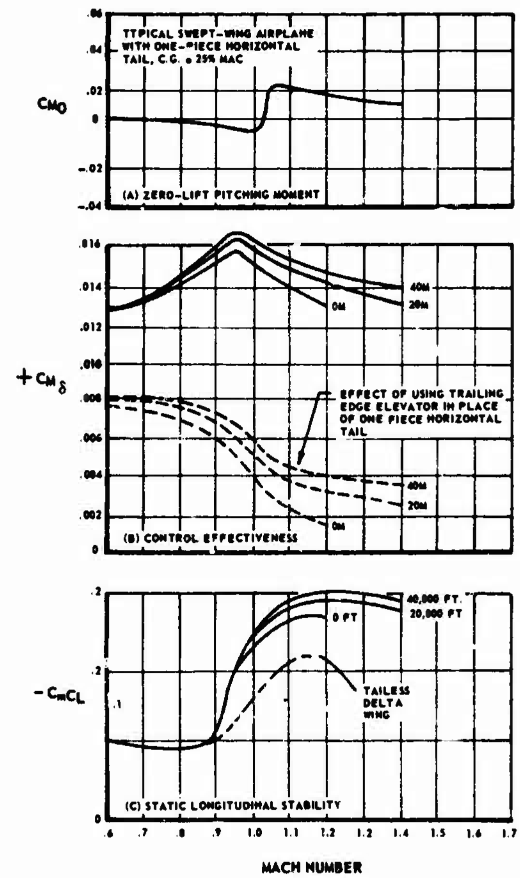
$$\Delta C_m = C_{m_o} + C_{m_\alpha}\Delta\alpha + C_{m_{\dot{\alpha}}}\Delta\dot{\alpha} + C_{m_{\delta_e}}\Delta\delta_e + C_{m_V}\Delta V + C_{m_q}\Delta q \qquad (2.109)$$

and since $C_{m_{C_L}} = \frac{1}{a} C_{m_\alpha}$, then: Assuming no change in speed or pitch rate, and since under compressibility C_{m_o} is not zero, the elevator required to maintain steady flight is:

$$\Delta\delta_e = -\frac{C_{m_o}}{C_{m_{\delta_e}}} - \frac{C_{m_{C_L}}}{C_{m_{\delta_e}}}\Delta C_L \qquad (2.110)$$

Speed stability depends on the variations of δ_e with transonic and supersonic speeds and according to equation 2.110, depends on how C_{m_o}, $C_{m_{\delta_e}}$, and $C_{m_{C_L}}$ vary.

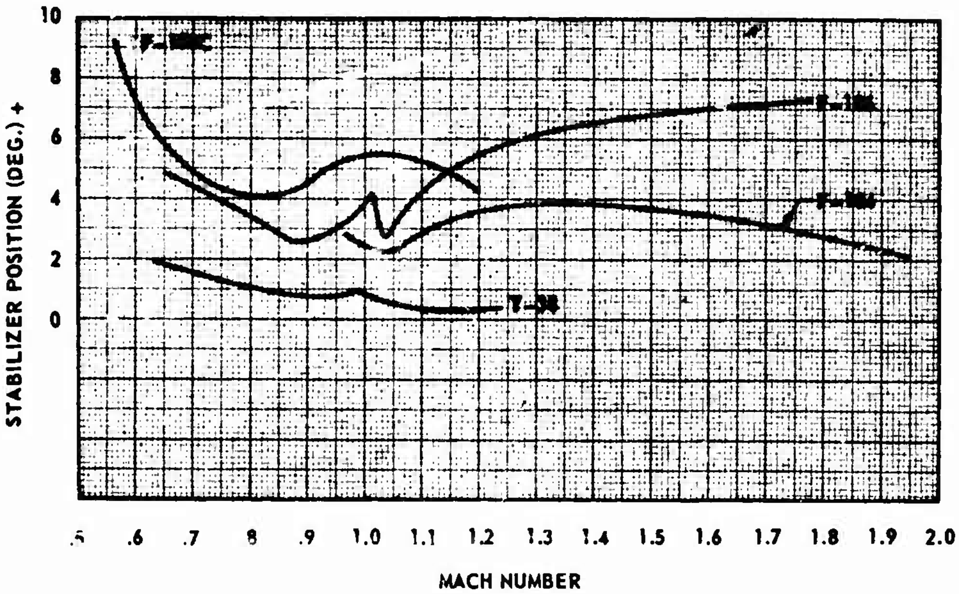
FIGURE 2.43
MACH VARIATIONS ON C_{M_O}, $C_{m_{\delta_e}}$ AND $C_{m_{C_L}}$



If equation 2.110 is analyzed using the plots in figure 2.43, speed instability during transonic flight becomes obvious. The value of $-C_{m_o}/C_{m_{\delta_e}}$ increases from approximately zero in the subsonic range to some positive value as the aircraft passes through Mach 1.0. The value of $C_{m_{C_L}}/C_{m_{\delta_e}}$ increases to a very large number in comparison to $C_{m_o}/C_{m_{\delta_e}}$ through this same range. The result is a positive $\Delta\delta_e$ or a reversal of elevator deflection with increasing airspeed. This manifests itself as a relaxation of forward pressure or even a pull force to maintain attitude or prevent a nose down tendency. As the aircraft speed increases to supersonic speed, $\Delta\delta_e$ again becomes nega-

tive and the pilot regains speed stability or decreasing δ_e with increasing airspeed. The actual results of some aircraft flown in this range are shown in figure 2.44.

FIGURE 2.44
δ_e vs MACH



Whether the speed instability or reversal in elevator deflections and stick forces are objectionable, depends on many factors such as magnitude of variation, length of time required to transverse the region of instability, control system characteristics, and conditions of flight. It is impossible for the engineer to determine from data plots if the degree of instability is acceptable. The pilot is the only one capable of evaluating these effects.

In the F-100C, a pull force of 14 pounds was required when accelerating from Mach 0.87 to Mach 1.0. The test pilot described this trim change as disconcerting while attempting to maneuver the aircraft in this region and recommended that a "q" or Mach sensing device be installed to eliminate this characteristic. Consequently, a mechanism was incorporated to automatically change the artificial feel gradient as the aircraft accelerates through the transonic range. Also, the longitudinal trim is automatically changed in this region by the use of a "Mach Trimmer."

In the F-104, the test pilot stated that transonic trim changes required an aft stick movement with increasing speed and a forward stick movement when decreasing speed, but described this speed instability as acceptable.

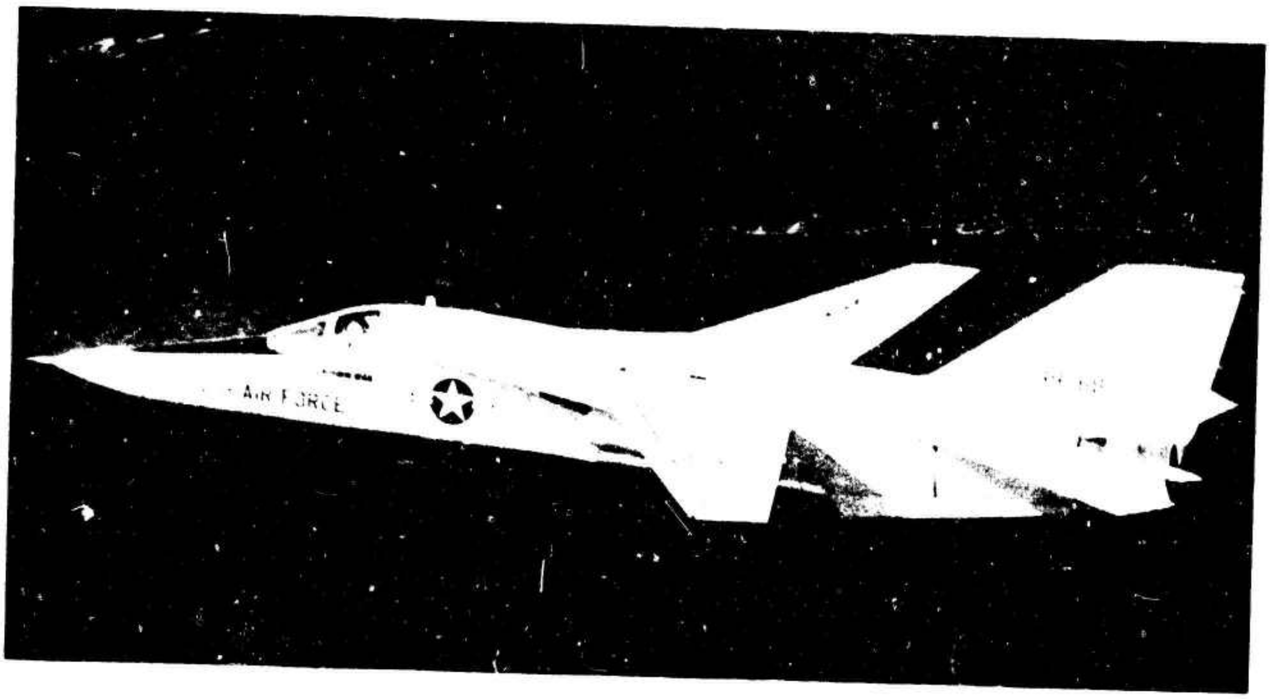
In the F-106 the pilot stated that the 1.0 to 1.1 Mach region is characterized by a moderate trim change necessitating pilot technique to avoid large variations in altitude during accelerations. Minor trim changes are encountered up to

Mach 1.35. His report concluded that the speed instabilities were not unsatisfactory.

In the T-38 which embodies the latest design concept, a departure is noted from the low tail configuration difficulties where the pilot described the transonic trim change as being hardly perceptible.

Aircraft design considerations are, of course, influenced by the stability aspects of high speed flight. It is desirable to design an aircraft where trim changes through transonic speeds are small. A flat wing without camber, twist, or incidence or a low aspect ratio wing and tail provide values of X_w/c, a_w, a_T, and $d\varepsilon/d\alpha$ which vary minimally over the Mach number range. An all-moving tail (slab) for control gives negligible variation of τ with Mach and maximum control effectiveness. A full power, irreversible control system is desirable to counteract the erratic changes in pressure distribution which affect Ch_α and Ch_{δ_e}.

In the transonic speed regime the meaning or importance of "neutral point" is reduced. At transonic speeds the variation of control angle and trim force with speed, although important, is not affected by cg position. Instead of relating trim gradients to a cg margin, it is more useful to view variation of control for trim as a function of compressibility and ignore cg position.



CHAPTER 3

MANEUVERABILITY

3.1 MANEUVERING FLIGHT

The method used to analyze maneuvering flight will be to determine stick-fixed maneuver points (h_m) and stick-free maneuver points (h'_m). It will be seen that these are analogous to their counterparts in static stability, the stick fixed and stick free neutral points. The maneuver points will also be defined in terms of the neutral points and the theory will help to predict which of these points will be critical as regards the aft center of gravity location. It will also be shown how the forward center of gravity is affected by the parameters that define the maneuver points.

Maneuvering flight will be analyzed much in the same manner used in determining a flight test relationship in longitudinal stability. For stick-fixed longitudinal stability, the flight test relationship was determined to be

$$\frac{d\delta_e}{dC_L} = -\frac{dC_m/dC_L}{C_{m_{\delta_e}}} \tag{3.1}$$

This equation gave the static longitudinal stability of the aircraft in terms that could easily be measured in flight test.

In maneuvering flight, a similar stick-fixed equation relating to easily measurable flight test quantities is desirable. Where in longitudinal stability, the elevator deflection was related to lift coefficient or angle of attack, one may surmise that in maneuvering flight elevator deflection will relate to load factor n.

To determine this expression, one must refer to the aircraft's basic equations of motion. As in longitudinal stability, the six equations of motion are the basis for all analysis of aircraft stability and control. In maneuvering an aircraft the same equations will hold true, but one additiona derivative will have to be added to the analysis. Recalling the pitching moment equation

$$M = \dot{q}I_y + pr\,(I_x - I_z) + (p^2 - r^2)\,I_{xz} \tag{3.2}$$

and the fact that in static stability analysis we have no roll rate, yaw rate, or pitch acceleration, equation 3.2 reduces to:

$$M = \dot{q}\,I_y = 0 \tag{3.3}$$

The variables that cause external pitching moments on an aircraft are infinite, i.e., speed brakes, canopy, elevator, etc. There are, however, five primary variables that we can consider.

$$M = f(V, \alpha, \dot{\alpha}, \delta_e, q) \tag{3.4}$$

If any or all of these variables change, there will be a change of total pitching moment that will equal the sum of the partial changes of all the variables. This is written as:

$$\Delta M = \frac{\partial M}{\partial \alpha}\Delta\alpha + \frac{\partial M}{\partial \dot{\alpha}}\Delta\dot{\alpha} + \frac{\partial M}{\partial \delta_e}\Delta\delta_e + \frac{\partial M}{\partial V}\Delta V + \frac{\partial M}{\partial q}\Delta q \tag{3.5}$$

Since in maneuvering flight, ΔV and $\Delta \dot{\alpha}$ are zero, equation 3.5 becomes:

$$\Delta M = \frac{\partial M}{\partial \alpha} \Delta \alpha + \frac{\partial M}{\partial \delta_e} \Delta \delta_e + \frac{\partial M}{\partial q} \Delta q = 0 \quad (3.6)$$

and since $M = qSc\, C_m$, then

$$\frac{\partial M}{\partial \alpha} = q\, Sc \frac{\partial C_m}{\partial \alpha} = q\, Sc\, C_{m_\alpha} \quad (3.7)$$

$$\frac{\partial M}{\partial \delta_e} = q\, Sc \frac{\partial C_m}{\partial \delta_e} = q\, Sc\, C_{m_{\delta_e}} \quad (3.8)$$

$$\frac{\partial M}{\partial q} = q\, Sc \frac{\partial C_m}{\partial q} \quad (3.9)$$

Substituting these values into equation 3.6, and multiplying by $1/q\, Sc$,

$$C_{m_\alpha} \Delta \alpha + C_{m_{\delta_e}} \Delta \delta_e + \frac{\partial C_m}{\partial q} \Delta q = 0 \quad (3.10)$$

The derivative $\partial C_m/\partial_q$ is carried instead of C_{m_q} since the compensating factor $c/2V$ is not used at this time.

Solving for the change in elevator deflection $\Delta \delta_e$,

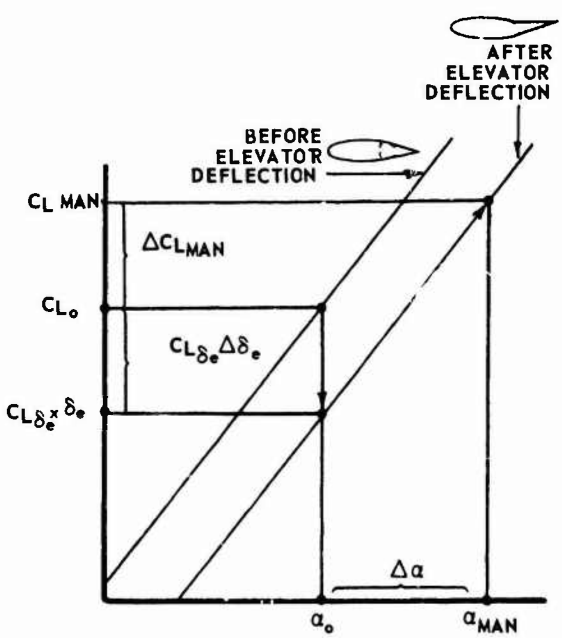
$$\Delta \delta_e = \frac{- C_{m_\alpha} \Delta \alpha - \partial C_m/\partial_q \,\Delta q}{C_{m_{\delta_e}}} \quad (3.11)$$

The analysis of equation 3.11 may be continued by substituting in values for $\Delta\alpha$ and Δq. The final equation obtained should be in the form of some flight test relationship. Since maneuvering is related to load factor, the elevator deflection required to obtain different load factors will define the stick-fixed maneuver point. The immediate goal then is to determine the change in angle of attack, $\Delta\alpha$, and change in pitch rate, Δq, in terms of load factor n.

3.2 THE PULL UP MANEUVER

3.3 In the pull up maneuver, the change in angle of attack of the aircraft, $\Delta\alpha$, may be related to the lift coefficient of the aircraft. In the pull up with constant velocity, the angle of attack of the whole aircraft will be changed since the aircraft has to fly at a higher C_L to obtain the load factor required. The change in C_L required to maneuver at high load factors at a constant velocity comes from two sources: (1) load factor increase, (2) elevator deflection. Although often ignored because of its small value when compared to total C_L, the change in lift with elevator deflection $C_{L_{\delta_e}} \Delta \delta_e$ will be carried along for a more general analysis.

Figure 3.1. LIFT COEFFICIENT VERSUS ANGLE OF ATTACK



Referring to figure 3.1, the aircraft is in equilibrum at some C_{L_o} corresponding to some α_o before the elevator is deflected to initiate the pull up. If the elevator is considered as a flap, its deflection will affect the lift curve as follows. When the elevator is deflected upward, the left curve shifts downward and does not change slope. This says that a certain amount of lift is initially lost when the elevator is deflected upward. The loss in lift because of elevator deflection is designated $C_{L_{\delta_e}} \Delta\delta_e$. The increase in downloading on the tail or increase in negative lift on the horizontal stabilizer causes a moment on the aircraft which creates a nose up pitch rate. The aircraft continues to pitch upward and increase its angle of attack until it reaches a new C_L and an equilibrium load factor. In other words a pitch rate is initiated and α increases until a maneuvering lift coefficient $C_{L_{MAN}}$ is reached for the deflected elevator δ_e. The change in angle of attack is $\Delta\alpha$. The change in C_L has come partially from the deflected elevator and mainly from the pitching maneuver. The change in C_L due to the maneuver is from C_{L_o} to $C_{L_{MAN}}$. Since it did not change the slope of the lift curve, if the change in lift caused by elevator deflection is included, the expression for $\Delta\alpha$ becomes:

$$C_L = a\alpha \tag{3.12}$$

$$\Delta C_L = a\Delta\alpha \tag{3.12a}$$

$$\Delta C_L = \Delta C_{L_{MAN}} - C_{L_{\delta_e}} \Delta\delta_e = a\Delta\alpha \tag{3.13}$$

$$\Delta\alpha = \frac{1}{a}\left[\Delta C_{L_{MAN}} - C_{L_{\delta_e}} \Delta\delta_e\right] \tag{3.14}$$

To put equation 3.14 in terms of load factor, $\Delta C_{L_{MAN}}$ must be defined. This is the change in lift from the initial condition to the final maneuvering condition. This change can occur from one g flight to some other load factor or it can start at 2 or 3 g's and progress to some new load factor. If C_L is at one g then

$$C_L = \frac{W}{qS} \tag{3.15}$$

$$C_{L_o} = \frac{n_o W}{qS} \qquad n_o \text{ - initial load factor} \tag{3.16}$$

$$C_{L_{MAN}} = \frac{nW}{qS} \qquad n \text{ - final load factor} \tag{3.17}$$

$$\Delta C_{L_{MAN}} = C_{L_{MAN}} - C_{L_o}$$

$$= \frac{nW}{qS} - \frac{n_o W}{qS} = C_L(n-n_o) \tag{3.18}$$

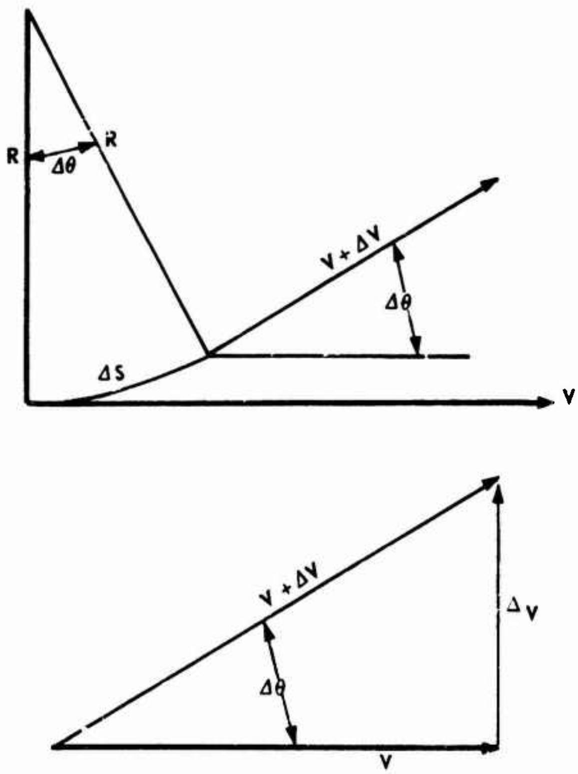
Finally substituting $\Delta C_{L_{MAN}}$ into equation 3.14,

$$\Delta\alpha = \frac{1}{a}\left[C_L\ (n-n_o) - C_{L_{\delta_e}} \Delta\delta_e\right] \tag{3.19}$$

Equation 3.19 is now ready for substitution into equation 3.11.

An expression for Δq in equation 3.11 will be derived using the pull up maneuver analysis.

Figure 3.2 CURVILINEAR MOTION



Referring to figure 3.2:

$$\Delta \theta = \frac{\Delta S}{R} \qquad (3.20)$$

$$\frac{d\theta}{dt} = \lim_{\Delta t \to o} \frac{\Delta \theta}{\Delta t} = \lim_{\Delta t \to o} \frac{\Delta S}{\Delta t} \frac{1}{R} \qquad (3.21)$$

$$\frac{d\theta}{dt} = \frac{V}{R} \qquad (3.22)$$

From figure 3.2

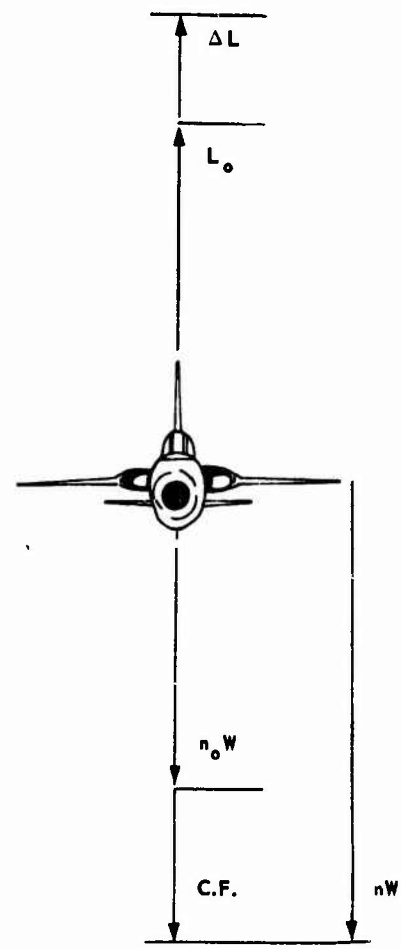
$$\frac{\Delta V}{V} = \Delta \theta \quad \text{(small angles)} \qquad (3.23)$$

$$\frac{d\theta}{dt} = \lim_{\Delta t \to o} \frac{\Delta V}{\Delta t} \frac{1}{V} = \frac{1}{V} \frac{dV}{dt} \qquad (3.24)$$

combining equations 3.24 and 3.22,

$$\frac{dV}{dt} = \frac{V^2}{R} \qquad (3.25$$

Figure 3.3 WINGS LEVEL PULL UP



From figure 3.3,

$$\Delta L = CF = nW - n_o W \qquad (3.26)$$

Again the factor n_oW indicates that the change may take place from any original load factor and is not limited to the straight and level flight condition. The centrifugal force that holds the aircraft in equilibrium can be expressed as:

$$CF = \frac{W}{g} a = \frac{W}{g} \frac{V^2}{R} \qquad (3.27)$$

Therefore:

$$\Delta L = W(n - n_o) = \frac{W}{g}\frac{V^2}{R} \tag{3.28}$$

$$\frac{g}{V}(n - n_o) = \frac{V}{R} \tag{3.29}$$

Substituting from equation 3.22,

$$\Delta q \approx \frac{d\theta}{dt} = \frac{V}{R} = \frac{g}{V}(n - n_o) \tag{3.30}$$

Now equations 3.30 and 3.19 may be substituted into equation 3.11.

$$\Delta \delta_e = \frac{-C_{m_\alpha} \frac{1}{a}\left[C_L (n - n_o) - C_{L_{\delta_e}} \Delta \delta_e\right]}{C_{m_{\delta_e}}}$$

$$\frac{-\frac{\partial C_m}{\partial q}\frac{g}{V}(n - n_o)}{C_{m_{\delta_e}}} \tag{3.31}$$

From longitudinal stability,

$$C_{m_\alpha} = \frac{\partial C_m}{\partial C_L}\frac{\partial C_L}{\partial \alpha} = a(h - h_n) \tag{3.32}$$

Also to help further in reducing the equation to its simplest terms,

$$V^2 = \frac{2W}{\rho S C_L} \tag{3.33}$$

and

$$\frac{\partial C_m}{\partial q} = \frac{c}{2V} C_{m_q} \tag{3.34}$$

Substituting equations 3.34, 3.33 and 3.22 into equation 3.31 and turning the algebra crank, results in,

$$\frac{\Delta \delta_e}{n - n_o} + \frac{a C_L}{C_{m_\alpha} C_{L_{\delta_e}} - C_{m_{\delta_e}} a}\left[h - h_n + \rho \frac{Sc}{4m} C_{m_q}\right] \tag{3.35}$$

Equation 3.35 is now in the form that will define the stick fixed maneuver point for the pull up. The definition of the maneuver point (h_m) is the cg position at which the elevator deflection per g goes to zero. Taking the limit of equation 3.35, where Δn is defined as $(n - n_o)$,

$$\lim_{\Delta n \to o} \frac{\Delta \delta_e}{\Delta n} = \frac{d\delta_e}{dn} \tag{3.36}$$

or

$$\frac{d\delta_e}{dn} = \frac{a C_L}{C_{m_\alpha} C_{L_{\delta_e}} - C_{m_{\delta_e}} a}\left[h - h_n + \rho \frac{Sc}{4m} C_{m_q}\right] \tag{3.37}$$

Setting equation 3.37 equal to zero will give the cg position (h) as the maneuver point (h_m).

$$h_m = h_n - \rho \frac{Sc}{4m} C_{m_q} \tag{3.38}$$

Solving equation 3.38 for h_n and substituting into equation 3.37,

$$\frac{d\delta_e}{dn} = \frac{a C_L}{C_{m_\alpha} C_{L_{\delta_e}} - C_{m_{\delta_e}} a}(h - h_m) \tag{3.39}$$

where $(h_m - h)$ is defined as the stick-fixed maneuver margin.

The significant points to be made about equation 3.39 are:

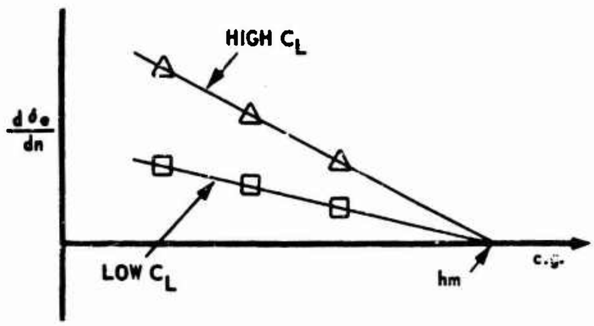
1. The derivative $d\delta_e/dn$ varies with the maneuver margin. The more forward the cg, the more elevator will be required to obtain the limit load factor. That is, as the cg moves forward, more elevator deflection is necessary to obtain a given load factor.

2. The higher the C_L, the more elevator will be required to obtain the limit load factor. That is, at low speeds (high

C_L) more elevator deflection is necessary to obtain a given load factor than is required to obtain the same load factor at a higher speed (lower C_L).

3. The derivative $d\delta_e/dn$ should be linear with respect to cg at a constant C_L.

Figure 3.4 $\frac{d\delta_e}{dn}$ vs cg



Another approach to solving for the maneuver point (h_m) is to return to the original stability equation.

$$\frac{dC_m}{dC_L} = h - \frac{X_{ac}}{c} + \frac{dC_m}{dC_{L_{Fus.}}} - \frac{a_t}{a_w} V_H \eta_T \left(1-\frac{d\epsilon}{d\alpha}\right) \tag{3.40}$$

The effect of pitch damping on the aircraft stability will be determined and added to equation 3.40. Recalling the relationship:

$$\frac{dC_m}{dq} = \frac{c}{2V} C_{m_q} \tag{3.41}$$

or

$$\Delta C_m = \frac{c}{2V} C_{m_q} \Delta q \tag{3.42}$$

Substituting the value obtained for Δq from equation 3.30,

$$\Delta C_m = \frac{cg}{2V^2} C_{m_q} (n - n_o) \tag{3.43}$$

Substituting the appropriate C_L expression for load factor,

$$\Delta C_m = \rho \frac{Sc}{4m} C_{m_q} (C_{L_{MAN}} - C_{L_o}) \tag{3.44}$$

if

$$\Delta C_L = C_{L_{MAN}} - C_{L_o}, \quad \text{then}$$

$$\lim_{\Delta C_L \to o} \frac{\Delta C_m}{\Delta C_L} = \frac{dC_m}{dC_L}_{\text{Pitch Damping}} = \rho \frac{Sc}{4m} C_{m_q} \tag{3.45}$$

This term may now be added to equation 3.45. If the sign of C_{m_q} is negative, then the term is a stabilizing contribution to the stability equation. C_{m_q} will be analyzed further.

$$\frac{dC_m}{dC_L} = h - \frac{X_{ac}}{o} + \frac{dC_m}{dC_{L_{Fus}}} - \frac{a_t}{a_w} V_H \eta_T \left(1 - \frac{d\epsilon}{d\alpha}\right) + \rho \frac{Sc}{4m} C_{m_q} \tag{3.46}$$

The maneuver point is found by setting dC_m/dC_L equal to zero and solving for the cg position where this occurs.

$$h_m = \frac{X_{ac}}{c} - \frac{dC_m}{dC_{L_{Fus}}} + \frac{a_t}{a_w} V_H \eta_T \left(1 - \frac{d\epsilon}{d\alpha}\right) - \rho \frac{Sc}{4m} C_{m_q} \tag{3.47}$$

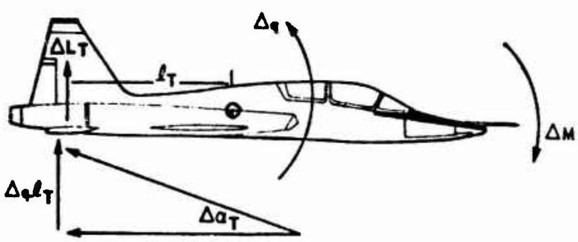
The first three terms on the right side of equation 3.47 may be identified as the expression for the neutral point h_n. If this substitution is made in equation 3.47, equation 3.38 is again obtained.

$$h_m = h_n - \rho \frac{Sc}{4m} C_{m_q} \tag{3.48}$$

The derivative C_{m_q} found in equation 3.37, 3.38, and 3.46 needs to be examined before proceeding with further discussion.

The damping that comes from the pitch rate established in a pull up, comes from the wing, tail, and fuselage components. The tail is the largest contributor to the pitch damping because of the long moment arm. For this reason it is usually used to derive the value of C_{m_q}. Sometimes an empirical value is added to account for the rest of the aircraft but more often than not, the value for the tail alone is used to estimate the derivative. The effect of the tail may be calculated in the following manner:

Figure 3.5 PITCH DAMPING



The pitching moment effect on the aircraft from the downward moving horizontal stabilizer is:

$$\Delta M = -\ell_T \, \Delta L_T = q_w \, S_w \, c_w \, \Delta C_m \qquad (3.49)$$

where

$$\Delta L_T = q_T \, S_T \, \Delta C_{L_T} \qquad (3.50)$$

Solving for ΔC_m,

$$\Delta C_m = -\frac{q_T \, \ell_T \, S_T}{q_w \, c_w \, S_w} \, \Delta C_{L_T} \qquad (3.51)$$

The combination $\ell_T S_T / c_w S_w$ can be recognized as the tail volume coefficient V_H. The term q_T/q_w is referred to as the tail efficiency factor η_T.

Equation 3.51 may then be written:

$$\Delta C_m = -V_H \, \eta_T \, \Delta C_{L_T} \qquad (3.52)$$

which can be further refined to:

$$\Delta C_m = -V_H \, \eta_T \, a_T \, \Delta \alpha_T \qquad (3.53)$$

From figure 3.5, the change in angle of attack at the tail caused by the pitch rate will be:

$$\Delta \alpha_T = \tan^{-1} \frac{\Delta q \, \ell_T}{V} \approx \Delta q \, \frac{\ell_T}{V} \qquad (3.54)$$

Substituting equation 3.54 into 3.53

$$\Delta C_m = -a_T \, V_H \, \eta_T \, \frac{\ell_T}{V} \, \Delta q \qquad (3.55)$$

Taking the limit of equation 3.55 gives

$$\frac{\partial C_m}{\partial q} = -a_T \, V_H \, \eta_T \, \frac{\ell_T}{V} \qquad (3.56)$$

Equation 3.56 shows that the damping expression dC_m/dq is a function of airspeed, i.e., this term is greater at lower speeds.

Solving for C_{m_q},

$$C_{m_q} = \frac{2V}{c} \frac{\partial C_m}{\partial q} = -2a_T \, V_H \, \eta_T \, \frac{\ell_T}{c} \qquad (3.57)$$

The damping derivative is not a function of airspeed but rather a value determined by design considerations only (subsonic flight). The damping in pitch derivative may be increased by increasing S_T or ℓ_T.

When this value for C_{m_q} is substituted into equation 3.48,

$$h_m = h_n' + \rho \frac{S\, a_T\, \eta_T\, \ell_T}{2m} V_H \qquad (3.58)$$

The following conclusions are apparent from equation 3.58.

1. The maneuver point should always be behind the neutral point. This is verified since the addition of a pitch rate increases the stability (C_{m_q} is negative in equation 3.46) of the aircraft. Therefore, the stability margin should increase.

2. Aircraft geometry is influential in locating the maneuver point aft of the neutral point.

3. As altitude increases, the distance between the neutral point and maneuver point decreases.

4. As weight decreases at any given altitude, the maneuver point moves further behind the neutral point and the maneuver stability margin increases.

5. The largest variation between maneuver point and neutral point occurs with a light aircraft flying at sea level.

3.3 AIRCRAFT BENDING

Before the pull up analysis is completed, one more subject should be covered. One of the assumptions made early in stability was that the aircraft was a rigid body. It is a well known fact that all aircraft bend when a load is applied. The bigger the aircraft, the more they bend. The effect on the aircraft bending is shown in figures 3.5, and 3.6.

Figure 3.5. RIGID AIRCRAFT UNDER HIGH LOAD FACTOR

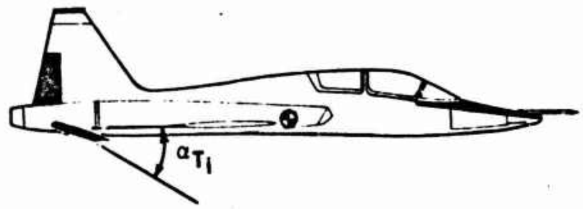
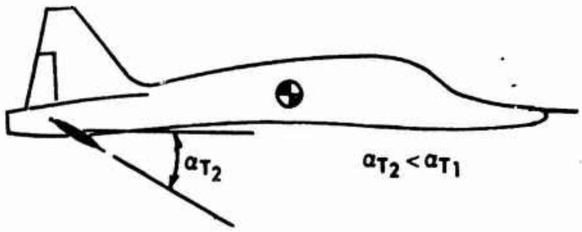


Figure 3.6. NON-RIGID AIRCRAFT UNDER HIGH LOAD FACTOR



The angle of attack of the tail is approximately the same as the angle of attack of the wing with the exception of downwash, incidence, etc., for a particular elevator deflection. As the non-rigid aircraft bends, the angle of attack α_T of the horizontal stabilizer decreases. In order to keep the aircraft at the same overall angle of attack, the original angle of attack of the tail must be reestablished. This requires an increase in the elevator (slab) deflection or a $\Delta\delta_e$ to reestablish the necessary α_T and to maintain the required maneuvering C_L. This additional elevator requirement under aircraft bending gives an apparent increase in the maneuvering stability of the aircraft or an additional $\Delta\delta_e$ per load factor.

3.4 THE TURN MANEUVER

The subject of maneuvering in pull ups has been thoroughly discussed and while it is the easiest method for a test pilot to perform, it is also the most time consuming.

Therefore, most maneuvering data is collected by turning. There are several methods used to collect data in a turn and these are discussed in the flight test portion of this text.

In order to analyze the maneuvering turn, equation 3.11 is recalled:

$$\Delta \delta_e = - \frac{C_{m_\alpha} \Delta \alpha - \partial C_m / \partial q \, \Delta q}{C_{m_{\delta_e}}} \qquad (3.59)$$

The expression for $\Delta\alpha$ in equation 3.19, derived for the pullup maneuver, is also applicable to the turning maneuver.

$$\Delta \alpha = \frac{1}{a} \left[C_L \, (n - n_o) - C_{L_{\delta_e}} \Delta \delta_e \right] \qquad (3.60)$$

Such is not the case for the Δq expression in equation 3.59. Another expression for Δq pertaining to the turn maneuver must be developed.

Referring to figure 3.7, the left vector will be statically balanced by the weight and centrifugal force. One component (L cos ϕ) balances the weight and the other (L sin ϕ) balances the centrifugal force.

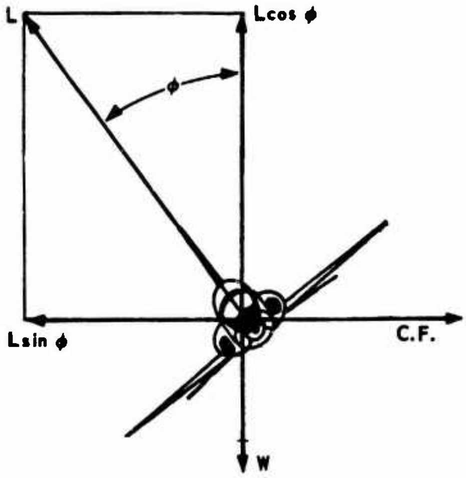
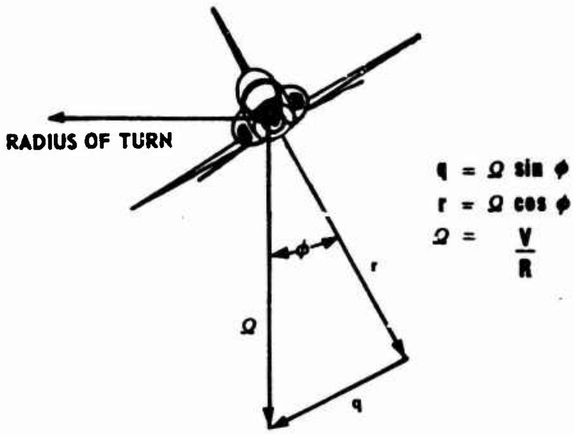


FIGURE 3.7a

Figure 3.7 AIRCRAFT IN THE TURN MANEUVER



$$L \sin \phi = CF = \frac{W}{g} \frac{V^2}{R} \qquad (3.61)$$

$$L \cos \phi = W \qquad (3.62)$$

$$n = L/W = \frac{1}{\cos \phi} \qquad (3.63)$$

Now dividing 3.61 by 3.62 and rearranging terms:

$$\frac{V}{R} = \frac{g}{V} \frac{\sin \phi}{\cos \phi} \qquad (3.64)$$

Referring to figure 3.7 where pitch rate is represented by a vector along the wings and yaw rate a vector vertically through the center of gravity, the following relationships can be derived.

$$\Omega = \frac{V}{R} \qquad (3.65)$$

$$q = \Omega \sin \phi \qquad (3.66)$$

$$q = \frac{V}{R} \sin \phi \qquad (3.67)$$

Substituting 3.64 into 3.67,

$$q = \frac{g}{V} \frac{\sin^2 \phi}{\cos \phi} \qquad (3.68)$$

$$q = \frac{g}{V} \frac{1 - \cos^2\phi}{\cos \phi} \qquad (3.69)$$

$$q = \frac{g}{V} \left(\frac{1}{\cos \phi} - \cos \phi\right) \qquad (3.70)$$

$$q = \frac{g}{V} \left(n - \frac{1}{n}\right) \qquad (3.71)$$

When maneuvering from initial conditions of n_o to n, the Δq equation becomes,

$$\Delta q = q - q_o = \frac{g}{V} \left(n - \frac{1}{n}\right) - \frac{g}{V} \left(n_o - \frac{1}{n_o}\right) \qquad (3.72)$$

$$\Delta q = \frac{g}{V} (n - n_o) \left(1 + \frac{1}{nn_o}\right) \qquad (3.73)$$

The general expression for Δq in equation 3.73 and the value for $\Delta\alpha$ in equation 3.60 may now be substituted into equation 3.59 to determine $\Delta\delta_e$

$$\Delta\delta_e = \frac{-C_{m_\alpha} \frac{1}{a} \left[C_L (n - n_o) - C_{L_{\delta_e}} \Delta\delta_e\right]}{C_{m_{\delta_e}}} \qquad (3.74)$$

$$- \frac{\partial C_m}{\partial q} \frac{g}{V} (n - n_o)\left(1 + \frac{1}{nn_o}\right)$$

Substituting

$$\frac{\partial C_m}{\partial q} = \frac{c}{2V} C_{m_q}$$

$$\left(C_{m_{\delta_e}} - \frac{C_{m_\alpha} C_{L_{\delta_e}}}{a}\right) \Delta\delta_e = - \frac{C_{m_\alpha} C_L}{a} (n - n_o) - C_{m_q} \frac{cg}{2V^2} (n - n_o)\left(1 + \frac{1}{nn_o}\right) \qquad (3.75)$$

$$\Delta\delta_e = \frac{C_{m_\alpha} C_L (n - n_o) + C_{m_q}\, a \frac{cg}{2V^2} (n - n_o)\left(1 + \frac{1}{nn_o}\right)}{C_{m_\alpha} C_{L_{\delta_e}} - C_{m_{\delta_e}}\, a} \qquad (3.76)$$

Now

$$C_{m_\alpha} = a\,(h - h_n) \text{ and } V^2 = \frac{2W}{C_L \rho S}$$

$$\frac{\Delta\delta_e}{(n - n_o)} = \frac{aC_L}{C_{m_\alpha} C_{L_{\delta_e}} - C_{m_{\delta_e}}\, a} \cdot \left[(h - h_n) + C_{m_q} \frac{\rho Sc}{4m}\left(1 + \frac{1}{nn_o}\right)\right] \qquad (3.77)$$

Taking the limit of $\Delta\delta_e/\Delta n$ in equation 3.77 and

$$\frac{d\delta_e}{dn} = \frac{aC_L}{C_{m_\alpha}C_{L_{\delta_e}} - C_{m_{\delta_e}}\, a} \cdot \left[h - h_n + \rho \frac{Sc}{4m} C_{m_q} \left(1 + \frac{1}{n^2}\right)\right] \qquad (3.78)$$

The maneuver point is determined by setting $d\delta_e/dn$ equal to zero and solving for the cg position at this point.

$$h_m = h_n - \rho \frac{Sc}{4m} C_{m_q} \left(1 + \frac{1}{n^2}\right) \qquad (3.79)$$

The maneuver point in a turn differs from the pullup by the factor $(1 + 1/n^2)$. This means that at low load factors the turn and pullup maneuver points will be very nearly the same. If equation (3.79) is solved for h_n and substituted back into equation 3.76, the result is:

$$\frac{d\delta_e}{dn} = \frac{aC_L}{C_{m_\alpha}C_{L_{\delta_e}} - C_{m_{\delta_e}}\, a} (h - h_m) \qquad (3.80)$$

The conclusion that $d\delta_e/dn$ is the same for both pullup and turn would be untrue since h_m in equation 3.80 for turns (includes the factor $(1 + 1/n^2)$ is different from the h_m found for the pullup maneuver. The same conclusions reached for 3.39 and 3.58 apply to 3.80 and 3.81 as well.

$$h_m = \frac{h_n + \rho S a_T \eta_T \ell_T}{2m} V_H (1 + 1/n^2) \qquad (3.81)$$

• 3.5 RECAPITULATION

Before looking further into the stick-free maneuverability case, it would be well to review the development in the preceding paragraphs and relate it to the results of chapter II.

The basic approach to longitudinal stability was centered around finding a value for dC_m/dC_L. It was found that a negative value for this derivative meant that the aircraft was statically stable. The derivative was analyzed for the stick-fixed case first and then the stick-free case. The cg position where this derivative was zero, was defined as the neutral point. Static margin was defined as the difference between the neutral point and the cg location. The stick-free case was determined by:

$$\left.\frac{dC_m}{dC_L}\right|_{\substack{\text{Stick Free}\\\text{Aircraft}}} = \left.\frac{dC_m}{dC_L}\right|_{\substack{\text{Stick Fixed}\\\text{Aircraft}}} + \Delta\left.\frac{dC_m}{dC_L}\right|_{\substack{\text{Effect of}\\\text{Free Elevator}}} \qquad (3.82)$$

The free elevator case was merely the basic stability of the aircraft with the effect of freeing the elevator added to it.

When the maneuvering case was introduced, it was shown that there was a new derivative to be discussed but the basic stability of the aircraft would not change - only the effect of pitch rate was added to it.

$$\left.\frac{dC_m}{dC_L}\right|_{\substack{\text{Stick Fixed}\\\text{Aircraft Pitching}}} = \left.\frac{dC_m}{dC_L}\right|_{\substack{\text{Stick Fixed}\\\text{Aircraft}}} + \Delta\left.\frac{dC_m}{dC_L}\right|_{\substack{\text{Effect of The}\\\text{Pitch Rate}}} \qquad (3.83)$$

For the stick-free case, the following must be true,

$$\left.\frac{dC_m}{dC_L}\right|_{\substack{\text{Stick Free}\\\text{Aircraft Pitching}}} = \left.\frac{dC_m}{dC_L}\right|_{\substack{\text{Stick Fixed}\\\text{Aircraft}}} + \Delta\left.\frac{dC_m}{dC_L}\right|_{\substack{\text{Effect of}\\\text{Free Elevator}}} + \Delta\left.\frac{dC_m}{dC_L}\right|_{\substack{\text{Effect of}\\\text{Pitch Rate}}} \qquad (3.84)$$

$$\left.\frac{dC_m}{dC_L}\right|_{\substack{\text{Stick Free}\\\text{Aircraft Pitching}}} = \left.\frac{dC_m}{dC_L}\right|_{\substack{\text{Stick Free}\\\text{Aircraft}}} + \Delta\left.\frac{dC_m}{dC_L}\right|_{\substack{\text{Effect of}\\\text{Pitch Rate}}} \qquad (3.85)$$

• 3.6 STICK FREE MANEUVERING

The first analysis of stick-free maneuvering requires a review of longitudinal stability. It was determined in chapter II that the effect of freeing the elevator was to multiply the tail term by the free elevator factor F which equaled $(1 - \tau\, C_{h_\alpha}/C_{h_\delta})$. Consequently, to free the elevator in the maneuvering case and find the stick-free maneuver point, the tail effect of stick-fixed maneuvering must be multiplied by this free elevator factor. Recalling equation 3.47 from the stick-fixed maneuvering discussion,

$$h_m = \frac{X_{ac}}{c} - \frac{dC_m}{dC_{L_{Fus}}} + \frac{a_t}{a_w} V_H \eta_T \cdot \left(1 - \frac{d\epsilon}{d\alpha}\right) - \rho \frac{Sc}{4m} C_{m_q} \qquad (3.86)$$

Multiplying the tail terms by F,

$$h_m' = \frac{X_{ac}}{c} - \frac{dC_m}{dC_{L_{Fus}}} + \frac{a_t}{a_w} V_H \eta_T \left(1 - \frac{d\epsilon}{d\alpha}\right)F - \rho \frac{Sc}{4m} C_{m_q} F \qquad (3.87)$$

The first three terms on the right are the expression for stick-free neutral point, h_n'.

$$h_m' = h_n' - \rho \frac{Sc}{4m} C_{m_q} F \qquad (3.88)$$

This is the stick free maneuver point in terms of the stick free neutral point for the pullup case. It may be extended to the turn case by using the term for the pitch rate of the tail in a turn.

$$h_m' = h_n' - \rho \frac{Sc}{4m} C_{m_q} F \left(1 + \frac{1}{n^2}\right) \qquad (3.89)$$

These equations do not give any flight test relationship and so it is necessary to derive this from stick forces, as was done in longitudinal static stability. The method used will be to relate the stick force-per-g to the stick free maneuver point since stick forces can be related to the freeing of the elevator. Starting with the relationship of stick force, gearing, and hinge moments that was derived in chapter II,

$$F_s = -GH_e \qquad (3.90)$$

$$H_e = q\, S_e\, c_e\, C_{h_e} \qquad (3.91)$$

$$F_s = -Gq\, S_e c_e\, C_{h_e} \qquad (3.92)$$

The change in stick force for a change in load factor becomes,

$$\frac{\Delta F_s}{\Delta n} = -Gq\, S_e\, c_e \frac{\Delta C_{h_e}}{\Delta n} \qquad (3.93)$$

where

$$\Delta C_{h_e} = C_{h_{\alpha_T}} \Delta \alpha_T + C_{h_{\delta_e}} \Delta \delta_e \qquad (3.94)$$

Stick-Free Pullup Maneuver:

ΔC_{h_e} must be written in terms of load factor and substituted back into equation 3.93. This will require defining $\Delta\alpha_T$ and $\Delta\delta_e$ in terms of load factor. The change in angle of attack of the tail comes partly from the change in angle of attack of the wing due to downwash and partly from the pitch rate.

$$\Delta \alpha_T = \Delta \alpha_W \left(1 - \frac{d\epsilon}{d\alpha}\right) + \Delta_q \frac{\ell_T}{V} \qquad (3.95)$$

where $\Delta\alpha_W + \Delta q$ in the above equation are

$$\Delta \alpha_W = \frac{1}{a}\left[C_L (n - n_o) - C_{L_{\delta_e}} \Delta \delta_e\right] \qquad (3.96)$$

$$\Delta_q = \frac{g}{V}(n - n_o) \qquad (3.97)$$

$$\Delta \delta_e = \frac{aC_L}{C_{m_\alpha} C_{L_{\delta_e}} - C_{m_{\delta_e}} a} (h - h_m)(n - n_o) \qquad (3.98)$$

If the equations above are substituted into 3.94, the results would be cumbersome at best. To simplify things $C_{L_{\delta_e}}$ will be assumed small enough to ignore. (Reasonable assumption since total change in lift of the aircraft when the elevator is deflected is small.) The above equations simplify to:

$$\Delta \alpha_T = \frac{C_L}{a}\left(1 - \frac{d\epsilon}{d\alpha}\right) \cdot (n - n_o) + \frac{g}{V^2} \ell_T (n - n_o) \qquad (3.99)$$

$$\Delta \delta_e = -\frac{C_L}{C_{m_{\delta_e}}} (h - h_m)(n - n_o) \qquad (3.100)$$

Substituting equations 3.99 and 3.100 into 3.94,

$$\frac{\Delta C_{h_e}}{n - n_o} = C_{h_\alpha} \frac{C_L}{a} \left(1 - \frac{d\epsilon}{d\alpha}\right) + C_{h_\alpha} g \frac{\ell_T}{V^2}$$

$$- C_{h_\delta} \frac{C_L}{C_{m_{\delta_e}}} (h - h_m) \qquad (3.101)$$

Substituting $V^2 = W/\rho S C_L$ and $C_{m_\delta} = -a_e V_H$ and isolating the maneuver margin $(h - h_m)$ by factoring out $(- C_{h_\delta} C_L/C_{m_{\delta_e}})$, the result is:

$$\frac{\Delta C_h}{n - n_o} = - \frac{C_{h_\delta} C_L}{C_{m_{\delta_e}}} \left[\frac{C_{h_\alpha}}{C_{h_\delta} a_W} \left(1 - \frac{d\epsilon}{d\alpha}\right) a_e V_H + \frac{C_{h_\alpha}}{C_{h_\delta}} \rho \frac{\ell_T}{2m} S a_e V_H + h - h_m \right] \qquad (3.102)$$

From longitudinal stability,

$$h_n - h_n' = \frac{C_{h_\alpha}}{C_{h_\delta}} \frac{a_\epsilon}{a_W} V_H \left(1 - \frac{d\epsilon}{d\alpha}\right) \qquad (3.103)$$

and if the second term in the parenthesis is multiplied by

$\frac{-2ca_T}{-2ca_T}$ and knowing that

$$C_{m_q} = -2a_T \frac{V_H}{c} \ell_T \qquad (3.104)$$

$$\tau = a_e/a_T \qquad (3.105)$$

$$F = 1 - \tau \frac{C_{h_\alpha}}{C_{h_\delta}} \qquad (3.106)$$

The second term becomes:

$$(F-1) \rho \frac{Sc}{4m} C_{m_q} \qquad (3.107)$$

Rewriting equation 3.102,

$$\frac{\Delta C_h}{n - n_o} = + \frac{C_{h_\delta}}{C_{m_{\delta_e}}} C_L \cdot \left[h_n' - h_n + (1 - F) \rho \frac{Sc}{4m} C_{m_q} - h + h_m \right] \qquad (3.108)$$

but

$$h_m = h_n - \rho \frac{Sc}{4m} C_{m_q} \qquad (3.109)$$

Therefore:

$$\frac{\Delta C_h}{n - n_o} = - \frac{C_{h_\delta}}{C_{m_{\delta_e}}} C_L \cdot \left[h - h_n' + \rho \frac{Sc}{4m} C_{m_q} F \right] \qquad (3.110)$$

Substituting equation 3.110 back into 3.93 and taking the limit

$$\frac{dF_s}{dn} = G\, 1/2\, \rho V^2 S_e c_e \frac{C_{h_\delta}}{C_{m_{\delta_e}}} C_L \cdot \left[h - h_n' + \rho \frac{Sc}{4m} C_{m_q} F \right] \qquad (3.111)$$

Defining the stick-free maneuver point as the cg position where dF_s/dn is equal to zero,

$$h_m' = h_n' - \rho \frac{Sc}{4m} C_{m_q} F \qquad (3.112)$$

which is the same equation as 3.88 previously derived. Equation 3.111 may be written,

$$\frac{dF_s}{dn} = G\, 1/2\, \rho V^2 S_e c_e \frac{C_{h_\delta}}{C_{m_{\delta_e}}} C_L \left[h - h_m' \right] \qquad (3.113)$$

Equation 3.113 may be rearranged if the following substitutions are made.

$$C_{m_{\delta_e}} = -a_e V_H = a_T \frac{\partial \alpha_T}{\partial \delta_e} V_H$$

$$= -C_{L_{\delta_e}} \frac{\ell_T S_T}{c_w S_w} \quad (3.114)$$

$$C_L = \frac{2W}{\rho V^2 S} \quad (3.115)$$

The stick-force-per-g equation becomes:

$$\frac{dF_s}{dn} = -G (S_e c_e W C_{h_\delta}) \frac{c_W}{\ell_T S_T C_{L_{\delta_e}}} \cdot$$

$$\left[h - h'_m\right] \quad (3.116)$$

Stick-Free Turn Maneuver:

The procedure used for determining the dF_s/dn equation and an expression for the stick-free maneuver point for the turning maneuver is practically identical to the pullup case. For the turn condition Δq is now,

$$\Delta q = \frac{g}{V} (n - n_o) (1 + \frac{1}{nn_o}) \quad (3.117)$$

The change in angle of attack of the tail, $\Delta\alpha_T$ and $\Delta\delta_e$ become

$$\Delta \alpha_T = \frac{C_L}{a} (n - n_o)(1 - \frac{d\epsilon}{d\alpha}) +$$

$$\frac{g \ell_T}{V^2} (n - n_o)(1 + \frac{1}{nn_o}) \quad (3.118)$$

$$\Delta\delta_e = \frac{-C_L}{C_{m_{\delta_e}}} \left[(h - h_n)(n - n_o) + \right.$$

$$\left. \rho \frac{Sc}{4m} C_{m_q} (n-n_o)(1 + \frac{1}{nn_o})\right] \quad (3.119)$$

Substituting equations 3.118 and 3.119 into equation 3.94 and performing the same factoring and substitutions as in the pullup case;

then,

$$\frac{\Delta C_h}{n-n_o} = -\frac{C_{h_\delta} C_L}{C_{m_{\delta_e}}} \cdot$$

$$\left[h - h'_n + \rho \frac{Sc}{4m} C_{m_q} F (1 + 1/n^2)\right]$$

$$(3.120)$$

Substituting 3.120 into 3.93

$$\frac{dF_s}{dn} = G\ 1/2\ \rho V^2 S_e c_e \frac{C_{h_\delta} C_L}{C_{m_{\delta_e}}} \cdot$$

$$\left[h - h'_n + \rho \frac{Sc}{4m} C_{m_q} F (1 + \frac{1}{n^2})\right]$$

$$(3.121)$$

And solving for the stick-free maneuver point,

$$h'_m = h'_n - \rho \frac{Sc}{4m} C_{m_q} F (1 + 1/n^2) \quad (3.122)$$

Further substitution puts equation 3.121 into the following form:

$$\frac{dF_s}{dn} = -G (S_e c_e W C_{h_\delta}) \frac{c_w}{\ell_T S_T C_{L_{\delta_e}}} \cdot$$

$$\left[h - h'_m\right] \quad (3.123)$$

Again, the turning stick-force-per-g equation 3.123 appears identical to the stick-free pullup equation. However, the expression for the maneuver point h'_m is different.

The term in the first parenthesis represents the hinge moment of the elevator. The second term is the elevator power and the last term is the negative value of the

stick-free maneuver margin. The following conclusions are drawn from this equation.

1. The stick-force-per-g appears to vary directly with the weight. However, weight also appears inversely in h_m'. Therefore, the full effect of weight cannot be truly analyzed since one effect could cancel the other.

2. Since airspeed does not appear in the equation, the stick-force-per-g will be the same at all airspeeds for a fixed cg.

From equation 3.112, come the following conclusions.

1. The difference between the stick-fixed and stick-free maneuver point is a function of the free elevator factor, F.

2. The stick-free maneuver point, h_m', varies directly with altitude, becoming closer to the stick-free neutral point, the higher the aircraft flies.

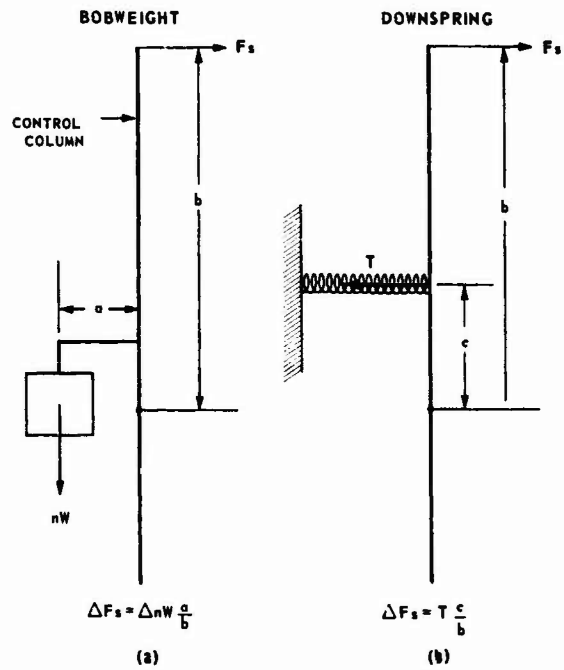
3. The location of the stick-free maneuver point is of academic interest only since it occurs at the point where $dF_s/dn = 0$. It is difficult to fly an aircraft with this type gradient. Consequently, military specifications limit the minimum value of dF_s/dn to three pounds per g.

4. The forward cg is limited by stick force per g. The maximum value is limited by the type aircraft (bomber, fighter, or trainer); i.e., heavier gradients in bomber type and lighter ones in fighters.

3.7 EFFECT OF BOBWEIGHTS AND SPRINGS

The effect of bobweights and springs on the stick-free maneuver point and stick-force gradients is of interest. The result of adding a spring or a bobweight to the control system adds an incremental force to the system. The effect of the spring is different from the effect of the bobweight. The spring exerts a constant force on the stick no matter what load factor is applied. The bobweight exerts a force on the stick proportional to the load factor.

Figure 3.8 BOBWEIGHT AND DOWNSPRING



The force increment for the downspring and bobweight are:

$$\Delta F_{s_{\text{Spring}}} = -\frac{G S_e \bar{c}_e W C_{h_\delta} \bar{c}_w}{\ell_T S_T C_{L_{\delta_e}}} (h - h_m')(n - n_o) + T \frac{c}{b} \quad (3.124)$$

$$\Delta F_{s_{\text{Bobweight}}} = - \frac{G\, S_e\, c_e\, W\, C_{h_\delta}}{\ell_T\, S_T\, C_{L_{\delta_e}}}\, c_w \cdot (h - h'_m)(n - n_o) + W \frac{a}{b}(n - n_o) \qquad (3.125)$$

When the derivative is taken with respect to load factor, the effect on dF_s/dn of the spring is zero. The stick force gradient is not affected by the spring nor is the stick-free maneuver point changed.

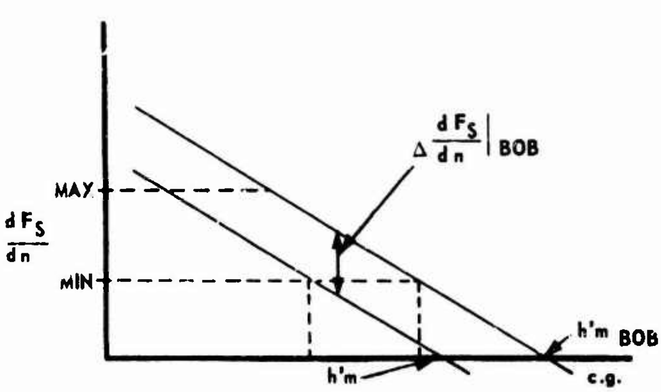
$$\left.\frac{dF_s}{dn}\right|_{\text{Spring}} = - \frac{G\, S_e\, c_e\, W\, C_{h_\delta}}{\ell_T\, S_T\, C_{L_{\delta_e}}}\, c_w\, (h - h'_m) \qquad (3.126)$$

For the bobweight, the stick force gradient dF_s/dn becomes:

$$\left.\frac{dF_s}{dn}\right|_{\text{Bobweight}} = - \frac{G\, S_e\, c_e\, W\, C_{h_\delta}}{\ell_T\, S_T\, C_{L_{\delta_e}}}\, c_w\, (h - h'_m) + W \frac{a}{b} \qquad (3.127)$$

Consequently, the addition of the bobweight (positive) increases the stick force gradient, moves the stick free maneuver point aft, and shifts the allowable cg spread aft (the minimum and maximum cg positions as specified by force gradients are moved aft). See figure 3.9.

Figure 3.9 EFFECTS OF ADDING A BOBWEIGHT



3.8 AERODYNAMIC BALANCING

Aerodynamic balancing is used to affect the stick force gradient and stick free maneuver point. Aerodyanmic balancing or varying values of C_{h_α} and C_{h_δ} affects the following stick free equations.

$$\frac{dF_s}{dn} = - \frac{G\, S_e\, c_e\, W\, C_{h_\delta}}{\ell_T\, S_T\, C_{L_{\delta_e}}}\, c_w\, (h - h'_m) \qquad (3.128)$$

$$h'_m = h'_n - \rho \frac{Sc}{4m}\, C_{m_q}\, F \qquad (3.129)$$

$$F = 1 - \tau \frac{C_{h_\alpha}}{C_{h_\delta}} \qquad (3.130)$$

Decreasing C_{h_δ} and/or increasing C_{h_α} by using two such aerodynamic balanced devices as an overhang balance or a lagging balance tab, does the following:

1. The free elevator factor, F, decreases.
2. The stick free maneuver point h'_m moves forward.
3. The maneuver margin term $(h - h'_m)$ decreases.
4. The stick force gradient decreases.
5. The forward and aft cg limits move forward.

Increasing C_{h_δ} and/or decreasing C_{h_α} by using a convex trailing edge or a leading balance tab does the following:

1. The free elevator factor, F, increases.
2. The stick free maneuver point h'_m moves aft.

3. The maneuver margin term $(h - h'_m)$ increases.

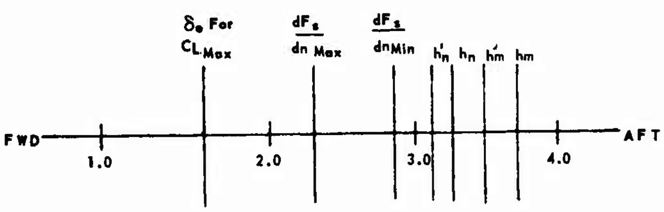
4. The stick force gradient increases.

5. The forward and aft cg limits move aft.

3.9 cg RESTRICTIONS

The restrictions on the aircraft's center of gravity location may be examined by referring to the mean aerodynamic chord in figure 3.10.

Figure 3.10 RESTRICTIONS TO CENTER OF GRAVITY LOCATIONS



The forward cg travel is normally limited by:

1. Maximum stick-force-per-g gradient - dF_s/dn.

or

2. Elevator required to land at $C_{L_{MAX}}$.

The aft cg travel is normally limited by:

1. Minimum stick-force-per-g - dF_s/dn.

or

2. Stick free neutral point-power on - h'_n.

Additional considerations:

1. Freeing the elevator causes a destabilizing moment that locates the stick free neutral and maneuver points ahead of their respective stick fixed points.

2. The stick-free neutral point, h'_n, can be moved aft artifically with a downspring. The stick free maneuver point, h'_m, can be moved aft with a bobweight but not a downspring.

3. The desired aft cg location may be unsatisfactory because it lies aft of the cg position giving minimum stick force gradient. The requirement for bobweight or a particular aerodynamic balancing would exist in order to shift the cg for minimum stick force gradient aft of the desired aft cg position.

The equations which pertain to maneuvering flight are repeated below:

<u>Pull Ups, Stick-Fixed</u>

$$h_m = h_n - \rho \frac{Sc}{4m} C_{m_q} \tag{3.131}$$

$$\frac{d\delta_e}{dn} = \frac{a\, C_L}{C_{m_\alpha} C_{L_{\delta_e}} - C_{m_{\delta_e}} a} (h - h_m) \tag{3.132}$$

<u>Pull Ups, Stick-Free</u>

$$h'_m = h'_n - \rho \frac{Sc}{4m} C_{m_q} F \tag{3.133}$$

$$\frac{dF_s}{dn} = - \frac{G\, S_e\, c_e\, W\, C_{h_\delta}\, c_w}{\ell_T\, S_T\, C_{L_{\delta_e}}} (h - h'_m) \tag{3.134}$$

$$\frac{dF_s}{dn} = -\frac{G\, S_e\, c_e\, W\, C_{h_\delta}}{\ell_T\, S_T\, C_{L_{\delta_e}}}\, c_w\, (h - h_m') + W\,\frac{a}{b} \qquad (3.135)$$

Bobweight

Turns; Stick-Fixed

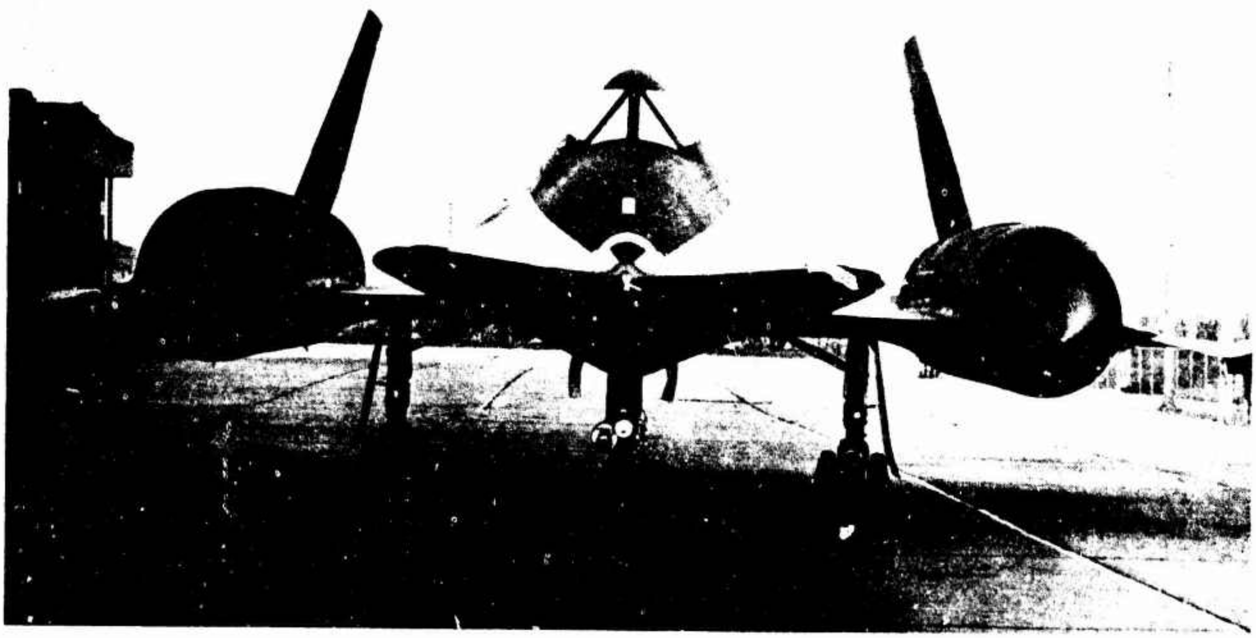
$$h_m = h_n - \rho\,\frac{Sc}{4m}\, C_{m_q}\left(1 + \frac{1}{n^2}\right) \qquad (3.136)$$

$$\frac{d\delta_e}{dn} = \frac{a\, C_L}{C_{m_\alpha}\, C_{L_{\delta_e}} - C_{m_{\delta_e}}\, a}\,(h - h_m) \qquad (3.137)$$

Turns; Stick-Free

$$h_m' = h_n' - \rho\,\frac{Sc}{4m}\, C_{m_q}\,(1 + 1/n^2) \qquad (3.138)$$

$$\frac{dF_s}{dn} = -\frac{G\, S_e\, c_e\, W\, C_{h_\delta}}{\ell_T\, S_T\, C_{L_{\delta_e}}}\, c_w\, (h - h_m') \qquad (3.139)$$



CHAPTER 4

LATERAL-DIRECTIONAL STATIC STABILITY

4.1 INTRODUCTION

An analysis of the equations of aircraft motion leads to the following mathematical description of aircraft lateral-directional motion:

$$F_y = m\dot{v} + mru - pwm \quad (4.1)$$

$$G_x = \dot{p}I_x + qr(I_z - I_y) - (\dot{r} + pq)I_{xz} \quad (4.2)$$

$$G_z = \dot{r}I_z + pq(I_y - I_x) + (qr - \dot{p})I_{xz} \quad (4.3)$$

These equations will yield a bounded or "stable" solution for a bounded or "stable" input. Thus, an analysis of aircraft lateral-directional motion need only consider the left side of the equations to determine aircraft stability. Further analysis of the aircraft equations of motion reveals the left side of the foregoing equations to be composed primarily of contributions from aerodynamic forces and moments, direct thrust, gravity, and gyroscopic moments. Of these, only the aerodynamic forces and moments (Y, L, N) will be analyzed because the other sources are usually eliminated through proper design.

It has been found from experience that when operating under a small disturbance assumption, aircraft lateral-directional motion can be considered independent of longitudinal motion and that it can be considered as a function of the following variables:

$$(Y, \mathcal{L}, \mathcal{N}) = f(\beta, \dot{\beta}, p, r, \delta_a, \delta_r) \quad (4.4)$$

The side forces, Y, acting on the aircraft, can be represented by a couple at the aircraft cg. The resulting moment is considered as a part of the aerodynamic moment about the cg, $\mathcal{N}$. The remaining force is useful only in determining the magnitude of the sideslip angle, β. In the treatment of aircraft lateral-directional motion, sideslip angle, β, is considered the independent variable, therefore, the side force equation will not be considered further.

The two remaining forcing functions can be expressed in terms of non-dimensional stability derivatives, angular rates and angular displacements:

$$C_n = C_{n_\beta}\beta + C_{n_{\dot{\beta}}}\hat{\dot{\beta}} + C_{n_p}\hat{p} + C_{n_r}\hat{r} + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r \quad (4.5)$$

$$C_\ell = C_{\ell_\beta}\beta + C_{\ell_{\dot{\beta}}}\hat{\dot{\beta}} + C_{\ell_p}\hat{p} + C_{\ell_r}\hat{r} + C_{\ell_{\delta_a}}\delta_a + C_{\ell_{\delta_r}}\delta_r \quad (4.6)$$

The analysis of aircraft lateral-directional motion is based on these two equations. A cursory examination of these equations reveals the presence of "cross-coupling" terms, e.g., $C_{n_p}p$ and $C_{n_{\delta_a}}\delta_a$ in the yawing moment equation (4.5). It is for this reason that aircraft lateral motions and directional motions must be considered together - each one influences the other.

Static directional stability will be considered first Each stability derivative in equation (4.5) will be discussed and its contribution to aircraft stability will be analyzed. A summary of these stability derivatives follows:

4.2 C_{n_β} -STATIC DIRECTIONAL STABILITY OR WEATHERCOCK STABILITY

Static directional stability is defined as the initial tendency of an aircraft to return to or depart from its equilibrium angle of sideslip when disturbed. Although the static directional stability of an aircraft is determined through consideration of all the terms in equation 4.5, C_{n_β} is often referred to as "static directional stability" because it is the predominant term.

When an aircraft is placed in a sideslip, aerodynamic forces develop which create moments about all three axis. The moments created about the Z axis tend to turn the nose of the aircraft into or away from the relative wind. The aircraft is statically directionally stable if the moments created by a sideslip angle tend to align the nose of the aircraft with the relative wind. By convention, sideslip angle is defined as positive if the relative wind is displaced to the right of the fuselage reference line.

FIGURE 4.2

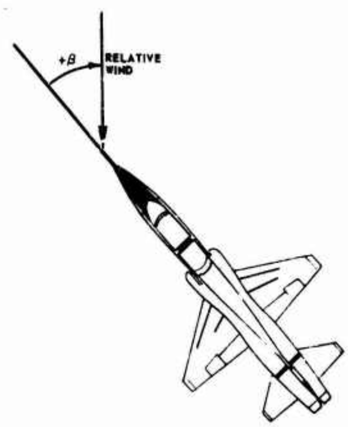
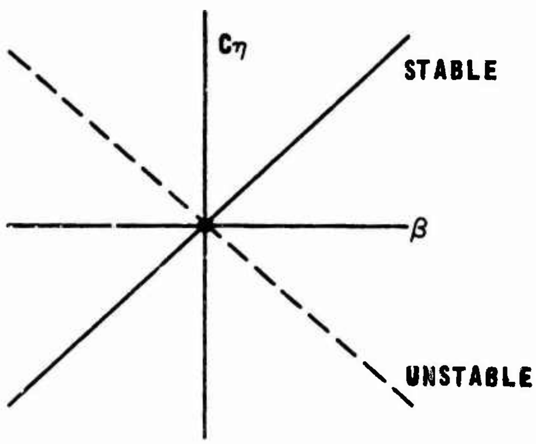


FIGURE 4.1

DERIVATIVE	NAME	SIGN FOR A STABLE AIRCRAFT	CONTRIBUTING PARTS OF AIRCRAFT
C_{n_β}	Static Directional Stability or Weathercock Stability	(+)	Tail, Fuselage, Wing
$C_{n_{\dot\beta}}$	Lag Effects	(-)	Tail
C_{n_p}	Cross-Coupling	(+)	Wing, Tail
C_{n_r}	Yaw Damping	(-)	Tail, Wing, Fuselage
$C_{n_{\delta_a}}$	Adverse or Complimentary Yaw	"0" or slightly (+)	Lateral Control
$C_{n_{\delta_r}}$	Rudder Power	(+)	Rudder Control

In figure 4.2 the aircraft is in a right sideslip. It is statically stable if it develops yawing moments that tend to align it with the relative wind, or, in this case, right (positive) yawing moments. Therefore, an aircraft is statically directionally stable if it develops positive yawing moments with a positive increase in sideslip. Thus, the slope of a plot of yawing moment coefficient, C_n, versus sideslip, β, is a quantitative measure of the static directional stability that an aircraft possesses. This plot would necessarily be determined from wind tunnel results.

FIGURE 4.3 WIND TUNNEL RESULTS OF YAWING MOMENT COEFFICIENT vs SIDESLIP

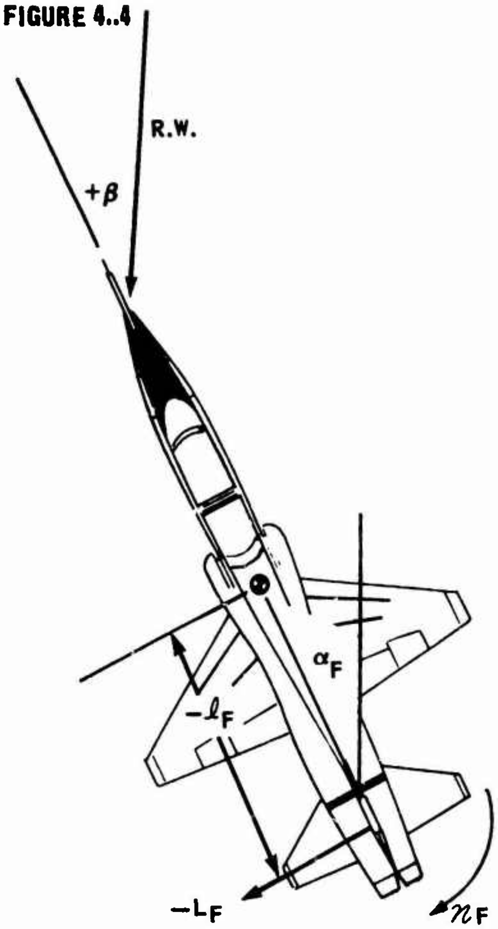


The total value of the directional stability derivative, C_{n_β}, at any sideslip angle, is determined by contributions from the vertical tail, the fuselage, and the wing. These contributions will be discussed separately.

Vertical Tail Contribution to C_{n_β}:

The vertical tail is the primary source of directional stability for virtually all aircraft. When the aircraft is yawed, the angle of attack of the vertical tail is changed. This change in angle of attack produces a change in lift on the vertical tail, and thus a yawing moment about the Z-axis.

FIGURE 4.4



Referring to figure 4.4, the yawing moment produced by the tail is:

$$n_F = (-\ell_F)(-L_F) = \ell_F L_F \tag{4.7}$$

The minus signs in this equation arise from the use of the sign convention adopted in the study of aircraft equations of motion. Forces to the left and distances behind the aircraft cg are negative.

As in other aerodynamic considerations, it is convenient to consider yawing moments in coefficient form so that static directional stability can be evaluated independent of weight, altitude and speed. Putting equation 4.7 in coefficient form:

$$C_{n_F} = \frac{\ell_F L_F}{q_w s_w b_w} = \frac{\ell_F C_{L_F}\, q_F\, S_F}{q_w s_w b_w} \tag{4.8}$$

Vertical tail volume ratio, V_V, is defined as:

$$V_v = \frac{S_F \ell_F}{S_w b_w} \qquad (4.9)$$

The sign of V_V may be either positive or negative. Making this substitution in equation 4.8:

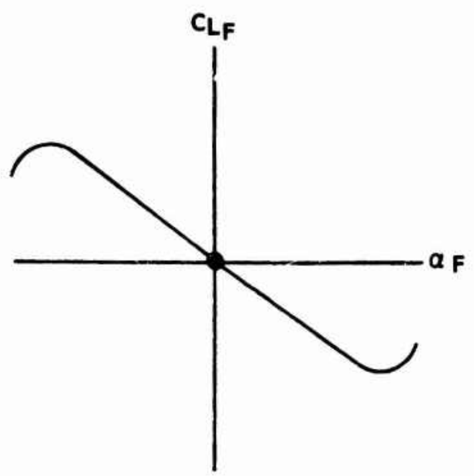
$$C_{n_F} = \frac{C_{L_F} q_F V_v}{q_w} \qquad (4.10)$$

For a propeller driven aircraft, q_W is greater than q_F. However, for a jet aircraft, these two quantities are equal. Thus, for a jet aircraft, equation 4.10 becomes:

$$C_{n_F} = C_{L_F} V_v \qquad (4.11)$$

The lift curve for a vertical tail is presented in figure 4.5.

FIGURE 4.5 LIFT CURVE FOR A VERTICAL TAIL



The negative slope of this curve is a result of the sign convention used. Reference figure 4.4. When the relative wind is displaced to the right of the fuselage reference line, the vertical tail is placed at a positive angle of attack. However, this results in a lift force to the left, or a negative lift. Thus, the sign of the lift curve slope of a vertical tail, a_F, will always be negative below the stall.

$$C_{L_F} = a_F \alpha_F \qquad (4.12)$$

Making this substitution in equation 4.11:

$$C_{n_F} = a_F \alpha_F V_v \qquad (4.13)$$

The angle of attack of the vertical tail, α_F, is not merely β. If the vertical tail were placed alone in an airstream, the α_F would be equal to β. However, when the tail is installed on an aircraft, changes in both magnitude and direction of the local flow at the tail take place. These changes may be caused by a propeller slipstream, or by the wing and the fuselage when the airplane is yawed. The angular deflection is allowed for by introducing the sidewash angle, σ, analogous to the downwash angle, ε. The value of σ is very difficult to predict, therefore suitable wind tunnel tests are required. The sign of σ is defined as positive if it causes α_F to be less than β. Thus,

$$\sigma = \beta - \alpha_F \qquad (4.14)$$

Substituting in equation 4.13:

$$C_{n_F} = a_F V_v (\beta - \sigma) \qquad (4.15)$$

The contribution of the vertical tail to weathercock stability is found by examining the change in C_{n_F} with a change in sideslip angle, β.

$$\frac{\partial C_{n_F}}{\partial \beta} = \left[C_{n_\beta (\text{Tail})} \right]_{\text{Fixed}} = V_v \, a_F \left(1 - \frac{\partial \sigma}{\partial \beta}\right) \qquad (4.16)$$

The subscript "fixed" is added to emphasize that, thus far, the vertical tail has been considered as a surface with no moveable parts, i.e., the rudder is "fixed."

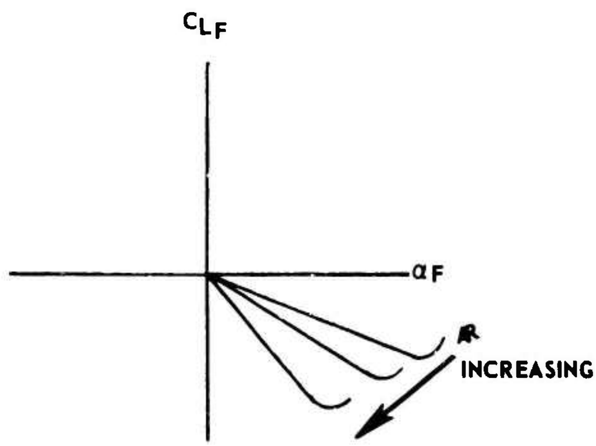
Equation 4.16 reveals that the vertical tail contribution to directional stability can only be changed by varying the vertical

tail volume ratio, V_V, or the vertical tail lift curve slope, a_F. The vertical tail volume ratio can be changed by varying the size of the vertical tail, or its distance from the aircraft cg. The vertical tail lift curve slope can be changed by altering the basic airfoil section of the vertical tail, or by end plating the vertical fin. An end plate on the top of the vertical tail is a relatively minor modification and yet it increases the directional stability of the aircraft significantly. This fact has been utilized in the case of the T-38 (figure 4.6). As can be seen in figure 4.7, the entire stabilator on the F-104 acts as an end plate and, therefore, adds greatly to the directional stability of the aircraft.

FIGURE 4.6

FIGURE 4.7

The effect of an end plate on the vertical stabilizer is to increase the effective aspect ratio of the vertical tail. As with any airfoil, this change in aspect ratio produces a change in the lift curve slope of the airfoil.

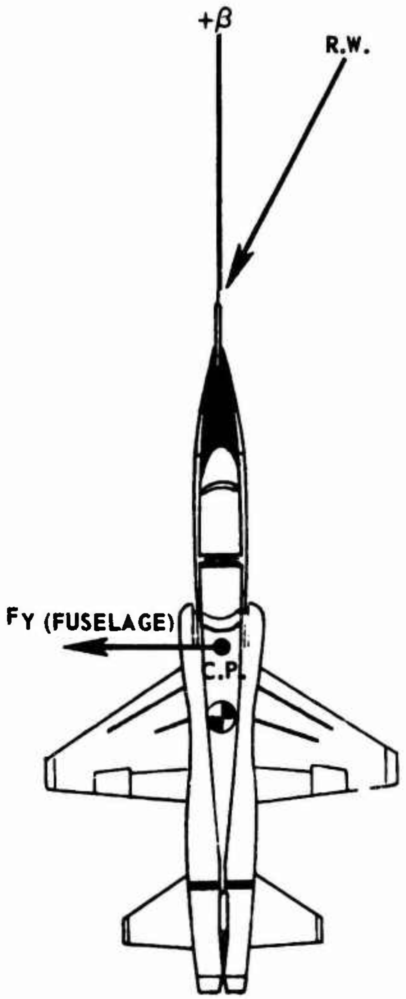


As the aspect ratio is increased, the α_F for stall is decreased. Thus, if the aspect ratio of the vertical tail is too high, the vertical tail will stall at low sideslip angles and a large decrease in directional stability will occur.

Fuselage Contribution to C_{n_β}:

The subsonic center of pressure of a typical fuselage occurs about one-fourth of the distance back from the nose. Since the aircraft center of gravity usually lies behind this point, the fuselage is generally de-stabilizing.

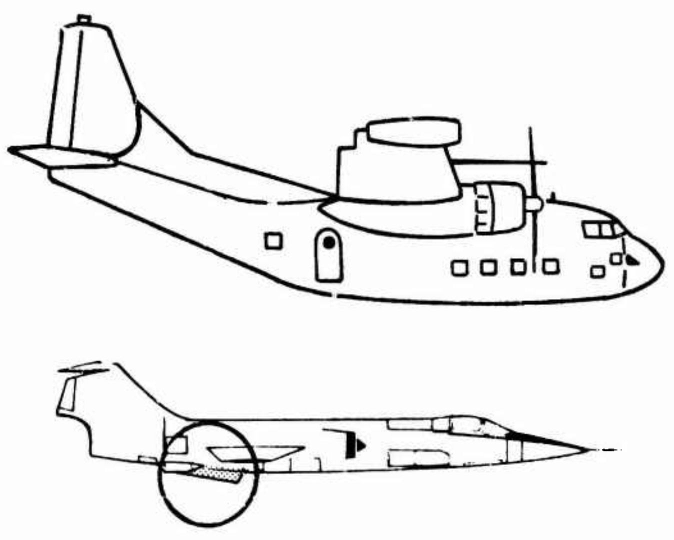
FIGURE 4.8



As can be seen from figure 4.8, a positive sideslip angle will produce a negative yawing moment about the cg, thus, $C_{n_{\beta}(\text{fuselage})}$ is negative or destablizing. The destabilizing influence of the fuselage diminishes at large sideslip angles due to a decrease in lift as the fuselage stall angle of attack is exceeded, and also due to an increase in parasite drag acting at the center of equivalent parasite area which is located aft of the cg.

If the overall directional stability of an aircraft becomes too low, the fuselage-tail combination can be made more stabilizing by adding a dorsal fin or a ventral fin. A dorsal fin was added to the C-123 and a ventral fin was added to the F-104 to improve static directional stability.

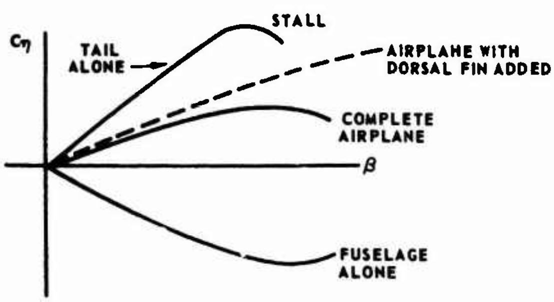
FIGURE 4.9



Since the addition of a dorsal fin decreases the effective aspect ratio of the tail, a higher sideslip angle can be attained before the vertical fin will stall. However, the major effect of the dorsal fin at large sideslip angles is to move the center of equivalent parasite area further aft of the cg, therefore producing a greater stabilizing moment at any given

sideslip angle. Thus, a dorsal fin greatly increases directional stability at large sideslip angles. Figure 4.10 shows the effect on directional stability of adding a dorsal fin.

FIGURE 4.10 EFFECT OF ADDING A DORSAL FIN



$C_{n_\beta(\text{fuselage})}$ is difficult to estimate, and although some empirical formulas exist, it is usually measured directly by wind tunnel tests using a model without a tail.

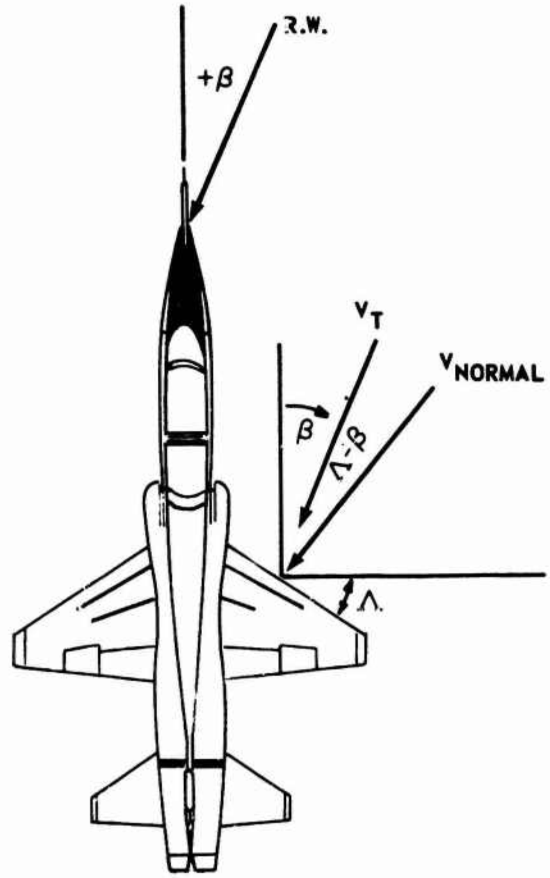
Wing Contribution to C_{n_β}:

The wing contribution to static directional stability is usually small. Straight wings make a slight positive contribution to static directional stability due to fuselage blanking in a sideslip. Effectively, the relative wind "sees" less of the downwind wing due to fuselage blanking. This reduces the lift of the downwind wing, and thus reduces the induced drag on the downwind wing. The difference in induced drag on the two wings tends to yaw the aircraft into the relative wind.

Swept back wings produce a greater positive contribution to stall directional stability than do straight wings.

Reference figure 4.11. The wing sweep angle, Λ is defined as the angle between a perpendicular to the fuselage reference line and the quarter cord line of the wing.

FIGURE 4.11



It can be seen that the component of free stream velocity normal to the wing is greater for swept back wings than for straight wings, and that is also greater on the upwind wing.

$$V_{N(\text{Upwind})} = V_T \cos(\Lambda - \beta) \qquad (4.17)$$

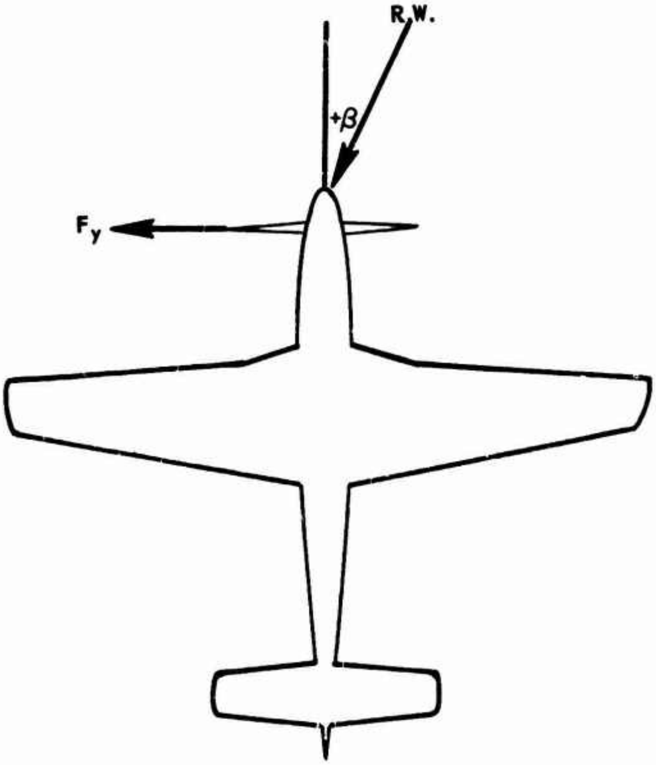
$$V_{N(\text{Downwind})} = V_T \cos(\Lambda + \beta) \qquad (4.18)$$

This difference in normal components creates a dissimilance of lift and therefore a disparity in induced drag on the two wings. Thus a stabilizing yawing moment is created. Similarly, forward swept wings would create an unstable contribution to static directional stability.

Miscellaneous Effects on C_{n_β}:

A propeller can have large effects on an aircraft's static directional stability. The propeller contribution to directional stability arises from the side force component at the propeller disc created as a result of yaw (see figure 4.12).

FIGURE 4.12



The propeller is destabilizing if a tractor and stabilizing if a pusher. Similarly, engine intakes have the same effects if they are located fore or aft of the aircraft cg.

Engine nacelles act like a small fuselage and can be stabilizing or destabilizing depending on whether their cp is located ahead or behind the cg.

Aircraft cg movement is restricted by longitudinal static stability considerations. However, within the relatively narrow limits established by longitudinal considerations, cg movements have no significant effects on static directional stability.

4.3 $C_{n_{\delta_r}}$ -RUDDER POWER

In most flight conditions, it is desired to maintain the sideslip angle equal to zero. If the aircraft has positive directional stability and is symmetrical, then it will tend to fly in this condition. However, yawing moments may act on the aircraft as a result of asymmetric thrust (one engine inoperative), slip stream rotation, or the unsymmetric flow field associated with turning flight. Under these conditions, sideslip angle can be kept to zero only by the application of a control moment. The control that provides this moment is the rudder.

Recall that,

$$C_{n_F} = a_F \, \alpha_F \, V_v \qquad (4.18)$$

$$\frac{\partial C_{n_F}}{\partial \delta_r} = \frac{\partial C_n}{\partial \delta_r} = a_F \, V_v \frac{\partial \alpha_F}{\partial \delta_r} \qquad (4.19)$$

Defining rudder effectiveness, τ, as:

$$\tau = \frac{\partial \alpha_F}{\partial \delta_r} \qquad (4.20)$$

$$\frac{\partial C_n}{\partial \delta_r} = C_{n_{\delta_r}} = a_F \, V_v \, \tau \qquad (4.21)$$

The derivative, $C_{n_{\delta_r}}$, is called "rudder power" and by definition, its algebraic sign is always positive. This is because a positive rudder deflection, $+\delta_r$ is defined as one that produces a positive moment about the cg, $+C_n$. The magnitude of the rudder power can be altered by varying the size of the vertical tail and its distance from the aircraft cg, or by using different airfoils for the tail and/or rudder, or by varying the size of the rudder.

•4.4 RUDDER FIXED STATIC DIRECTIONAL STABILITY

Having some knowledge of both C_{n_β} and $C_{n_{\delta_r}}$, it is now possible to work toward some relationship that can be used in flight to measure the static directional stability of the aircraft. In flight, the maneuver that will be used to determine the static directional stability of the aircraft is the "steady straight sideslip." In a steady straight sideslip, equation 4.5 reduces to,

$$C_{n_\beta}\beta + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r = 0 \qquad (4.22)$$

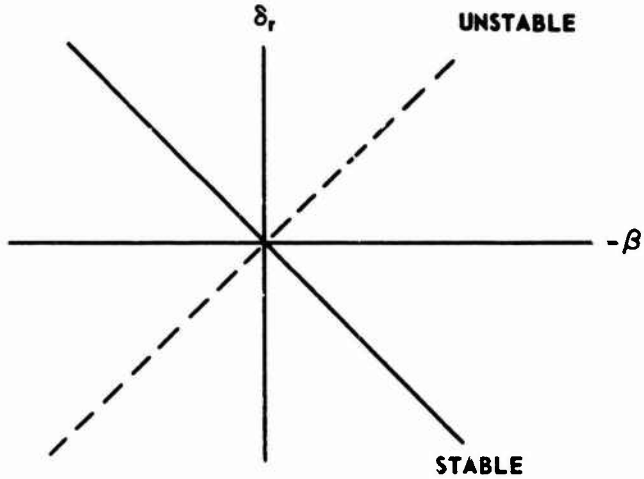
Thus,

$$\delta_r = -\frac{C_{n_\beta}}{C_{\delta_r}}\beta - \frac{C_{n_{\delta_a}}}{C_{n_{\delta_r}}}\delta_a \qquad (4.23)$$

$$\frac{\partial\delta_r}{\partial\beta} = -\frac{C_{n_\beta(\text{Fixed})}}{C_{n_{\delta_r}}} \qquad (4.24)$$

Again, the subscript "fixed" is added as a reminder that equation 4.24 is an expression for the static directional stability of an aircraft if the rudder is not free to float. Looking at equation 4.24, $C_{n_{\delta_r}}$ is a known quantity once an aircraft is built, therefore, $\partial\delta_r/\partial\beta$ can be taken as a direct indication of the rudder fixed static directional stability of an aircraft. The relationship, $\partial\delta_r/\partial\beta$, can easily be measured in flight. Since C_{n_β} has to be positive in order to have positive directional stability, and $C_{n_{\delta_r}}$ is positive by definition, $\partial\delta_r/\partial\beta$ must be negative to obtain positive directional stability.

FIGURE 4.13



RUDDER DEFLECTION vs SLIDESLIP

•4.5 RUDDER FREE STATIC DIRECTIONAL STABILITY

On aircraft with reversible control systems, the rudder is free to float in response to its hinge moments, and this floating can have large effects on the directional stability of the airplane. In fact, a plot of $\partial\delta_r/\partial\beta$ may be stable while an examination of the rudder free static directional stability reveals the aircraft to be unstable. Thus, if the rudder is free to float, there will be a change in the tail contribution to static directional stability. To analyze the nature of this change, recall that hinge moments are produced by the pressure distribution caused by angle of attack and control surface deflection. In the case of the rudder,

$$H_m = \cancel{H_{m_0}}^{\,0} + \frac{\partial H_m}{\partial\alpha_F}\alpha_F + \frac{\partial H_m}{\partial\delta_r}\delta_r \qquad (4.25)$$

In coefficient form

$$C_h = C_{h_{\alpha_F}}\cdot\alpha_F + C_{h_{\delta_r}}\cdot\delta_r \qquad (4.26)$$

It can be seen that when the vertical tail is placed at some angle of attack, α_F, the rudder will start to "float." However, as soon as it deflects, restoring moments are set up, and an equilibrium floating angle will be reached where the floating tendency is just balanced by the restoring tendency and $C_h = 0$. At this point,

$$C_{h_{\alpha_F}} \cdot \alpha_F = -C_{h_{\delta_r}} \cdot \delta_{r_{(Float)}} \qquad (4.27)$$

Thus,

$$\delta_{r_{(Float)}} = - \frac{C_{h_{\alpha_F}}}{C_{h_{\delta_r}}} \alpha_F \qquad (4.28)$$

With this background, it is now possible to develop a relationship that expresses the static directional stability of an aircraft with the rudder free to float.

Recall that,

$$C_{n_F} = V_v\, a_F\, \alpha_F \qquad (4.13)$$

$$\alpha_F = \left(\beta - \sigma + \frac{\partial \alpha_F}{\partial \delta_r}\, \delta_{r_{(Float)}}\right) \qquad (4.29)$$

Therefore,

$$C_{n_F} = V_v\, a_F \left(\beta - \sigma + \frac{\partial \alpha_F}{\partial \delta_r} \delta_{r_{(Float)}}\right) \qquad (4.30)$$

$$C_{n_{\beta_{(Free)}}} = \frac{\partial C_{n_F}}{\partial \beta} = V_v\, a_F \left(1 - \frac{\partial \sigma}{\partial \beta} + \tau \frac{\partial \delta_{r_{(Float)}}}{\partial \beta}\right) \qquad (4.31)$$

$$C_{n_{\beta_{(Free)}}} = V_v\, a_F \left(1 - \frac{\partial \sigma}{\partial \beta}\right) \cdot \left(1 + \tau \frac{\partial \delta_{r_{(Float)}}}{\partial \beta} \cdot \frac{1}{1 - \frac{\partial \sigma}{\partial \beta}}\right) \qquad (4.32)$$

From equation 4.14,

$$\frac{\partial \alpha_F}{\partial \beta} = 1 - \frac{\partial \sigma}{\partial \beta} \qquad (4.33)$$

$$C_{n_{\beta_{(Free)}}} = V_v\, a_F \left(1 - \frac{\partial \sigma}{\partial \beta}\right) \left(1 + \tau \frac{\partial \delta_{r_{(Float)}}}{\partial \beta} \cdot \frac{1}{\frac{\partial \alpha_F}{\partial \beta}}\right) \qquad (4.34)$$

$$C_{n_{\beta_{(Free)}}} = V_v\, a_F \left(1 - \frac{\partial \sigma}{\partial \beta}\right) \cdot \left(1 + \tau \frac{\partial \delta_{r_{(Float)}}}{\partial \alpha_F}\right) \qquad (4.35)$$

Recall that,

$$\delta_{r_{(Float)}} = - \frac{C_{h_{\alpha_F}}}{C_{h_{\delta_r}}} \alpha_F \qquad (4.28)$$

Therefore,

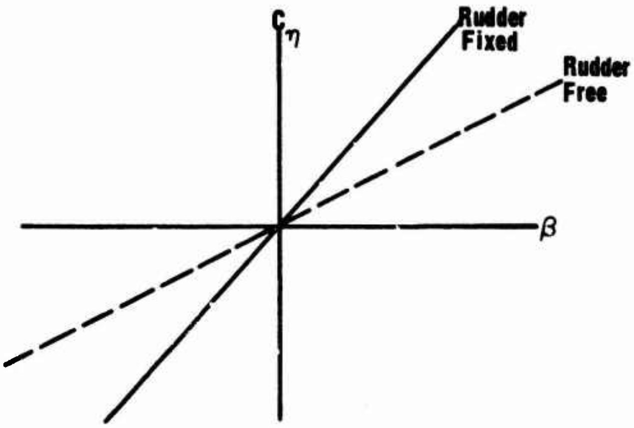
$$\frac{\partial \delta_{r_{(Float)}}}{\partial \alpha_F} = - \frac{C_{h_{\alpha_F}}}{C_{h_{\delta_r}}} \qquad (4.36)$$

Thus,

$$C_{n_{\beta_{(Free)}}} = V_v\, a_F \left(1 - \frac{\partial \sigma}{\partial \beta}\right) \cdot \left(1 - \tau \frac{C_{h_{\alpha_F}}}{C_{h_{\delta_r}}}\right) \qquad (4.37)$$

It can be seen that this expression differs from equation 4.16, the expression for rudder fixed static directional stability by the term $(1 - \tau\, C_{h_{\alpha_F}}/C_{h_{\delta_r}})$. Since this term will always result in a quantity less than one, it can be stated that the effect of rudder float is to reduce the slope of the static directional stability curve.

FIGURE 4.14

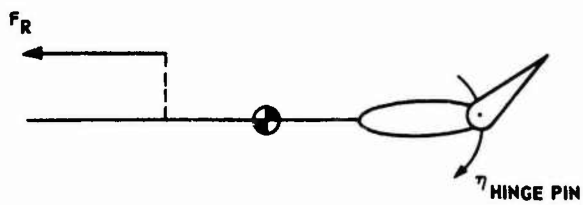


THE EFFECT OF FREEING THE RUDDER ON STATIC DIRECTIONAL STABILITY

Equation 4.37 does not contain parameters that are easily measured in flight, therefore it is necessary to develop an expression that will be useful in flight test work.

Assuming a steady straight sideslip, figure 4.15 schematically represents the forces and moments at work.

FIGURE 4.15



In a steady straight sideslip, $\Sigma\mathcal{N} = 0$. Therefore, it follows that $\Sigma\mathcal{N}_{\text{Hinge Pin}} = 0$. Now if moments are summed about the rudder hinge pin, the rudder force exerted by the pilot, F_R, acts through a moment arm and gearing mechanism, both accounted for by some constant, K, and must balance the other aerodynamic yawing moments so that $\Sigma\mathcal{N}_{\text{Hinge Pin}} = 0$. The pilot is hindered in his task by the fact that the rudder floats. Thus, in steady straight flight,

$$\sum \eta_{\text{Hinge Pin}} = 0 = F_r \cdot K + H_{\mathcal{M}} \qquad (4.38)$$

$$F_r = -G.H_M \qquad (4.39)$$

Where G is merely 1/K.
Knowing,

$$H_{\mathcal{M}} = C_h\, q_r\, S_r\, c_r \qquad (4.40)$$

From equation 4.26,

$$H_{\mathcal{M}} = q_r\, S_r\, c_r\, (C_{h_{\alpha_F}} \cdot \alpha_F + C_{h_{\delta_r}} \cdot \delta_r) \qquad (4.41)$$

Thus, equation 4.39 becomes,

$$F_r = -Gq_r\, S_r\, c_r\, (C_{h_{\alpha_F}} \cdot \alpha_F + C_{h_{\delta_r}} \cdot \delta_r) \qquad (4.42)$$

Applying equation 4.27,

$$F_r = -Gq_r\, S_r\, c_r\, (-C_{h_{\delta_r}} \cdot \delta_{r_{(Float)}} + C_{h_{\delta_r}} \cdot \delta_r) \qquad (4.43)$$

$$F_r = -Gq_r\, S_r\, c_r\, C_{h_{\delta_r}}\, (\delta_r - \delta_{r_{(Float)}}) \qquad (4.44)$$

The difference between where the pilot pushes the rudder, δ_r, and the amount it floats, $\delta_{r(Float)}$, is the free position of the rudder, $\delta_{r(Free)}$.

Therefore,

$$F_r = -Gq_r\, S_r\, c_r\, C_{h_{\delta_r}}\, \delta_{r(Free)} \qquad (4.45)$$

$$\frac{\partial F_R}{\partial \beta} = -Gq_r\, S_r\, c_r\, C_{h_{\delta_r}} \cdot \frac{\partial \delta_{r(Free)}}{\partial \beta} \qquad (4.46)$$

From equation 4.24, it can be shown that,

$$\frac{\partial \delta_{r(\text{Free})}}{\partial \beta} = -\frac{C_{n\beta(\text{Free})}}{C_{n\delta_r}} \qquad (4.47)$$

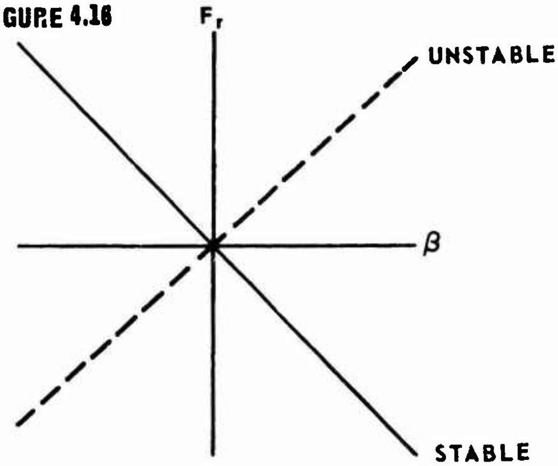
Thus,

$$\frac{\partial F_r}{\partial \beta} = G q_r S_r c_r \frac{C_{h\delta_r}}{C_{n\delta_r}} C_{n\beta(\text{Free})} \qquad (4.48)$$

This equation shows that the parameter, $\partial F_r/\partial \beta$, can be taken as an indication of the rudder free static directional stability of an aircraft. This parameter can be readily measured in flight.

An analysis of the components of equation 4.48 reveals that for static directional stability, the sign of $\partial F_r/\partial \beta$ should be negative.

FIGURE 4.16



RUDDER FORCE VERSUS SIDESLIP ANGLE

4.6 $C_{n\delta a}$ YAWING MOMENT DUE TO LATERAL CONTROL DEFLECTION

The remaining derivatives in equation 4.5 that have not been studied thus far are called "cross derivatives." It is the existence of these cross derivatives that causes the rolling and yawing motions to be so closely coupled.

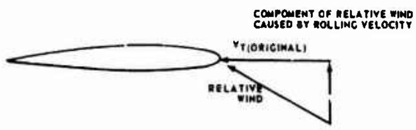
The first of these cross derivatives to be covered will be $C_{n\delta_a}$, and is the yawing moment due to lateral control deflection. In order for a lateral control to produce a rolling moment, it must create an unbalanced lift condition on the wings. The wing with the most lift will also produce the most induced drag according to the equation $C_{D_i} = C_L^2/\pi e\, AR$. Also, any change in the profile of the wing due to a lateral control deflection will cause a change in profile drag. Thus, any lateral control deflection will produce a change in both induced and profile drag. The predominate effect will be dependent on the particular aircraft configuration and the flight condition. If induced drag predominates, the aircraft will tend to yaw away from the direction of roll. This phenomenon is known as "adverse yaw." The sign of $C_{n\delta_a}$ for adverse yaw is negative. If profile drag predominates, the aircraft will tend to yaw into the direction of roll. This is known as "complimentary" of "proverse" yaw. The sign of $C_{n\delta_a}$ for complimentary yaw is positive. Both ailerons and spoilers are capable of producing either adverse or complimentary yaw. To determine which condition will prevail, the particular aircraft configuration and flight condition must be analyzed. If design permits, it is desirable to have $C_{n\delta_a} = 0$ or be slightly positive. A slight positive value will ease the pilot's turn coordination task.

4.7 C_{n_p} - YAWING MOMENT DUE TO ROLL RATE

The derivative C_{n_p} is called yawing moment due to roll rate. Both the wing and the tail contribute to C_{n_p}. The wing contribution arises from two sources. The first comes from the change in profile drag associated with the

change in wing angle of attack due to rolling. As an aircraft is rolled, the angle of attack on the downgoing wing is increased. Refer to figure 4.17. Conversely, the angle of attack on the upgoing wing is decreased.

FIGURE 4.17



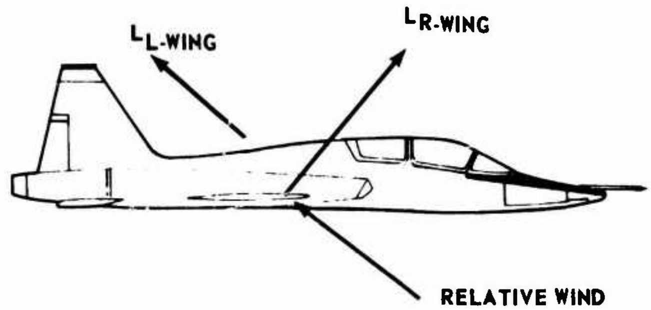
CHANGE OF RELATIVE WIND ON RIGHT WING DURING A RIGHT ROLL

This increase in angle of attack on the downgoing wing means that the relative wind "sees" more of the downgoing wing and that therefore the profile drag will be greater on this wing than on the upgoing wing. For the right roll depicted in figure 4.17, the increased profile drag would cause a yaw to the right. Thus, the sign of C_{n_p} due to this effect only is positive. However, the second wing effect is predominant and the foregoing effect exerts only a mitigating influence.

The local lift vector is always perpendicular to the local relative wind. As already discussed, the inclination of the relative wind is different on the wings during a roll. Thus, there will be a diference in the inclination of the two wing lift vectors. The lift vector on the downgoing wing will be tilted forward, and the lift vector on the upgoing wing will be tilted aft. Refer to figure 4.18.

Since each lift vector has a component in the X-direction, a yawing moment will result. In the case depicted, for a right roll the yaw will be to the left. Thus, the sign of C_{n_p} due to this effect will be negative. As previously mentioned, this is the predominant wing effect and thus, overall, the sign of the wing contribution to C_{n_p} is negative.

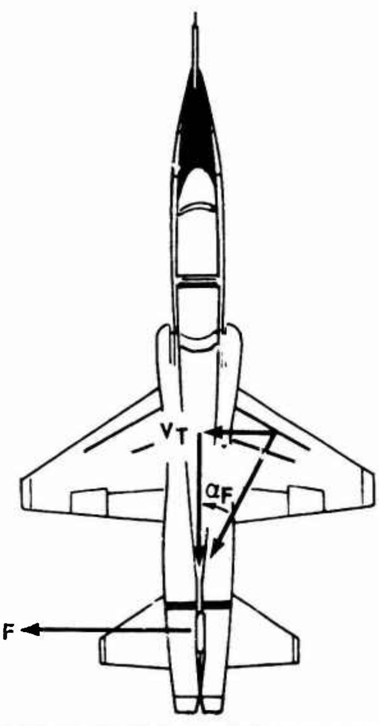
FIGURE 4.18



INCLINATION OF WING LIFT VECTORS DURING A RIGHT ROLL

The vertical tail makes a larger contribution to C_{n_p} than does either wing effect. Rolling changes the angle of attack on the vertical tail. Refer to figure 4.19.

FIGURE 4.19



CHANGE IN ANGLE OF ATTACK OF THE VERTICAL TAIL DUE TO A RIGHT ROLL RATE

This change in angle of attack on the vertical tail will generate a lift force. In the situation depicted in figure 4.19, the change in angle of attack will generate a lift force, L_F, to the left. This will create a positive yawing moment. Thus, C_{n_p} for the vertical tail is positive.

Considering both wing and tail, a slight positive value of C_{n_p} is desired to aid in Dutch roll damping.

4.8 C_{n_r} YAW DAMPING

The derivative C_{n_r}, is called yaw damping and, by definition, its sign is always negative. The aircraft fuselage adds a negligible amount to C_{n_r} except when it is very large. The important contributions are those of the wing and tail.

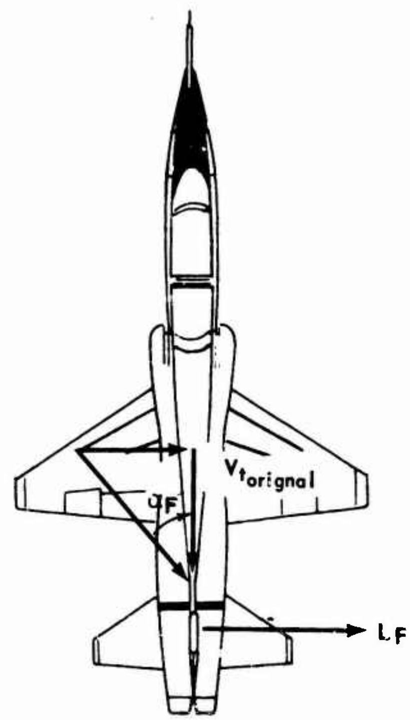
The tail contribution to C_{n_r} arises from the fact that there is a change in angle of attack on the vertical tail whenever the aircraft is yawed. This change in α_F produces a lift force, L_F, that in turn produces a yawing moment that opposes the original yawing moment. Refer to figure 4.20. The tail contribution to C_{n_r} accounts for 80-90% of the total "yaw damping" on most aircraft.

The wing contribution to C_{n_r} arises from the fact that in a yaw, the outside wing experiences an increase in both induced drag and profile drag due to the increased dynamic pressure on the wing. An increase in drag on the outside wing creates a yawing moment that opposes the original direction of yaw.

4.9 $C_{n_{\dot{\beta}}}$—YAW DAMPING DUE TO LAG EFFECTS IN SIDEWASH

The derivative $C_{n_{\dot{\beta}}}$ is yaw damping due to lag effects in sidewash, σ. Very little can be authoritatively stated about the magnitude of algebraic sign of $C_{n_{\dot{\beta}}}$ due to the wide variations of opinion in interpreting the experimental data concerning it.

FIGURE 4.20



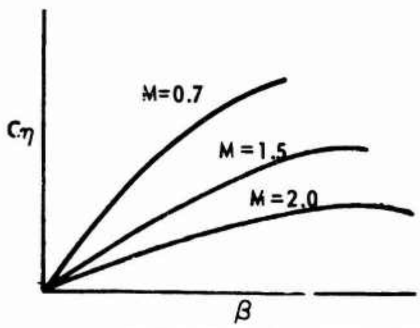
CHANGE IN ANGLE OF ATTACK OF VERTICAL FIN DUE TO YAWING RATE

During any change in β, the angle of attack of the vertical fin will always be less than it will be at steady state. This is due to lag effects is sidewash. Since this phenomenon reduces the angle of attack of the vertical tail, it also reduces the yawing moment created by the vertical tail. This reduction in yawing moment is, effectively, a contribution to yaw damping. Thus the description, "yaw damping due to lag effects in sidewash."

●4.10 HIGH SPEED ASPECTS OF STATIC DIRECTIONAL STABILITY

C_{n_β} - The effectiveness of an airfoil decreases as the velocity increases supersonically. Thus, for a given β, as Mach increases, the restoring moment generated by the tail diminishes. The wing-fuselage combination continues to be destabilizing throughout the flight envelope. Thus, the overall C_{n_β} of the aircraft will decrease with increasing Mach.

FIGURE 4.21



CHANGE IN C_{η_β} WITH MACH NUMBER

The requirement for large values of C_{n_β} is compounded by the tendency of high speed aerodynamic designs toward divergencies in yaw due to roll coupling effects. This problem can be combated by designing an extremely large tail (F-104, F-111, T-38), by endplating the tail (F-104, T-38), by using ventral fins (F-104), or by using fore body strakes.

The F-104 employs a ventral fin in addition to a sizeable vertical stabilizer to increase supersonic directional stability. The efficiency of underbody surfaces is not affected by wing wake at high angles of attack, and supersonically, they are located in a high energy compression pattern.

Fore body strakes located radially along the horizontal center line in the x-y plane of the aircraft have also been employed effectively to increase directional stability at supersonic speeds. This increase in C_{n_β} by the employment of strakes is a result of a more favorable pressure distribution over the fore body surface, and in addition, the creation of improved flow effects at the vertical tail location by virtue of diminished flow circulation. In addition, even small sideslip angles will produce fuselage blanking of the downwind strake and create a dissimilance of induced drag, and thus a stable contribution to C_{n_β}.

$C_{n_{\delta_r}}$ - In the transonic region, flow separation will decrease the effectiveness of any trailing edge control surface. On most aircraft however, this is offset by an increase in the C_{L_α} curve in the transonic region. As a result, flight controls are usually the most effective in this region. However, as Mach number continues to increase, the C_{L_α} curve will decrease, and thus, control surface effectiveness will continue to decrease. In addition, once the flow over the surface is supersonic, a trailing edge control cannot influence the pressure distribution on the surface itself, due to the fact that pressure disturbances cannot be transmitted forward in a supersonic environment. Thus, the rudder power will decrease as Mach increases above the transonic region.

$C_{n_{\delta_a}}$ - For the same reasons discussed under rudder power, a given aileron deflection will not produce as much lift at high Mach number as it did transonically. Therefore, induced drag will be less. In addition, the profile drag, for a given aileron deflection, increases with Mach number. Thus, the tendency toward complimentary yaw increases with Mach.

C_{n_r} - The development of yaw damping depends on the ability of the wing and tail to develop lift. Thus, as Mach number increases and the ability of all surfaces to develop lift decreases, yaw damping will also decrease.

C_{n_p} - The slope of a curve of C_{n_p} normally doesn't change with Mach number. However, the magnitude of attainable roll rate will decrease with decreasing aileron effectiveness. Therefore, the magnitude of C_{n_p} encountered at higher Mach numbers will normally be less.

$C_{n_{\dot\beta}}$ - This derivative normally will not change with Mach number.

4.11 STATIC LATERAL STABILITY

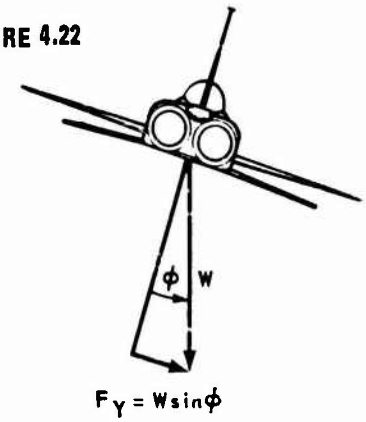
The analysis of aircraft lateral static stability is based on equation 4.6, which is repeated here for reference.

$$C_\ell = C_{\ell_\beta}\beta + C_{\ell_{\dot\beta}}\dot{\hat\beta} + C_{\ell_p}\hat p + C_{\ell_r}\hat r + C_{\ell_{\delta_a}}\delta_a + C_{\ell_{\delta_r}}\delta_r \qquad (4.6)$$

It can be seen that the rolling moment, C_ℓ, is not a function of bank angle, ϕ. In other words, a change in bank angle will produce no change in rolling moment. In fact, ϕ produces no moment at all. Thus, $C_{\ell_\phi} = 0$, and although it is analogous to C_{m_α} and C_{n_β}, it contributes nothing to the lateral static stability analysis.

Bank angle, ϕ, does have an indirect effect on rolling moment. As the aircraft is rolled into a bank angle, a component of aircraft weight will act along the Y-axis, and will thus produce an unbalanced force. Refer to figure 4.22. This unbalanced force in the Y direction, F_Y, will produce a sideslip, β, and as seen from equation 4.6, this will influence the rolling moment produced.

FIGURE 4.22



SIDE FORCE PRODUCED BY BANK ANGLE

Each stability derivative in equation 4.6 will be discussed and its contribution to aircraft stability will be analyzed. A summary of these stability derivatives follows:

FIGURE 4.23

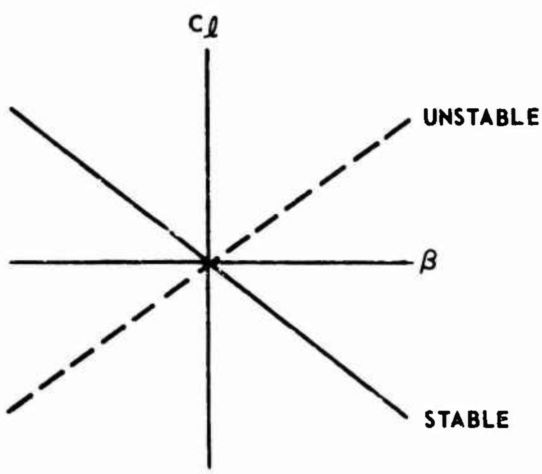
DERIVATIVE	NAME	SIGN FOR A STABLE AIRCRAFT	CONTRIBUTING PARTS OF AIRCRAFT
C_{ℓ_β}	Dihedral Effect	(-)	Wing, Tail
$C_{\ell_{\dot\beta}}$	C_ℓ due to $\dot\beta$	(+)	Wing, Tail
C_{ℓ_p}	Roll Damping	(-)	Wing, Tail
C_{ℓ_r}	C_ℓ due to Yaw Rate	(+)	Wing, Tail
$C_{\ell_{\delta_a}}$	Lateral Control Power	(+)	Lateral Control
$C_{\ell_{\delta_r}}$	C_ℓ due to Rudder Deflection	(-)	Rudder

4.12 C_{ℓ_β} - DIHEDRAL EFFECT

The tendency of an aircraft to fly wings level is related to the derivative C_{ℓ_β}, which is known as "Dihedral Effect." Although the static lateral stability of an aircraft is a function of all the derivatives in equation 4.6, C_{ℓ_β} is the predominant term. Therefore, static lateral stability is often referred to as "Stable Dihedral Effect."

An aircraft has stable dihedral effect if a positive sideslip produces a negative rolling moment. Thus, the algebraic sign of C_{ℓ_β} must be negative for stable dihedral effect.

FIGURE 4.24



WIND TUNNEL RESULTS OF ROLLING MOMENT COEFFICIENT vs SIDESLIP

It is possible to have too much or too little dihedral effect. High values of dihedral effect give good spiral stability. If an aircraft has a large amount of positive dihedral effect, the pilot is able to pick up a wing with top rudder. This also means that in level flight a small amount of sideslip will cause the aircraft to roll and this can be annoying to the pilot. This is known as a high ϕ/β ratio. In multi-engine aircraft, an engine failure will normally produce a large sideslip angle. If the aircraft has a great deal of dihedral effect, the pilot must supply an excessive amount of aileron force and deflection to overcome the rolling moment due to sideslip. Still another detrimental effect of too much dihedral effect may be encountered when the pilot rolls an aircraft. If an aircraft in rolling to the right tends to yaw to the left, the resulting right sideslip, together with stable dihedral effect, creates a rolling moment to the left. This effect could materially reduce the maximum roll rate available. The pilot, then wants a certain amount of dihedral effect, but not too much. The end result is usually a design compromise.

Both the wing and the tail exert an influence on C_{ℓ_β}. The various effects on C_{ℓ_β} can be classified as "direct" or "indirect." A direct effect actually produces some increment of C_{ℓ_β} while an indirect effect merely alters the value of the existing C_{ℓ_β}.

The discrete wing and tail effects that will be considered are classified as follows:

FIGURE 4.25

Effects on C_{ℓ_β}

DIRECT	INDIRECT
Geometric Dihedral	Aspect Ratio
Wing Sweep	Taper Ratio
Wing-Fuselage Interference	Tip Tanks
Vertical Tail	Wing Flaps

Geometric dihedral, γ, is defined as positive when the cord lines of the wing tip are above those at the wing root. To understand the effect of geometric dihedral on static lateral stability, consider figure 4.26.

FIGURE 4.26a TOP VIEW

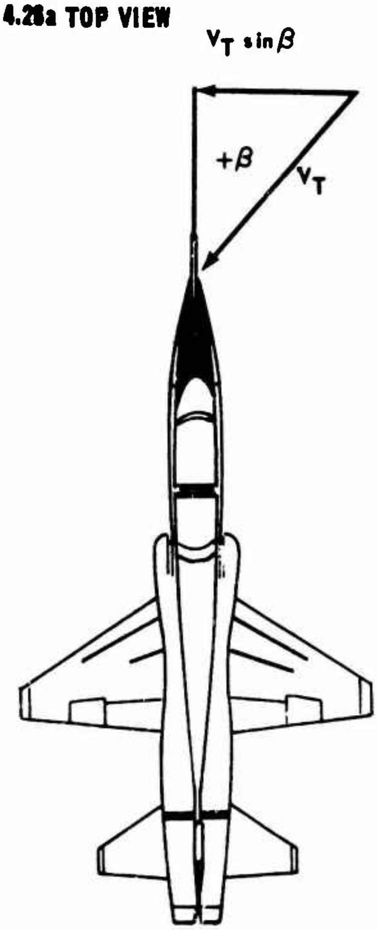


FIGURE 4.26b REAR VIEW

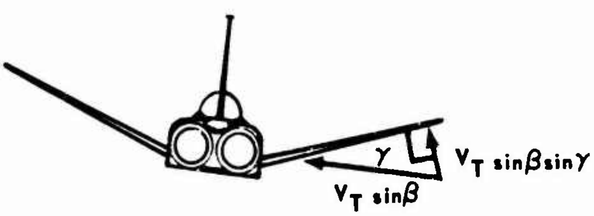
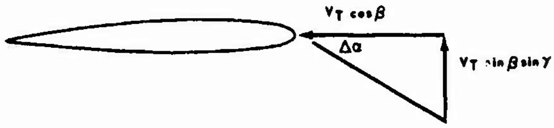


FIGURE 4.26c END VIEW OF UPWIND WING



It can be seen that when an aircraft is placed in a sideslip, positive geometric dihedral causes the component, $V_T \sin\beta \sin\gamma$ to be added to the lift producing component of the relative wind, $V_T \cos\beta$. Thus, geometric dihedral causes the angle of attack on the upwind wing to be increased by $\Delta\alpha$.

$$\tan\Delta\alpha = \frac{V_T \sin\beta\sin\gamma}{V_T \cos\beta} \tag{4.49}$$
$$= \tan\beta\sin\gamma$$

Making the small angle assumption,

$$\Delta\alpha = \tan\beta\sin\gamma \tag{4.50}$$

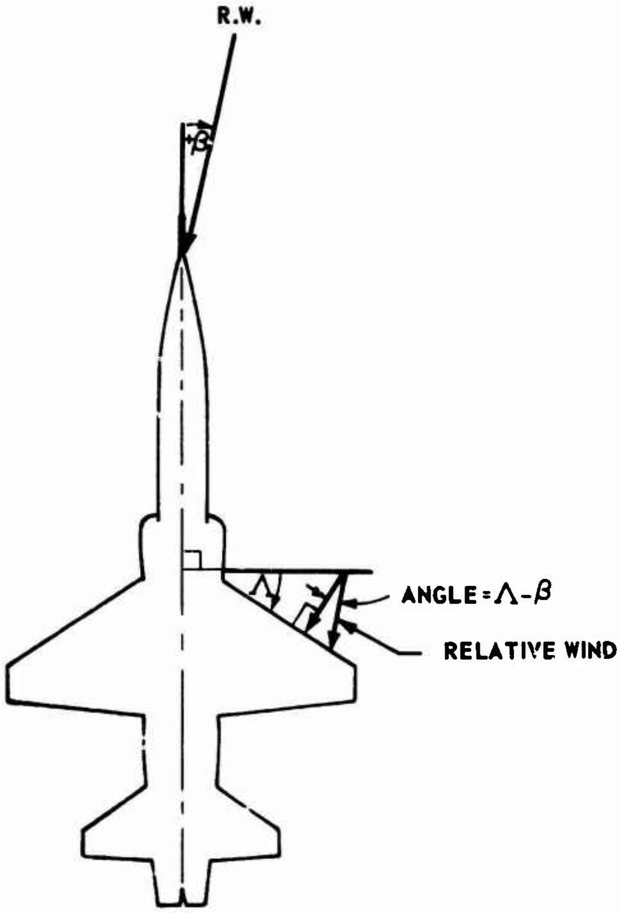
Conversely, the angle of attack on the downwind wing will be reduced. These changes in angle of attack tend to increase the lift on the upwind wing and decrease the lift on the downwind wing, thus producing a roll away from the sideslip. In figure 4.26, for example, a positive sideslip, $+\beta$, will increase the angle of attack on the upwind, or right, wing, thus producing a roll to the left. Therefore, it can be seen that this effect produces a stable, or negative, contribution to C_{ℓ_β}.

Wing Sweep:

The wing sweep angle, Λ, is measured from a perpendicular to the aircraft x-axis at the forward wing root, to a line connecting the quarter cord points of the wing. Wing sweep back is defined as positive.

Aerodynamic theory shows that the lift of a yawed wing is determined by the component of the free stream velocity normal to wing. That is, $L = C_L \, 1/2 \, \rho V_N^2 S$ where, V_N is the normal velocity.

FIGURE 4.27 EFFECT OF WING SWEEP ON C_{ℓ_β}



It can be seen from figure 4.27 that on a swept wing aircraft, the normal component of free stream velocity on the upwind wing is,

$$V_N = V_T \cos(\Lambda - \beta) \tag{4.51}$$

Conversely, on the downwind wing,

$$V_N = V_T \cos(\Lambda + \beta) \tag{4.52}$$

Therefore, V_N will be greater on the upwind wing. This will cause the upwind wing to produce more lift and will thus create a roll away from the direction of the sideslip. In other words, a right sideslip will produce a roll to the left. Thus, wing sweep makes a stable contribution to C_{ℓ_β} and produces the same effect as geometric dihedral.

To fully appreciate the effect of wing sweep on static lateral stability, it will be necessary to develop an equation relating the two.

$$L_{(\text{Upwind Wing})} = C_L \frac{S}{2} 1/2 \rho V_N^2 \tag{4.53}$$

$$L_{(\text{Upwind Wing})} = C_L \frac{S}{2} 1/2 \rho \left[V_T \cos(\Lambda - \beta)\right]^2 \tag{4.54}$$

$$\Delta L = C_L \frac{S}{2} 1/2 \rho \left[V_T \cos(\Lambda - \beta)\right]^2 - C_L \frac{S}{2} 1/2 \rho \left[V_T \cos(\Lambda + \beta)\right]^2 \tag{4.55}$$

$$\Delta L = C_L \frac{S}{2} 1/2 \rho V_T^2 \left[\cos^2(\Lambda - \beta) - \cos^2(\Lambda + \beta)\right] \tag{4.56}$$

Applying a trigonometric identity,

$$\left[\cos^2(\Lambda - \beta) - \cos^2(\Lambda + \beta)\right] = \sin 2\Lambda \sin 2\beta \tag{4.57}$$

Making the assumption of a small sideslip angle,

$$\left[\cos^2(\Lambda - \beta) - \cos^2(\Lambda + \beta)\right] = 2\beta \sin 2\Lambda \tag{4.58}$$

Therefore, equation 4.56 becomes,

$$\Delta L = C_L \frac{S}{2} 1/2 \rho V_T^2 2\beta \sin 2\Lambda \qquad (4.59)$$

$$= C_L S\, 1/2 \rho V_T^2 \beta \sin 2\Lambda$$

The rolling moment produced by this change in lift is,

$$\mathcal{L} = - \Delta L \cdot \bar{Y} \qquad (4.60)$$

Where $\bar{Y}$ is the distance from the wing cp to the aircraft cg. The minus sign arises from the fact that equation 4.59 assumes a positive sideslip, $+\beta$, and for an aircraft with stable dihedral effect, this will produce a negative rolling moment.

$$C_{\ell} = \frac{\mathcal{L}}{q_w S_w b_w} \qquad (4.61)$$

$$C_{\ell} = - \frac{\bar{Y} C_L S \rho V_T^2 \beta \sin \Lambda}{\rho V_T^2 S b}$$

$$= - \frac{C_L \bar{Y} \beta}{b} \sin \Lambda \qquad (4.62)$$

$$\frac{\partial C_{\ell}}{\partial \beta} = C_{\ell_\beta} = - \frac{\bar{Y}}{b} C_L \sin 2\Lambda \qquad (4.63)$$

$$= - \text{CONST} \,(C_L \sin \Lambda)$$

Where the constant will be on the order of 0.2. Equation 4.63 should not be used above $\Lambda = 45°$ because highly swept wings are subject to leading edge separation at high angles of attack, and this can result in reversal of the dihedral effect. Therefore, it's best to use empirical results above $\Lambda = 45°$.

From equation 4.63, it can be seen that at low speeds, high C_L, sweepback makes a large contribution to stable dihedral effect. However, at high speeds, low C_L, sweepback makes a relatively small contribution to stable dihedral effect.

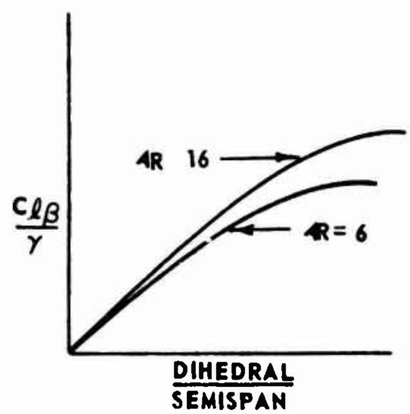
For angles of sweep on the order of 45°, the wing sweep contribution to C_{ℓ_β} may be on the order of $- 1/5\ C_L$. For large values of C_L, this is a very large contribution, equivalent to nearly ten degrees of geometric dihedral. At very high angles of attack, such as during landing and takeoff, this effect can be very helpful to a swept wing fighter encountering downwash.

Since the effect of sweepback varies with C_L, becoming extremely small at high speeds, it can help keep the proper ratio of C_{ℓ_β} to C_{n_β} at high speeds and reduce poor Dutch roll characteristics at these speeds.

Wing Aspect Ratio:

The wing aspect ratio exerts an indirect effect on dihedral effect. On a high aspect ratio wing, the center of pressure is further from the cg than on a low aspect ratio wing. This results in high aspect ratio planforms having a longer moment arm and thus, greater rolling moments for a given asymmetric lift distribution. Refer to figure 4.28. It should be noted that aspect ratio, in itself, does not create dihedral effect, but that it merely alters the magnitude of the existing dihedral effect.

FIGURE 4.28

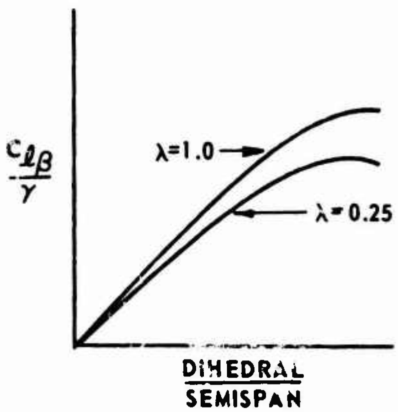


CONTRIBUTION OF ASPECT RATO TO DIHEDRAL EFFECT

Wing Taper Ratio:

Taper ratio, λ, is a measure of how fast the wing cord shortens. Taper ratio is the ratio of the tip cord to the root cord. Therefore, the lower the taper ratio, the faster the cord shortens. On highly tapered wings, the center of pressure is closer to the aircraft cg than on untapered wings. This results in a shorter moment arm and thus, less rolling moment for a given asymmetric lift distribution. Refer to figure 4.29. Taper ratio does not create dihedral effect, but merely alters the magnitude of the existing dihedral effect. Thus it has an "indirect" effect on dihedral effect.

FIGURE 4.29

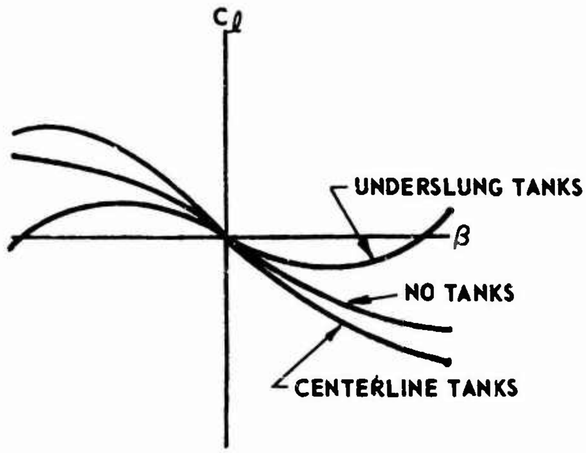


CONTRIBUTION OF TAPER RATIO TO DIHEDRAL EFFECT

Tip Tanks:

Tip tanks, pylon tanks and other external stores will generally exert an indirect influence on $C_{\ell\beta}$. Adding external stores creates an end-plating effect on the wing, and this, in turn, alters the effective aspect ratio of the wing. The effect of a given external store configuration is hard to predict analytically, and it is usually necessary to rely on empirical results. To illustrate the effect of various external store configurations, data for the F-80 is presented in figure 4.30. The data is for a clean F-80 230 gallon centerline tip tanks, and 165 gallon underslung tanks. This data shows that the centerline tanks increase dihedral effect while the underslung tanks reduce stable dihedral effect considerably.

FIGURE 4.30

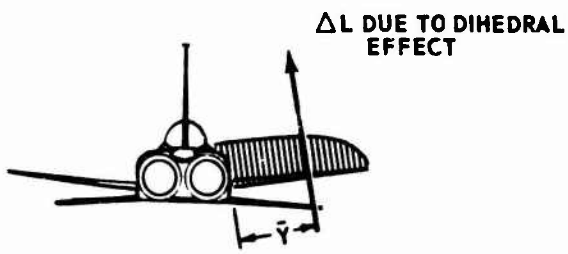


EFFECT OF TIP TANKS ON $C_{\ell\beta}$ OF F-80

Partial Span Flaps:

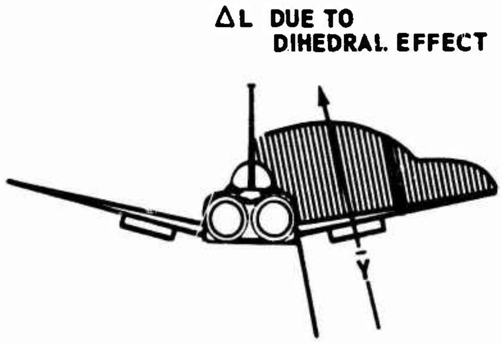
Partial-span flaps indirectly exert a detrimental effect on static lateral stability. Refer to figure 4.31.

FIGURE 4.31a



WING LIFT DISTRIBUTION, NO FLAPS

FIGURE 4.31b



WING LIFT DISTRIBUTION, FLAPS EXTENDED

Partial-span flaps shift the center of lift of the wing inboard, reducing the effective moment arm $\bar{Y}$. Therefore, although the values of ΔL remain the same, the rolling moment will decrease. The higher the effectiveness of the flaps in increasing the lift coefficient, the greater will be the change in span lift distribution and the more detrimental will be the effect of the flaps. Therefore, the decrease in lateral stability due to flap deflection may be large.

Deflected flaps cause a secondary variation in the effective dihedral that depends on the planform of the flaps themselves. If the shape of the wing gives a sweepback to the leading edge of the flaps, a slight positive dihedral effect results when the flaps are deflected. If the leading edge of the flaps are swept forward, flap deflection causes a slight negative dihedral effect. These effects are produced by the same phenomenon that produced a change in $C\ell_\beta$ with wing sweep. The effect of flap platform on $C\ell_\beta$ is generally small.

Wing - Fuselage Interference:

Of the various interference effects between parts of the aircraft, probably the most important is the change in angle of attack of the wing near the root due to the flow pattern about the fuselage in a sideslip. To visualize the change in angle of attack, refer to figure 4.32.

FIGURE 4.32

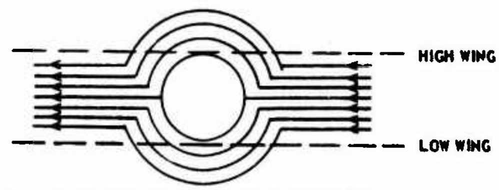


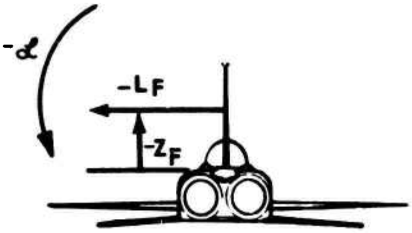
Figure 4.32. INFLUENCE OF WING-FUSELAGE INTERFERENCE ON $C_{\ell\beta}$

The fuselage induces vertical velocities in a sideslip which, when combined with the mainstream velocity, alter the local angle of attack of the wing. When the wing is located at the top of the fuselage (high-wing), then the angle of attack will be increased at the wing root, and a positive sideslip will produce a negative rolling moment: i.e., the dihedral effect will be enhanced. Conversely, when the aircraft has a low wing, the dihedral effect will be diminished by the fuselage interference. Generally, this explains why high-wing airplanes often have little or no geometric dihedral, whereas low-wing aircraft may have a great deal of geometric dihedral.

Vertical Tail:

When an aircraft sideslips, the angle of attack of the vertical tail is changed. This change in angle of attack produces a lift force on the vertical tail. If the center of pressure of the vertical tail is above the aircraft cg, this lift force will produce a rolling moment. Refer to figure 4.33.

FIGURE 4.33



ROLLING MOMENT CREATED BY VERTICAL TAIL AT A POSITIVE ANGLE OF SIDESLIPE

In the situation depicted in figure 4.33, the negative rolling moment was created by a positive sideslip angle, thus, the vertical tail contributes a stable increment to dihedral effect. This contribution can be quite large. In fact, it can be the major contribution to C_{ℓ_β} on aircraft with large vertical tails such as the F-104 and the T-38. This effect can be calculated in the same manner yawing moments were calculated in the directional case.

Assuming a positive sideslip angle,

$$-\mathcal{L}_F = (-L_F)(-Z_F) \qquad (4.64)$$

$$C_{\ell_F} = \frac{-Z_F L_F}{q_w S_w b_w} \qquad (4.65)$$

$$C_{\ell_F} = \frac{-Z_F C_{L_F} q_F S_F}{q_w S_w b_w} \qquad (4.66)$$

Define V_F as,

$$V_F = \frac{S_F Z_F}{S_w b_w} \qquad (4.67)$$

Assume that for a jet aircraft,

$$q_F = q_w \qquad (4.68)$$

And equation 4.66 becomes,

$$C_{\ell_F} = -C_{L_F} V_F = -a_F \alpha_F V_F \qquad (4.69)$$

Knowing

$$\alpha_F = (\beta - \sigma) \qquad (4.14)$$

$$C_{\ell_F} = -a_F V_F (\beta - \sigma) \qquad (4.70)$$

$$C_{\ell_{\beta_{\text{Vertical Tail}}}} = \frac{\partial C_{\ell_F}}{\partial \beta} \qquad (4.71)$$

$$= -a_F V_F \left(1 - \frac{\partial \sigma}{\partial \beta}\right)$$

Equation 4.71 reveals that a vertical tail contributes a stable increment to C_{ℓ_β}, whereas a ventral fin $[V_F = (+)]$ would contribute an unstable increment to C_{ℓ_β}. Also, if the lift curve slope of the vertical tail is increased, by end plating for example, the stable dihedral effect would be greatly increased. For example, the F-104 has a high horizontal stabilizer that acts as an end plate on the vertical tail and this increases the stable dihedral effect. In fact, the increase is so large that it is necessary to add negative geometric dihedral to the wings and a ventral fin to maintain a reasonable value of stable dihedral effect.

4.13 $C_{\ell_{\delta_a}}$ - LATERAL CONTROL POWER

Lateral control is achieved by altering the lift distribution so that the total lift on the two wings differ, thereby creating a rolling moment. This may be done simply by destroying a certain amount of lift on one wing by means of a spoiler, or by altering the

lift on both wings by means of ailerons. This discussion will be limited to the use of ailerons as the means of lateral control.

Since the purpose of the ailerons is to create a rolling moment, a logical measure of aileron power would be the rolling moment created by a given aileron deflection. Before progressing to the actual development of this relationship, it is necessary to make several definitions. A positive deflection of either aileron, $+\delta_a$, is defined as one which produces a positive rolling moment, (right wing down). Thus, by definition, $C_{\ell_{\delta_a}}$ is positive. Also, in this discussion, total aileron deflection is defined as the sum of the two individual aileron deflections. Thus,

$$\delta_{a_{Total}} = \delta_{a_{Left}} + \delta_{a_{Right}} \qquad (4.72)$$

The assumption will be made that the wing cp shift due to aileron deflection will not alter the value of C_{ℓ_β}. The distance from the x-axis to the cp of the wing will be labeled $\bar{Y}$. When the ailerons are deflected, they produce a change in lift on both wings. This total change in lift, ΔL, produces a rolling moment, $\mathcal{L}$.

$$\mathcal{L} = \Delta L \cdot \bar{Y} \qquad (4.73)$$

$$\mathcal{L} = \frac{\partial C_{L_a}}{\partial \alpha_a} \cdot \Delta \alpha_a \cdot q_a \cdot S_a \cdot \bar{Y} \qquad (4.74)$$

Where the "a" subscripts refer to "aileron" values.

$$\mathcal{L} = a_a \Delta \alpha_a q_a S_a \bar{Y} \qquad (4.75)$$

$$C_\ell = \frac{a_a \Delta \alpha_a S_a \bar{Y}}{S_w b_w} \qquad (4.76)$$

Where $\Delta\alpha_a = \delta_{a_{Total}}$

$$C_\ell = \frac{a_a \delta_{a_{Total}} S_a \bar{Y}}{S_w b_w} \qquad (4.77)$$

$$\frac{\partial C_\ell}{\partial \delta_a} = C_{\ell_{\delta_a}} = \frac{a_a S_a \bar{Y}}{b_w S_w} \qquad (4.78)$$

Thus, from equation 4.78, it can be seen that lateral control power is a function of the aileron airfoil section, the area of the aileron in relation to the area of the wing, and the location of the wing cp.

4.14 IRREVERSIBLE CONTROL SYSTEMS

Now that both C_{ℓ_β} and $C_{\ell_{\delta_a}}$ have been discussed, it is possible to develop a parameter which can be measured in flight to determine the static lateral stability of an aircraft. As in the directional case, the maneuver that will be flown will be a steady straight sideslip. Considering this maneuver, equation 4.6 reduces to,

$$C_\ell = C_{\ell_\beta}\beta + C_{\ell_{\delta_a}}\delta_a + C_{\ell_{\delta_r}}\delta_r = 0 \qquad (4.79)$$

$$\delta_a = -\frac{C_{\ell_\beta}\beta}{C_{\ell_{\delta_a}}} - \frac{C_{\ell_{\delta_r}}}{C_{\ell_{\delta_a}}}\delta_r \qquad (4.80)$$

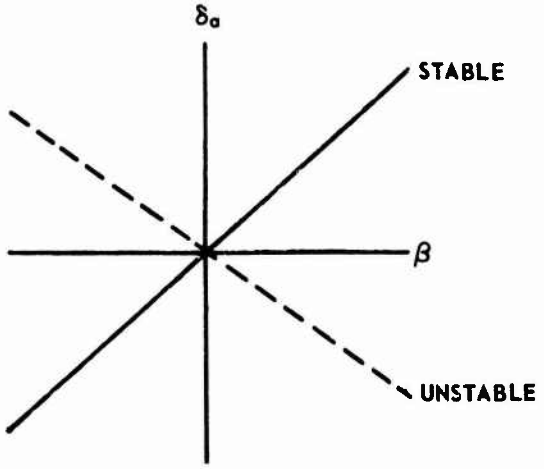
$$\frac{\partial \delta_{a(Fixed)}}{\partial \beta} = -\frac{C_{\ell_{\beta(Fixed)}}}{C_{\ell_{\delta_a}}} \qquad (4.81)$$

Thus, since $C_{\ell_{\delta_a}}$ is known once the aircraft is built, $\partial\delta_a/\partial\beta$, can be taken as a direct measure of the static lateral stability of an aircraft. Again, the subscript "Fixed" has been added as a reminder that in this development the aileron has not been free to "float."

Equation 4.81 reveals that for static lateral stability, a

plot of $\partial\delta_a/\partial\beta$ should have a positive slope. Refer to figure 4.34.

FIGURE 4.34



AILERON DEFLECTION VERSUS SIDESLIP ANGLE

4.15 REVERSIBLE CONTROL SYSTEMS

It is now necessary to consider an aircraft with a reversible control system. On this type aircraft, the ailerons are free to float in response to their hinge moments. Using the same approach as in the directional case, it is possible to derive an expression that will relate the "Aileron Free" static lateral stability to parameters that can be easily measured in flight.

In a steady straight sideslip, $\Sigma\mathcal{L} = 0$. Therefore, it follows that $\Sigma\mathcal{L}_{\substack{\text{Aileron}\\ \text{Hinge Pin}}} = 0$. Now if moments are summed about the aileron hinge pin, the aileron force exerted by the pilot, F_a, acts through a moment arm and gearing mechanism, both accounted for by some constant, K, and must balance the other aerodynamic rolling moments so that $\Sigma\mathcal{L}_{\substack{\text{Aileron}\\ \text{Hinge Pin}}} = 0$. Thus, in steady straight flight,

$$\Sigma\mathcal{L}_{\substack{\text{Aileron}\\ \text{Hinge Pin}}} = 0 = F_a \cdot K + H_a \tag{4.82}$$

$$F_a = -G \cdot H_a \tag{4.83}$$

Where G is merely 1/K.

Knowing.

$$H_a = C_h\, q_a\, S_a\, c_a \tag{4.84}$$

From equation 4.26

$$H_a = q_a\, S_a\, c_a\, (C_{h_{\alpha_a}} \cdot \alpha_a + C_{h_{\delta_a}} \cdot \delta_a) \tag{4.85}$$

Thus, equation 4.83 becomes,

$$F_a = -G\, q_a\, S_a\, c_a\, (C_{h_{\alpha_a}} \cdot \alpha_a + C_{h_{\delta_a}} \cdot \delta_a) \tag{4.86}$$

From equation 4.27,

$$C_{h_{\alpha_a}} \cdot \alpha_a = -C_{h_{\delta_a}} \cdot \delta_{a(\text{Float})} \tag{4.87}$$

Equation 4.86 becomes,

$$F_a = -G\, q_a\, S_a\, c_a\, (-C_{h_{\delta_a}} \cdot \delta_{a(\text{Float})} + C_{h_{\delta_a}} \cdot \delta_a) \tag{4.88}$$

$$F_a = -G\, q_a\, S_a\, c_a\, C_{h_{\delta_a}} \left(\delta_a - \delta_{a(\text{Float})}\right) \tag{4.89}$$

The difference between where the pilot pushes the aileron, δ_a, and the amount it floats, $\delta_{a(\text{Float})}$, is the free position of the aileron, $\delta_{a(\text{Free})}$.

Therefore,

$$F_a = -G\, q_a\, S_a\, c_a\, C_{h_{\delta_a}}\, \delta_{a(\text{Free})} \tag{4.90}$$

$$\frac{\partial F_a}{\partial \beta} = - G\, q_a\, S_a\, c_a\, C_{h_{\delta_a}} \frac{\partial \delta_{a(Free)}}{\partial \beta} \qquad (4.91)$$

From equation 4.81, it can be shown that,

$$\frac{\partial \delta_{a(Free)}}{\partial \beta} = - \frac{C_{\ell_{\beta(Free)}}}{C_{\ell_{\delta_a}}} \qquad (4.92)$$

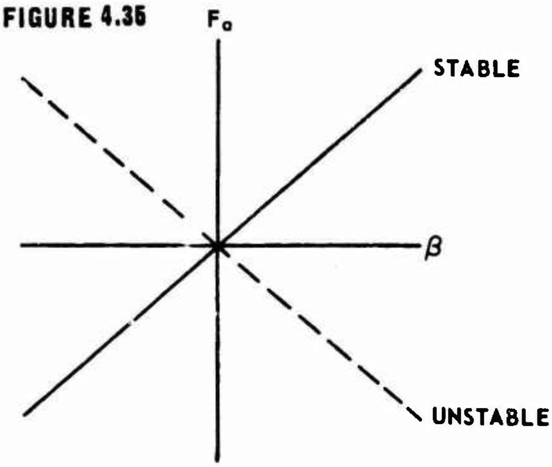
Thus,

$$\frac{\partial F_a}{\partial \beta} = G\, q_a\, S_a\, c_a \frac{C_{h_{\delta_a}}}{C_{\ell_{\delta_a}}} C_{\ell_{\beta(Free)}} \qquad (4.93)$$

This equation shows that the parameter $\partial F_a/\partial \beta$, can be taken as an indication of the aileron free static lateral stability of an aircraft. This parameter can be readily measured in flight.

An analysis of equation 4.93 reveals that for stable dihedral effect, a plot of $\partial F_a/\partial \beta$ would have a positive slope. Refer to figure 4.35.

FIGURE 4.35



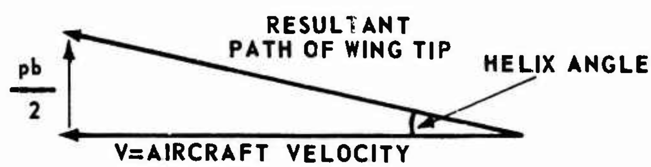
AILERON FORCE VERSUS SIDESLIP ANGLE

4.16 ROLLING PERFORMANCE

It has been shown how aileron force and aileron deflection can be used as a measure of the stable dihedral effect of an aircraft. However, it is now necessary to consider how aileron force and aileron deflection affect the rolling capability of the aircraft. For example, full aileron deflection may produce excellent rolling characteristics on certain aircraft, however, because of the large aileron forces required, the pilot may not be able to fully deflect the ailerons, thus making the overall rolling performance unsatisfactory. Thus, it is necessary to evaluate the rolling performance of the aircraft.

The rolling qualities of an aircraft can be evaluated by examining the parameters F_a, δ_a, p and $(pb/2V)$. Although the importance of the first three parameters is readily apparent, the parameter $(pb/2V)$ needs some additional explanation. Physically, $(pb/2V)$ may be described as the helix angle that the wing tip of a rolling aircraft describes. Refer to figure 4.36.

FIGURE 4.36



WING TIP HELIX ANGLE

It can be seen that,

$$\tan(\text{Helix Angle}) = \frac{pb}{2V} \qquad (4.94)$$

Assuming a small angle,

$$\text{Helix Angle} = \frac{pb}{2V} \qquad (4.95)$$

Figure 4.36 is a vectorial presentation of the wind forces acting on the downgoing wing during a roll. It shows that the angle of attack on the downgoing wing is increased due to roll rate. Thus $(pb/2V)$ represents a damping term.

With the foregoing background, it is possible to discuss the effect of the parameters, F_a, δ_a, p, (pb/2V) throughout the flight envelope of an aircraft.

From equation 4.90, it can be seen that

$$F_a = (f)\ V^2\ \delta_{a_{(Free)}} \qquad (4.96)$$

$$\delta_{a_{(Free)}} = (f)\ F_a\ \frac{1}{V^2} \qquad (4.97)$$

To derive a functional relationship for (pb/2V), it is necessary to start with,

$$C_\ell = C_{\ell_\beta}\beta + C_{\ell_{\dot\beta}}\hat{\dot\beta} + C_{\ell_p}\hat{p} + C_{\ell_r}\hat{r} + C_{\ell_{\delta_a}}\delta_a + C_{\ell_{\delta_r}}\delta_r \qquad (4.6)$$

and examine the effects of roll terms only, i.e., assume that the roll moment developed is due to the interaction of moments due to δ_a and roll damping only. Therefore, equation 4.6 becomes,

$$C_\ell = C_{\ell_p}\hat{p} + C_{\ell_{\delta_a}}\delta_a = C_{\ell_p}\left(\frac{pb}{2V}\right) + C_{\ell_{\delta_a}}\delta_a \qquad (4.98)$$

Below Mach or aerolastic effects, $C_{\ell_{Max}}$ = constant, so if it is desired to evaluate an aircraft's maximum rolling performance, equation 4.98 becomes,

$$C_{\ell_p}\left(\frac{pb}{2V}\right) + C_{\ell_{\delta_a}}\delta_a = \text{constant} \qquad (4.99)$$

$$\left(\frac{pb}{2V}\right) = \frac{\text{Constant} - C_{\ell_{\delta_a}}\delta_a}{C_{\ell_p}} \qquad (4.100)$$

$$\left(\frac{pb}{2V}\right) = (f)\ \delta_a \qquad (4.101)$$

From equation 4.97,

$$\left(\frac{pb}{2V}\right) = (f)\ \delta_a = (f)\ F_a\ \frac{1}{V^2} \qquad (4.102)$$

A function relationship for roll rate, p, can be derived from equation 4.100,

$$p = \frac{\text{Constant} - C_{\ell_{\delta_a}}\delta_a}{C_{\ell_p}} \cdot \frac{2}{b} \cdot V \qquad (4.103)$$

$$p = (f)\ V\ \delta_a \qquad (4.104)$$

From equation 4.97,

$$p = (f)\ V\ \delta_a = (f)\ F_a\ \frac{1}{V} \qquad (4.105)$$

To summarize, the rolling performance of an aircraft can be evaluated by examining the parameters, F_a, δ_a, p, and (pb/2V). Functional relationships have been developed in order to look at the variance of these parameters below Mach or aeroelastic effects. These functional relationships are:

$$F_a = (f)\ V^2\ \delta_a \qquad (4.96)$$

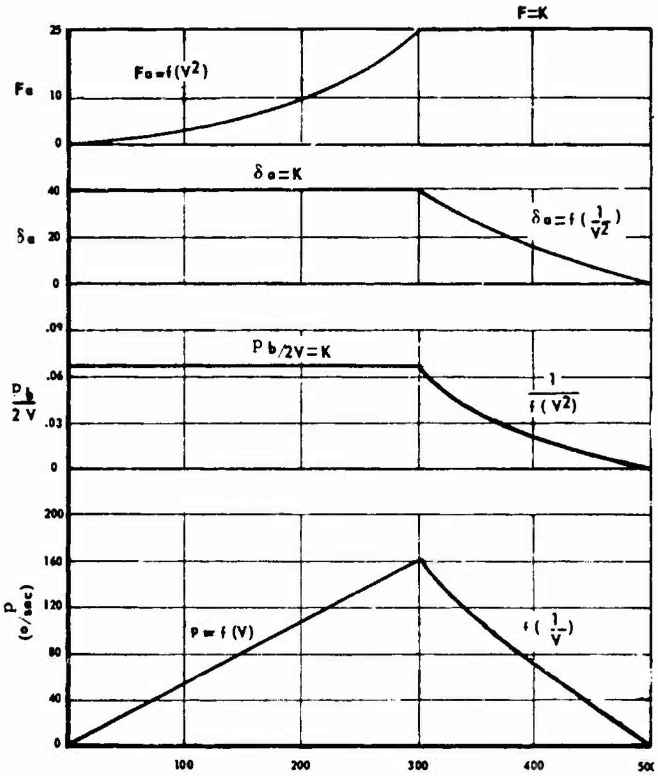
$$\delta_a = (f)\ F_a\ \frac{1}{V^2} \qquad (4.97)$$

$$\left(\frac{pb}{2V}\right) = (f)\ \delta_a = (f)\ F_a\ \frac{1}{V^2} \qquad (4.102)$$

$$p = (f)\ V\ \delta_a = (f)\ F_a\ \frac{1}{V} \qquad (4.105)$$

These relationships are expressed graphically in figure 4.37 for a case in which the pilot desires the maximum roll rate at all airspeeds.

FIGURE 4.37



ROLLING PERFORMANCE

As indicated in equation 4.96, the force required to hold a constant aileron deflection will vary as the square of the airspeed. The force required by the pilot to hold full aileron deflection will increase in this manner until the aircraft reaches V_{Max} or until the pilot is unable to apply any more force. In figure 4.37, it is assumed that the pilot can supply a maximum of 25 pounds force and that this force is reached at 300 knots. If the speed is increased further, the aileron force will remain at this 25 pound maximum value. The curve of aileron deflection versus airspeed shows that the pilot is able to maintain full aileron deflection out to 300 knots. Inspection of equation 4.97 shows that if aileron force is constant beyond 300 knots, then aileron deflection will be proportional to $(1/V^2)$. Equation 4.102 shows that $(pb/2V)$ will vary in the same manner as aileron deflection. Inspection of equation 4.105 shows that the maximum roll rate available will increase linearly as long as the pilot can maintain maximum aileron deflection; up to 300 knots in this case. Beyond this point, the maximum roll rate will fall off hyperbolically. That is, above 300 knots, p is proportional to 1/V. It follows, then, that at high speeds the maximum roll rate may become unacceptably low. One method of combating this problem is to increase the pilot's mechanical advantage by adding boosted or fully powered ailerons.

4.17 ROLL DAMPING C_{ℓ_p}

Aircraft roll damping comes from the wing and the vertical tail. The algebraic sign of C_{ℓ_p} is negative as long as the local angle of attack remains below the local stall angle of attack.

The wing contribution to C_{ℓ_p} arises from the change in wing angle of attack that results from the rolling velocity. It has already been shown that the downgoing wing in a rolling maneuver experiences an increase in angle of attack and that this increased α tends to develop a rolling moment that opposes the original rolling moment. However, when the wing is near the aerodynamic stall, a rolling motion may cause the downgoing wing to exceed the stall angle of attack. In this case, the local lift curve slope may fall to zero or even reverse sign. The algebraic sign of the wing contribution to C_{ℓ_p} may then become positive. This is what occurs when a wing "autorotates," as in spinning.

The vertical tail contribution to C_{ℓ_p} arises from the fact that when the aircraft is rolled, the angle of attack on the vertical tail is changed. This change in

angle of attack develops a lift force. If the vertical tail cp is above or below the aircraft cg, the rolling moment developed will oppose the original rolling moment and C_{ℓ_p} due to a conventional vertical tail or a ventral fin will be negative.

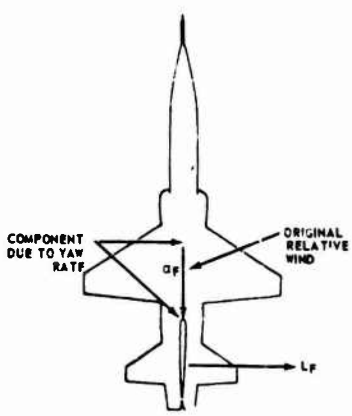
● 4.18 ROLLING MOMENT DUE TO YAW RATE C_{ℓ_r}

The contributions to this derivative come from two sources, the wings and the vertical tail.

As the aircraft yaws, the velocity of the relative wind is increased on the outboard wing and decreased on the inboard wing. This causes the outboard wing to produce more lift and thus produces a rolling moment. A right yaw would produce more lift on the left wing and thus a rolling moment to the right. Thus, the algebraic sign of the wing contribution to C_{ℓ_r} is positive.

The tail contribution to C_{ℓ_r} arises from the fact that as the aircraft is yawed, the angle of attack on the vertical tail is changed. Refer to figure 4.38.

FIGURE 4.38



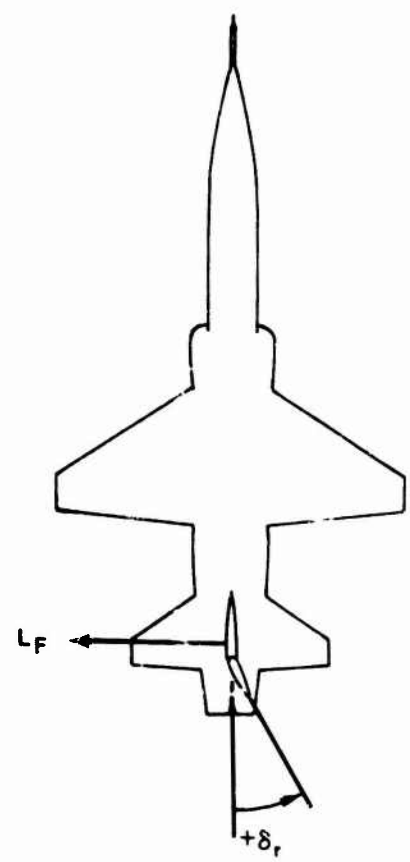
CHANGE IN α_F DUE TO YAW RATE

The lift force thus produced, L_F, will create a rolling moment if the vertical tail cp is above or below the cg. For a conventional vertical tail, the sign of C_{ℓ_r} will be positive while for a ventral fin the sign will be negative.

● 4.19 ROLLING MOMENT DUE TO RUDDER DEFLECTION - $C_{\ell_{\delta_r}}$

When the rudder is deflected, it creates a lift force on the vertical tail. If the cp of the vertical tail is above or below the aircraft cg a rolling moment will result. Refer to figure 4.39.

FIGURE 4.39



LIFT FORCE DEVELOPED AS A RESULT OF δ_r

It can be seen that if the cp of the vertical tail is above the cg, as with a conventional vertical tail, the sign of $C_{\ell_{\delta_r}}$ will be negative. However, with a ventral fin, the sign would be positive.

It is interesting to note that the effects of $C_{\ell_{\delta_r}}$ and C_{ℓ_β} are opposite in nature. When the rudder is deflected to the right, initially, a rolling moment to the left is created due to $C_{\ell_{\delta_r}}$. However, as sideslip develops due to the rudder deflection, dihedral effect, C_{ℓ_β}, comes into play and causes a resulting rolling moment to the right. Therefore, when a pilot applies right rudder to pick up a left wing, he initially creates a rolling moment to the left and finally, to the right.

4.20 ROLLING MOMENT DUE TO LAG EFFECTS IN SIDEWASH - $C_{\ell_{\dot\beta}}$

In the discussion of $C_{n_{\dot\beta}}$, it was pointed out that during an increase in β, the angle of attack of the vertical tail will be less than it will finally be in steady state conditions. If the cp of the vertical tail is displaced from the aircraft cg, this change in α_F due to lag effects will alter the rolling moment created during the β build up period. Because of lag effects, C_{ℓ_F} will be less during the β build up period than at steady state. Thus, for a conventional vertical tail, the algebraic sign of $C_{\ell_{\dot\beta}}$ is positive.

Again, it should be pointed out that there is widespread disagreement over the interpretation of data concerning lag effects in sidewash and that the foregoing is only one basic approach to a many faceted and complex problem.

4.21 HIGH SPEED CONSIDERATIONS OF STATIC LATERAL STABILITY

C_{ℓ_β} - Generally, C_{ℓ_β} is not greatly affected by Mach number. However, in the transonic region the increase in the lift curve slope of the vertical tail increases this contribution to C_{ℓ_β} and usually results in an overall increase in C_{ℓ_β} in the transonic region.

$C_{\ell_{\delta_a}}$ - Because of the decrease in the lift curve slope of all aerodynamic surfaces in supersonic flight, lateral control power decreases as Mach number increases supersonically.

Aeroelasticity problems have been quite predominant in the lateral control system, since in flight at very high dynamic pressures the wing torsional deflections which occur with aileron usage are considerable and cause noticeable changes in aileron effectiveness. At some high dynamic pressures, dependent upon the given wing structural integrity, the twisting deformation might be great enough to nullify the effect of aileron deflection and the aileron effectiveness will be reduced to zero. Since at speeds above the point where this phenomenon occurs, rolling moments are created which are opposite in direction to the control deflection, this speed is termed "aileron reversal speed." In order to alleviate this characteristic the wing must have a high torsional stiffness which presents a significant design problem in sweptwing aircraft. For an aircraft design of the B-47 type, it is easy to visualize how aeroelastic distortion might result in a considerable reduction in lateral control capability at high speeds. In addition, lateral control effectiveness at transonic Mach numbers may be reduced seriously by flow separation effects as a result of shock formation.

However, modern high speed fighter designs have been so successful in introducing sufficient rigidity into wing structures and employing such design modifications as split ailerons, inboard ailerons, spoiler systems, etc., that the resulting high control power, coupled with the low C_{ℓ_p} of low aspect ratio planforms, has resulted in the lateral control becoming an accelerating device rather than a rate control. That is to say, a steady state rolling velocity is normally not reached prior to attaining the desired bank angle. Consequently, many high speed aircraft have a type of differential aileron system to provide the pilot with much more control surface during approach and landings and to restrict his degree of control in other areas of flight.

Spoiler controls are quite effective in reducing aeroelastic distortions since the pitching moment changes due to spoilers are generally smaller than those for a flap type control surface. However, a problem associated with spoilers is their tendency to reverse the roll direction for small stick inputs during transonic flight. This occurs as a result of re-energizing the boundary layer by a vortex generator effect for very small deflections of the spoiler, which can reduce the magnitude of the shock induced separation and actually increase the lift on the wing. This difficulty can be eliminated by proper design techniques.

C_{ℓ_p} - Since the development of "damping" requires the development of lift on either the wing or the tail, it is dependent on the value of the lift curve slope. Thus, as the lift curve slope of both the wing and tail decrease supersonically, C_{ℓ_p} will decrease. Also, since most supersonic designs make use of low aspect ratio surfaces, C_{ℓ_p} will tend to be less for these designs.

C_{ℓ_r} and $C_{\ell_{\delta_r}}$ - Both of these derivatives depend on the development of lift and will decrease as the lift curve slope decreases supersonically.

$C_{\ell_{\dot{\beta}}}$ - Data on the supersonic variation of this derivative is sketchy, but it probably will not change significantly with Mach number.

CHAPTER 5

DIFFERENTIAL EQUATIONS

ABBREVIATIONS AND SYMBOLS FOR THIS CHAPTER

x, y, z:	dependent or independent variables
t:	time in seconds
p:	differential operator with dimensions of seconds^{-1}
j:	constant equal to $\sqrt{-1}$
ϕ:	angular constant in radians
e:	constant equal to $\lim\limits_{x \to 0} (1 + x)^{\frac{1}{x}} = 2.71828.....$
x_t, y_t, z_t:	transient solution to differential equation
x_p, y_p, z_p:	particular (steady state) solution to differential equation
$\dot{x}$:	the dot notation indicated differentiation with respect to time; i.e., $\dot{x} = \frac{dx}{dt}$
τ :	time constant in seconds
T_1:	time to half amplitude in seconds
ζ :	damping ratio
ω_n:	undamped natural frequency in radians per second
ω_d:	damped frequency in radians per second
s:	Laplace variable in seconds^{-1}
L:	Laplace Transform
L^{-1}:	inverse Laplace Transform
$X(s), Y(s), Z(s)$:	Laplace Transform of $x(t)$, $y(t)$, $z(t)$
$\triangleq$:	symbol used for definitions; e.g., $\dot{x} \triangleq \frac{dx}{dt}$ means $\dot{x}$ is defined as $\frac{dx}{dt}$

● 5.1 INTRODUCTION

The theory of differential equations is a subject of considerable scope, ranging from the rather simple and obvious through the abstract and not so obvious. One can spend a lifetime studying the subject, and a few people have. We have neither the time, nor perhaps the inclination for such devotions. Our purpose is to cover those aspects of the theory of differential equations which are of direct application to work at the School.

This chapter deals with the tools and techniques required to analyze differential equations. Such techniques are easily extended for use in the study of aircraft dynamics. An aircraft in flight displays motions similar to a mass-spring-damper system. The static stability of the airplane is similar to the spring, the moments of inertia similar to the mass, and the airflow serves to damp the aircraft motion.

The first section in the chapter provides a review of basic differential equation theory. Succeeding sections deal with operator techniques, analysis of first and second order systems, use of Laplace Transforms, and solution of simultaneous equations.

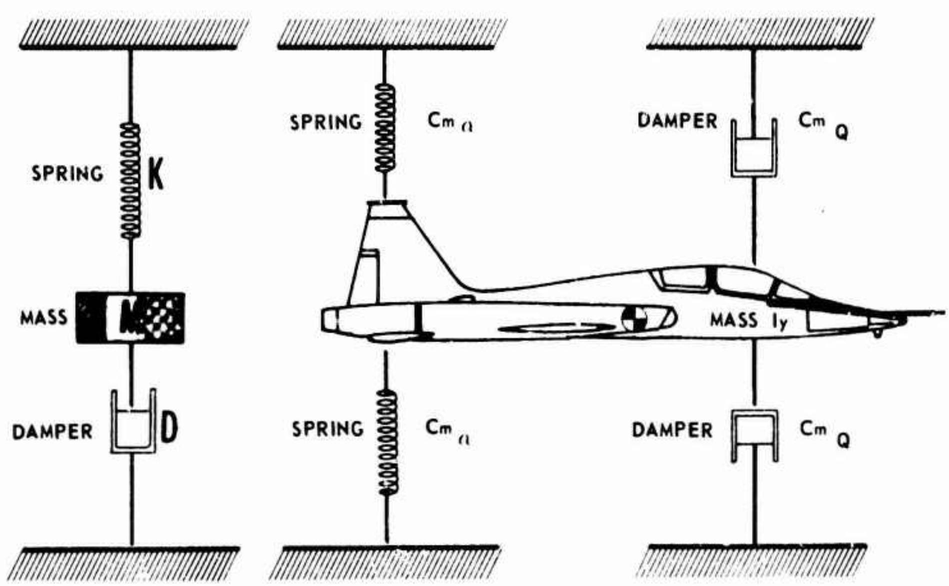
Before proceeding with our study, we shall define several terms which will be used in this chapter.

Definitions:

1. Differential Equation: A differential equation is an equation which involved a dependent variable (or variables) together with one or more of its derivatives with respect to an independent variable (or variables).

2. Solution: Any function, free of derivatives, which satisfies a differential equation is said to be a solution of the differential equation.

FIGURE 5.1



3. Ordinary Differential Equation: A differential equation which involves derivatives with respect to a single independent variable is called an ordinary differential equation.

4. Order: The n^{th} derivative of a dependent variable is called a derivative or order n, or an n^{th} order derivative. The order of a differential equation is the order of the highest order derivative present.

5. Degree: The exponent of the highest order derivative is called the degree of the differential equation --- if the differential equation can be rationalized or cleared of fractions with regard to derivatives appearing in the differential equation.

6. Linear Differential Equation (ordinary, single dependent variable): A differential equation in which the dependent variable and its derivatives appear in no higher than the 1st degree, and the coefficients are either constants or functions of the independent variable is called a linear differential equation.

7. Linear System: Any physical system described by a linear differential equation is called a linear system.

8. General Solution: A solution of a differential equation of order n which contains n arbitrary constants will be called a general solution of the differential equation.

5.2 REVIEW OF BASIC PRINCIPLES

Before investigating Operator Notation and Laplace Transforms, we would do well to review the more basic methods of solving differential equations.

Direct Integration:

To solve a differential equation we seek a mathematical expression, relating the variables appearing in the differential equation, which qualifies as a solution under the definitions given above. A first thought or inspiration may be: Since we are presented with an equation containing derivatives, then a solution may be obtained by antidifferentiating or integrating. This process removes derivatives and provides arbitrary constants.

EXAMPLE:

Given

$$\frac{dy}{dx} = x + 4$$

rewriting

$$dy = (x + 4)dx$$

integrating

$$\int dy = \int (x + 4)dx + c$$

gives us

$$y = \frac{x^2}{2} + 4x + c \qquad (5.1)$$

....

EXAMPLE:

Given

$$\frac{d^2y}{dx^2} = x + 4$$

Assume

$$\frac{d^2y}{dx^2} = \frac{dy'}{dx}$$

where

$$y' = \frac{dy}{dx}$$

then

$$\frac{dy'}{dx} = x + 4$$

or

$$\int dy' = \int (x + 4)dx + c_1$$

then

$$y' = \frac{dy}{dx} = \frac{x^2}{2} + 4x + c_1$$

integrating

$$\int dy = \int \left(\frac{x^2}{2} + 4x + c_1 \right) dx + c_2$$

giving

$$y = \frac{x^3}{6} + 2x^2 + c_1 x + c_2 \tag{5.2}$$

....

Equations (5.1) and (5.2) qualify as general solutions under our definitions stated above.

Life is full of disappointments and we would soon learn that this direct application of the integration process would fail to work in many cases.

EXAMPLE:

$$2xy + (x^2 + \cos y)\frac{dy}{dx} = 0 \tag{5.3}$$

or

$$dy = \frac{2xy}{x^2 + \cos y} dx$$

$$\int dy = \int \frac{2xy}{x^2 + \cos y} dx + c \tag{5.4}$$

....

We cannot perform the integration of the term to the right of the equal sign in equation (5.4). The equation (5.3) can be solved, however, using straight forward techniques. ($x^2y + \sin y = c$ is a general solution.) We emphasize the word "technique" since the solution may rely upon novel approaches, special groupings or "judicious arrangements" and, perhaps, witchcraft or conjuring. The former require extensive experience and maturity within the discipline, and the latter talents are rarely endowed by nature. We shall study a few special differential equations which are easy to solve and have wide application in the analysis of physical problems.

<u>First Order Equations</u>:

We shall consider briefly the first order ordinary differential equation. Suppose we represent such an equation by:

$$F(y', y, x) = 0$$

where

$$y' = \frac{dy}{dx}$$

This is concise notation used by mathematicians to denote a differential equation containing an independent variable x, a dependent variable y, and the derivative of y with respect to x. The equation may contain the derivative in differential form.

EXAMPLES:

$$\frac{dy}{dx} = x + y$$

$$3x\,dx + 4y\,dy = 0$$

$$y' = \frac{x - y}{x + y}$$

$$\frac{dy}{dx} = \frac{x - y\cos x}{\sin x + y}$$

....

First order differential equations may be solved by:

1. Separating variables and integrating directly
2. Recognizing exact forms and integrating directly
3. Finding an integrating factor (fudge factor) which will make the equation exact.
4. Inspection, rearrangement of terms, etc. to use method (1) or (2), or a combination of the two.

These methods are thoroughly treated in all elementary differential equations texts. A brief review of methods (1) and (2) are given below.

1. Separation of variables: When a differential equation can be put in the form

$$f_1(x)dx + f_2(y)dy = 0 \quad (5.5)$$

Where one term contains x and dx only, and the other y and dy only, the variables are said to be separated. A solution of equation (5.5) can then be obtained by direct integration.

$$\int f_1(x)dx + \int f_2(y)dy = c \quad (5.6)$$

where c is an arbitrary constant. Note, that for a differential equation of the first order there is one arbitrary constant. In general, the number of arbitrary constants is equal to the order of the differential equation.

The general form of a first order equation with variables separable is

$$M\,dx + N\,dy = 0 \quad (5.7)$$

where M and N are functions of only one variable.

EXAMPLE:

$$\frac{dy}{dx} = \frac{x^2 + 3x + 4}{y + 6}$$

$$(y + 6)\,dy = (x^2 + 3x + 4)\,dx$$

$$\int (y + 6)dy = \int (x^2 + 3x + 4)dx + c$$

$$\frac{y^2}{2} + 6y = \frac{x^3}{3} + \frac{3x^2}{2} + 4x + c$$

....

2. Exact differential equations: If we have

$$F(x, y) = c$$

where c is a constant, then

$$dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy = 0 \quad (5.8)$$

If we let $M = \partial F/\partial x$ and $N = \partial F/\partial y$, then equation (5.8) becomes

$$dF = M\,dx + N\,dy = 0 \quad (5.9)$$

A necessary and sufficient condition for (5.9) to be an exact differential equation is that

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad (5.10)$$

Since

$$\frac{\partial M}{\partial y} = \frac{\partial^2 F}{\partial y\,\partial x}$$

and

$$\frac{\partial N}{\partial x} = \frac{\partial^2 F}{\partial x\,\partial y}$$

we have assumed, without proof, that the order of differentiation is immaterial.

We have now shown that an equation of the form

$$dF = M\,dx + N\,dy = 0 \quad (5.9)$$

is exact when

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

and has a solution

$$F = c$$

EXAMPLE:

$$(2xy + 3x^2)\,dx + x^2\,dy = 0$$

$$M = 2xy + 3x^2$$

$$\frac{\partial M}{\partial y} = 2x$$

$$N = x^2$$

$$\frac{\partial N}{\partial x} = 2x$$

so that

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

and the equation is exact with solution

$$F = c$$

but what is F equal to?

....

Previously we said that $M = \partial F/\partial x$, indicating that we should be able to find F by integrating M with respect to x. The constant of integration which we must add is independent of x, but may be dependent on y. Thus,

$$F = \int M\,\partial x + f(y) = c \quad (5.11)$$

where the symbol ∂x indicates that integration is with respect to x, keeping y constant. Similarly,

$$F = \int N\,\partial y + f(x) = c \quad (5.12)$$

Solving our example

$$F = \int M\,\partial x + f(y) = c$$

$$F = \int (2xy + 3x^2)\,\partial x + f(y)$$

$$= x^2y + x^3 + f(y)$$

and

$$F = \int N\,\partial y + f(x) = c$$

$$F = \int x^2\,\partial y + f(x)$$

$$= x^2y + f(x)$$

Equating the two solutions

$$x^2y + x^3 + f(y) = x^2y + f(x)$$

by observation

$$f(x) = x^3$$

$$f(y) = 0$$

and

$$F = x^2y + x^3 = c$$

$\therefore\ x^2y + x^3 = c$ is the solution desired.

3. First order linear differential equations: We conclude the discussion of 1st order equations by considering the following form

$$\frac{dy}{dx} + R(x)\, y = 0 \qquad (5.13)$$

where $R(x)$ may be a constant. To solve, merely separate variables.

$$\frac{dy}{y} + R(x)\, dx = 0$$

integrating

$$\int \frac{dy}{y} = - \int R(x)\, dx + c'$$

where

$$c' = \ln c$$

Thus

$$\ln y = - \int R(x)\, dx + \ln c$$

or

$$y = ce^{- \int R(x)\, dx}$$

If R is a constant then

$$y = ce^{-Rx} \qquad (5.14)$$

We might conclude from this result that a 1st order differential equation of form (5.13) with constant coefficients may be solved quite simply. This is true and the solution will always have the form (5.14).

EXAMPLE:

$$\frac{dy}{dx} + 2y = 0 \qquad (5.14a)$$

then we have directly

$$y = ce^{\boxed{-2}\, x} \qquad (5.14b)$$

which is the general solution. It is quickly recognized that the solution is simply obtained by plugging the negative of the coefficient of y into the position indicated by the small square.

....

● 5.3 LINEAR DIFFERENTIAL EQUATIONS AND OPERATOR TECHNIQUES

A form of the differential equation that is of particular interest is:

$$f(x) = A_n \frac{d^n y}{dx^n} + A_{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \ldots + A_1 \frac{dy}{dx} + A_o y \qquad (5.15)$$

If the coefficient expressions A_n, A_{n-1}, ..., A_o are all functions of x only, then equation (5.15) is called a linear differential equation. If the coefficient expressions A_n, ..., A_o are all constants, then (5.15) is called a linear differential equation with constant coefficients.

EXAMPLE:

$$x^2 \frac{d^2y}{dx^2} + 3\frac{dy}{dx} + xy = \sin x$$

is a linear differential equation.

EXAMPLE:

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = e^x$$

is a linear differential equation with constant coefficients. Linear differential equations with constant coefficients occur frequently in the analysis of physical systems. Mathematicians and engineers have developed simple and effective techniques to solve this type of

equation by using either "classical" or operational methods. When attempting to solve a linear differential equation of the form:

$$A_n \frac{d^n y}{dx^n} + A_{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + A_1 \frac{dy}{dx} + A_o y = f(x) \qquad (5.16)$$

it is helpful to examine the equation

$$A_n \frac{d^n y}{dx^n} + A_{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + A_1 \frac{dy}{dx} + A_o y = 0 \qquad (5.17)$$

(5.17) is the same as (5.16) with the right hand side zero. We shall refer to (5.16) as the general equation and equation (5.17) as the complementary or homogeneous equation. Solutions of equation (5.17) possess a useful property known as superposition, which may be briefly stated as follows: Suppose $y_1(x)$ and $y_2(x)$ are distinct solutions of (5.17). Then any linear combination of $y_1(x)$ and $y_2(x)$ are also solutions of (5.17). A linear combination would be $c_1y_1(x) + c_2y_2(x)$.

EXAMPLE:

$$\frac{d^2 y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

It can be verified that $y_1(x) = e^{3x}$ is a solution, and that $y_2(x) = e^{2x}$ is another solution which is distinct from $y_1(x)$. Using superposition, then $y(x) = c_1e^{3x} + c_2e^{2x}$ is also a solution.

....

Equation (5.16) may be interpreted as representing a physical system where the left side of the equation describes the natural or designed state of the system, and where the right side of the equation represents the input or forcing function.

One might logically pursue the following line of reasoning in attempting to find a solution to the problem described by equation (5.16).

1. A general solution of (5.16) must contain n arbitrary constants and must satisfy the equation.

2. The following statements are justified by experience:

   a. It is reasonably straight forward to find a solution to the complementary equation (5.17), containing n arbitrary constants. Such a solution will be called the homogeneous or transient solution. Physically, it represents the response present in the system regardless of input.

   b. There are varied techniques for finding a solution to (5.16). Such solutions do not, in general, contain arbitrary constants but instead represent the response of the system due to a particular input or forcing function. This solution will be called the particular or steady state solution.

3. If we take the transient solution which describes response already existing in the system and then add on the response due to the forcing function, it would appear that a solution so written would blend the two responses

and describe the total response of the system represented by (5.16). In fact, the definition of a general solution is satisfied under such an arrangement. The transient solution contains the correct number of arbitrary constants, and the particular solution guarantees that the combined solutions satisfy the general equation (5.16). Call the transient solution y_t and the particular solution y_p. A general solution of (5.16) is then given by:

$$y = y_t + y_p \tag{5.18}$$

<u>Transient Solution</u>:

Equation (5.13a) above is a complementary or homogeneous first order linear differential equation with constant coefficients. We recognized a quick and simple method of finding a solution to this equation. We also recognized that the solution was always of exponential form. We might hope that solutions of higher order equations of the same family would take the same form. There is one quick way to find out - we shall assume that

$$y = e^{\lambda x} \tag{5.19}$$

is a solution of (5.17) and see what happens. Let λ represent a constant which we do not yet know. Substituting in (5.17) gives:

$$A_n \lambda^n e^{\lambda x} + A_{n-1} \lambda^{n-1} e^{\lambda x} + \ldots + A_1 \lambda e^{\lambda x} + A_o e^{\lambda x} = 0 \tag{5.20}$$

which may be factored:

$$e^{\lambda x} (A_n \lambda^n + A_{n-1} \lambda^{n-1} + \ldots + A_1 \lambda + A_o) = 0$$

Now we can assert that:

$$A_n \lambda^n + A_{n-1} \lambda^{n-1} + \ldots + A_1 \lambda + A_o = 0 \tag{5.21}$$

since $e^{\lambda x} \neq 0$. Equation (5.21) is called the characteristic equation. (Sometimes called auxiliary or indicial equation.) Equation (5.21) has n possible solutions. Agree to call them $\lambda_1, \lambda_2, \ldots \lambda_n$. Then for λ_i, $y = e^{\lambda_i x}$ is a solution, and so is $c_i e^{\lambda_i x}$, where c_i is an arbitrary constant. From the superposition principle stated above, it also follows that

$$y_t = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x} \cdots + c_n e^{\lambda_n x} \tag{5.22}$$

is a solution containing n arbitrary constants. We have included the subscript "t" on y to indicate that (5.22) represents the transient solution. From the foregoing it is seen that we have succeeded in extending the method for 1st order complementary equations to higher order complementary or homogeneous equations. Again we note that we have traded off an integration problem for an algebra problem (solving equation (5.22) for the λ's).

Differential or derivative operators can be defined and manipulated to play the same role as λ above. It is immediately confirmed by checking (5.22) that the exponents of λ are the same as the

order of the derivatives in the same position . [E.g., compare (5.17) and (5.20).]

$$A_n \frac{d^n y}{dx^n} + A_{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \ldots + A_1 \frac{dy}{dx} + A_o y = 0 \tag{5.17}$$

$$A_n \lambda^n e^{\lambda x} + A_{n-1} \lambda^{n-1} e^{\lambda x} + \ldots A_1 \lambda e^{\lambda x} + A_o e^{\lambda x} = 0 \tag{5.20}$$

If we designate an operator p, p^2, ..., p^n as follows:

$$p = \frac{d}{dx}, \; p^2 = \frac{d^2}{dx^2}, \ldots, p^n = \frac{d^n}{dx^n} \tag{5.23}$$

$$p(y) = \frac{dy}{dx}, \; p^2 y = \frac{d^2 y}{dx^2}, \ldots . \; p^n y = \frac{d^n y}{dx^n} \tag{5.24}$$

then (5.17) may be written:

$$A_n p^n(y) + A_{n-1} p^{n-1}(y) + \ldots + A_1 p(y) + A_o y = 0 \tag{5.25}$$

or, since the derivative operates linearly (each term in succession),

$$(A_n p^n + A_{n-1} p^{n-1} + \ldots + A_1 p + A_o) y = 0 \tag{5.26}$$

and the operator expression ($A_n p^n$ + ... + A_1 p + A_o) has the same algebraic structure as (5.21). The operator expression in (5.26) is a polynomial with precisely the same form as the polynomial on the left side of (5.21), hence it is often solved directly for the constants required in the solution of (5.19). In this case, the transient solution (5.22) would appear[1]

$$y_t = c_1 e^{p_1 x} + c_2 e^{p_2 x} + \ldots + c_n e^{p_n x} \tag{5.27}$$

There are cases for which (5.22) and (5.27) are not entirely satisfactory in providing a solution, but this will be discussed later. The λ's or p's may be real, imaginary, or complex numbers.

EXAMPLE:

$$\frac{d^2 y}{dx^2} + \frac{dy}{dx} - 2y = 0$$

By the characteristic equation method:

$$\lambda^2 + \lambda - 2 = 0$$

$$\lambda = +1, -2$$

$$y = c_1 e^x + c_2 e^{-2x}$$

Using operator notation:

$$(p^2 + p - 2) y = 0$$

$$p^2 + p - 2 = 0$$

$$p = 1, -2$$

$$y = c_1 e^x + c_2 e^{-2x}$$

....

[1] To the mathematical purist, this equation is incorrect. By our definition, px = 1. We shall continue to use this notation, however, knowing that it will aid us in solving differential equations.

We snall now restrict the discussion to 2nd order linear differential equations with real constant coefficients, and consider the various cases for solutions of the complementary (homogeneous) equation.

Consider the equation:

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0 \qquad (5.28)$$

We have seen above that the solution of this differential equation is equivalent to solving the quadratic equation:

$$ap^2 + bp + c = 0 \qquad (5.29)$$

The general solution of (5.28) is of the form:

$$y = c_1e^{p_1x} + c_2e^{p_2x} \qquad (5.30)$$

where c_1 and c_2 are arbitrary constants, and p_1 and p_2 are solutions of the quadratic equation (5.29). Recall from algebra that a quadratic equation can yield complex roots, imaginary roots, or real roots (i.e., $p_{1,2} = [-b \pm\sqrt{b^2 - 4ac}\,]/2a$). We consider the solution (5.30) for various values of equation (5.29), and consider changes in the form of the solution which may be desirable or necessary.

....

Case 1: Roots Real and Unequal: If p_1 and p_2 are real and unequal the desired form of solution is just as is.

EXAMPLE:

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 12y = 0$$

$$(p^2 + 4p - 12)y = 0 \text{ (in operator form)}$$

solving

$$\left(p^2 + 4p - 12\right) = 0$$

gives

$$p = \frac{-4 \pm\sqrt{16 + 48}}{2}$$

$$= \frac{-4 \pm 8}{2}$$

or

$$p = -6,\ 2$$

and

$$y = c_1e^{-6x} + c_2e^{2x}$$

is the required solution.

....

Case 2: Roots Real and Equal: If p_1 and p_2 are real and equal we run into trouble.

EXAMPLE:

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$$

$$(p^2 - 4p + 4)y = 0 \text{ (in operator form)}$$

solving,

$$p = \frac{4 \pm\sqrt{16 - 16}}{2} = \frac{4}{2} = 2$$

or $p = 2$. But this gives only one value of p. If we try to use (5.30) all we get is $y = c_1e^{2x}$, but we need two arbitrary constants to have a transient solution of (5.28). If we are really alert, we may notice that the operator expression $(p^2 - 4p + 4)$ can be written $(p - 2)\cdot(p - 2)$, or $(p - 2)^2$, which is a polynominal expression with a repeated factor. (i.e., $p = 2, 2$ is the solution.) We can then write $y = c_1e^{2x} + c_2e^{2x}$ as the transient

solution. This is really no better than our first attempt, $y = c_1e^{2x}$, since c_1 and c_2 can be combined into a single arbitrary constant.

$$y = c_1e^{2x} + c_2e^{2x} = (c_1 + c_2)e^{2x}$$

$$= c_3e^{2x}$$

To solve this problem, simply multiply one of the arbitrary constants by x.[2] Now write: $y = c_1e^{2x} + c_2xe^{2x}$. We can no longer "lump" the two coefficients of e^{2x} together. The solution now contains two arbitrary constants, and it is easily verified that

$$y = c_1e^{2x} + c_2xe^{2x}$$

is a transient solution of the problem above.

....

Case 3: Roots Pure Imaginary

EXAMPLE:

$$\frac{d^2y}{dx^2} + y = 0$$

in operator form,

$$(p^2 + 1)y = 0$$

solving,

$$p = \frac{0 \pm \sqrt{0 - 4}}{2} = \pm \sqrt{-1}$$

In most engineering work we refer to $\sqrt{-1}$ as j. (In mathematical texts it is denoted by i.) Now,

$$p = \pm j$$

and the solution is written:

$$y = c_1e^{jx} + c_2e^{-jx} \qquad (5.31)$$

....

[2] See, for example, C.R. Wylie, Chapter 2.

This is a perfectly good solution from a mathematical standpoint, but is unwieldy and unsuggestive to engineers. A mathematician by the name of Euler worked out this puzzle for us by developing an equation called Euler's identity.

$$e^{jx} = \cos x + j \sin x \qquad (5.32)$$

This equation can be restated in many ways geometrically and analytically, and can be verified by adding the series expansion of cos x to the series expansion of j sin x. Now (5.31) may be expressed:

$$y = c_1(\cos x + j \sin x) + c_2[\cos(-x) + j \sin(-x)] = (c_1 + c_2)(\cos x) + [j(c_1 - c_2)] (\sin x) \qquad (5.33)$$

or

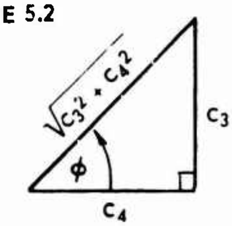
$$y = c_3 \cos x + c_4 \sin x \qquad (5.34)$$

Equation (5.34) has another interesting form. Let

$$y = \sqrt{c_3^2 + c_4^2}\left[\frac{c_3}{\sqrt{c_3^2 + c_4^2}} \cdot \cos x + \frac{c_4}{\sqrt{c_3^2 + c_4^2}} \sin x\right] \qquad (5.35)$$

Now consider a right triangle with sides labeled as follows:

FIGURE 5.2



Now,

$$\frac{c_3}{\sqrt{c_3^2 + c_4^2}} = \sin \emptyset ,$$

$$\frac{c_4}{\sqrt{c_3^2 + c_4^2}} = \cos \emptyset ,$$

and

$$\sqrt{c_3^2 + c_4^2} = A.$$

A and Ø are arbitrary constants, and (5.35) becomes:

$$y = A(\sin \emptyset \cos x + \cos \emptyset \sin x)$$

or

$$y = A \sin (x + \emptyset) \qquad (5.36)$$

To summarize, if the roots of the operator polynomial are pure imaginary, they will be numerically equal but opposite in sign, and the solution will have the form (5.34) or (5.36).

....

Case 4: Roots Complex

EXAMPLE:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

in operator form,

$$(p^2 + 2p + 2)y = 0$$

solving,

$$p = \frac{-2 \pm \sqrt{4 - 8}}{2} = -1 \pm \sqrt{-1}$$

or

$$p = -1 + j , -1 -j$$

and

$$y = c_1e^{(-1 + j)x} + c_2e^{(-1 -j)x} \qquad (5.37)$$

Equation (5.37) may be written:

$$y = e^{-x} \left[c_1e^{jx} + c_2e^{-jx}\right]$$

or using the results (5.34) and (5.36):

$$y = e^{-x} \left[c_3 \cos x + c_4 \sin x\right] \qquad (5.38)$$

or

$$y = e^{-x} \left[A \sin (x + \emptyset)\right] \qquad (5.39)$$

Note also, that (5.36) could be written in the form

$$y = A \cos (x + \theta) , \text{ where } \theta = \emptyset - 90^{\circ}.$$

....

Particular Solution:

The particular solution, for our work here, will be obtained by the method of undetermined coefficients. This method consists of assuming a solution of the same general form as the input (forcing function), but with undetermined coefficients. Substitution of this assumed solution into the differential equation then enables us to evaluate these coefficients. The method of undetermined coefficients is applicable when the forcing function or input is a polynomial, terms of the form sin ax, cos ax, e^{ax}, or combinations of sums and products of these. The complete solution of the linear differential equation with constant coefficients is then given by (5.18) (i.e., the solution to the complementary equation (transient solution), plus the particular solution).

A few remarks are appropriate regarding the 2nd order linear differential equation with constant coefficients. Although the equation

is interesting in its own right, it is of particular value to us because it is a mathematical model for several problems of physical interest.

$$\left.\begin{array}{ll} a\,\dfrac{dy^2}{dx^2} + b\,\dfrac{dy}{dx} + cy = F(x) & \text{(mathematical model)} \\[2ex] m\,\dfrac{d^2x}{dt^2} + \beta\,\dfrac{dx}{dt} + Kx = F(t) & \text{(describes a mass spring damper system)} \\[2ex] L\,\dfrac{d^2Q}{dt^2} + R\,\dfrac{dQ}{dt} + \dfrac{Q}{C} = E(t) & \text{(describes a series LRC electrical circuit)} \end{array}\right\} \quad (5.40)$$

Equations (5.40) are all the same mathematically but are expressed in different notation. Different notations or symbols are employed to emphasize the physical parameters involved, or to force the solution to appear in a form that is easy to interpret. In fact, the similarity of these last two equations may suggest how one might design an electrical circuit to simulate the operation of a mechanical system.

Consider the equation:

$$a\,\frac{d^2y}{dx^2} + b\,\frac{dy}{dx} + cy = f(x) \qquad (5.41)$$

We now must solve for the special solution (particular solution) which results from a given input, $f(x)$. This particular solution can be found using various techniques, but we will consider only one, the method of undetermined coefficients. This method consists of assuming a solution form with unspecified constants (undetermined coefficients), and solving for the values of the constants which will satisfy the given differential equation. The method is best described by considering examples.

EXAMPLE:

$$\frac{d^2y}{dx^2} + 4\,\frac{dy}{dx} + 3y = 6 \qquad (5.42)$$

The input is a constant (trivial polynomial), so we assume a solution of form $y_p = K$. Obviously, $d^2K/dx^2 = 0$, and $dK/dx = 0$. Substituting,

$$0 + 4(0) + 3K = 6$$

$$y_p = K = 2$$

Therefore, $y_p = 2$ is a particular solution. We note that we can solve the equation:

$$\frac{d^2y}{dx^2} + 4\,\frac{dy}{dx} + 3y = 0$$

in operator form

$$(p^2 + 4p + 3)y = 0$$

or

$$p = -1, -3$$

and the transient solution is:

$$y_t = c_1 e^{-x} + c_2 e^{-3x}$$

The general solution of (5.42) may be written:

$$y = \underbrace{c_1 e^{-x} + c_2 e^{-3x}}_{\substack{\text{transient}\\ \text{solution}}} + \underbrace{2}_{\substack{\text{particular}\\ \text{(or steady}\\ \text{state)}\\ \text{solution}}}$$

....

EXAMPLE:

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = x^2 + 2x \quad (5.43)$$

Now the form of f(x) for (5.43) is a polynomial of 2nd degree, so we assume a particular solution for y of 2nd degree. (i.e., let $y_p = Ax^2 + Bx + C$.) Then,

$$\frac{dy_p}{dx} = 2Ax + B$$

and

$$\frac{d^2y_p}{dx^2} = 2A$$

Substituting into (5.43),

$$(2A) - 4(2Ax + B) + 3(Ax^2 + Bx + C) = x^2 + 2x$$

or

$$(3A)x^2 + (8A + 3B)x + (2A + 4B + 3C) = x^2 + 2x$$

Equating like powers of x,

x^2: $3A = 1$

$A = 1/3$

x: $8A + 3B = 2$

$3B = 2 - \frac{8}{3}$

$B = -2/9$

x^o: $2A + 4B + 3C = 0$

$3C = 8/9 - 2/3$

$C = 2/27$

Therefore,

$$y_p = 1/3\, x^2 - 2/9\, x + 2/27$$

The general solution of (5.43) is given by

$$y = c_1 e^{-x} + c_2 e^{-3x} + 1/3\, x^2 - 2/9\, x + 2/27$$

since the transient solution is the same as for (5.42). As a general rule, if the forcing function is a polynomial of degree n, assume a polynomial solution of degree n.

....

EXAMPLE:

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{2x} \quad (5.44)$$

The forcing function is e^{2x} so we assume a solution of the form

$$y = Ae^{2x}$$

$$\frac{d}{dx}(Ae^{2x}) = 2Ae^{2x}$$

$$\frac{d^2}{dx^2}(Ae^{2x}) = 4Ae^{2x}$$

Substituting in (5.44),

$$4Ae^{2x} + 4(2Ae^{2x}) + 3(Ae^{2x}) = e^{2x}$$

$$e^{2x}(4A + 8A + 3A) = e^{2x}$$

The coefficients on both sides of the equation must be the same. Therefore, $4A + 8A + 3A = 1$, or $15A = 1$, and $A = 1/15$. The particular solution of (5.44) then is $y_p = 1/15\, e^{2x}$. The transient solution is still the same as for (5.42). A final example will illustrate a pitfall sometimes encountered using this method.

....

EXAMPLE:

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{-x} \quad (5.45)$$

The forcing function is e^{-x}, so we assume a solution of the form $y = Ae^{-x}$. Then

$$\frac{d}{dx}(Ae^{-x}) = -Ae^{-x}$$

and

$$\frac{d^2}{dx^2}(Ae^{-x}) = Ae^{-x}$$

Substituting:

$$Ae^{-x} + 4(-Ae^{-x}) + 3(Ae^{-x}) = e^{-x}$$

$$(A - 4A + 3A)e^{-x} = e^{-x}$$

$$(0)e^{-x} = e^{-x}$$

Obviously, this is an incorrect statement. To find where we made our mistake, let's review our procedures.

....

To solve an equation of the form

$$(p + a)(p + b)y = e^{-ax}$$

we solve the homogeneous equation to get

$$(p + a)(p + b)y = 0$$

$$p = -a, -b$$

$$y_t = c_1e^{-ax} + c_2e^{-bx}$$

If we assume $y_p = Ae^{-ax}$ then

$$y = y_t + y_p = c_1e^{-ax} + c_2e^{-bx} + Ae^{-ax}$$

$$= (c_1 + A)e^{-ax} + c_2e^{-bx}$$

$$= c_3e^{-ax} + c_2e^{-bx}$$

$$= y_t$$

However, we have already seen that y_t is the solution only when the right side of the equation is zero, and will not solve the equation when we have a forcing function. Therefore, we assume a particular solution.

$$y_p = Axe^{-ax}$$

then

$$y = y_p + y_t = c_1e^{-ax} + c_2e^{-bx} + Axe^{-ax}$$

$$= (c_1 + Ax)e^{-ax} + c_2e^{-bx}$$

$$\neq y_t$$

Similarly, we could have the equation

$$(p + aj)(p - aj)y = \sin ax$$

with transient solution

$$y_t = c_1 \sin ax + c_2 \cos ax$$

If we assume $y_p = A \sin ax + B \cos ax$

then

$$y = y_t + y_p = (c_1 + A) \sin ax + (c_2 + B) \cos ax$$

$$= c_3 \sin ax + c_4 \cos ax$$

$$= y_t$$

Therefore, we assume

$$y_p = Ax \sin ax + Bx \cos ax$$

and

$$y = (c_1 + Ax) \sin ax + (c_2 + Bx) \cos ax$$

$$\neq y_t$$

Note the following, however, with the equation

$$(p + a - jb)(p + a + jb)y = \sin bx$$

$$y_t = e^{-ax}(c_1 \sin bx + c_2 \cos bx)$$

we can assume $y_p = B \sin bx + C \cos bx$

then

$$y = c_1 e^{-ax} \sin bx + c_2^{-ax} \cos bx + B \sin bx + C \cos bx$$

$$y = (c_1 e^{-ax} + B) \sin bx + (c_2 e^{-ax} + C) \cos bx$$

$$\neq y_t$$

Similarly, if

$$(p + a - jb)(p + a + jb)y = e^{-ax}$$

we could assume

$$y_p = Ae^{-ax}$$

In our example above (equation 5.45), a valid solution can be found by assuming $Y_p = Axe^{-x}$, then

$$\frac{d}{dx}(Axe^{-x}) = A(-xe^{-x} + e^{-x})$$

and

$$\frac{d^2}{dx}(Axe^{-x}) = A(xe^{-x} - 2e^{-x})$$

Substituting:

$$A(xe^{-x} - 2e^{-x}) + 4A(-xe^{-x} + e^{-x}) + 3(Axe^{-x}) = e^{-x}$$

$$(A - 4A + 3A)xe^{-x} + (-2A + 4A)e^{-x} = e^{-x}$$

$$(0)xe^{-x} + 2Ae^{-x} = e^{-x}$$

and

$$A = 1/2$$

Thus,

$$y_p = (1/2)xe^{-x}$$

is a particular solution of (5.45), and the general solution is given by:

$$y = c_1 e^{-x} + c_2 e^{-3x} + 1/2\ xe^{-x}$$

The key to successful application of the method of undetermined coefficients is to assume the proper form for a trial particular solution. Table 5.1 summarizes the results of this discussion.

(Table 5.1)

Differential equation: $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$

	$f(x)$ *	Assume y_p **
1.	β	A
2.	βx^n (n a positive integer)	$A_o x^n + A_1 x^{n-1} + \dots + A_{n-1}x + A_n$
3.	βe^{rx} (r either real or complex)	Ae^{rx}
4.	$\beta \cos kx$	$A \cos kx + B \sin kx$
5.	$\beta \sin kx$	
6.	$\beta x^n e^{rx} \cos kx$	$(A_o x^n + \dots + A_{n-1}x + A_n)e^{rx} \cos kx +$
7.	$\beta x^n e^{rx} \sin kx$	$+ (B_o x^n + \dots + B_{n-1}x + B_n)\, e^{rx} \sin kx$

* When $f(x)$ consists of a sum of several terms, the appropriate choice for y_p is the sum of y_p expressions corresponding to these terms individually.

** Whenever a term in any of the y_p's listed in this column duplicates a term already in the complementary function, all terms in that y_p must be multiplied by the lowest positive integral power of x sufficient to eliminate the duplication.

Solving for Constants of Integration:

As discussed in section 5.2, the number of arbitrary constants in the solution of our linear differential equation is equal to the order of the equation. These constants of integration may be determined by initial or boundary conditions. That is, we must know the physical state (position, velocity, etc.) of the system at some time in order to evaluate these constants. Many times these conditions are given at t = 0 (initial conditions).

It should be emphasized at this point, that the arbitrary constants of the solution are evaluated from the complete solution (transient plus steady state) of the equation.

We shall illustrate this method with an example.

EXAMPLE:

$$\ddot{x} + 4\dot{x} + 13x = 3 \qquad (5.46)$$

where the dot notation indicates derivatives with respect to time (i.e., $\dot{x} = dx/dt$, $\ddot{x} = d^2x/dt^2$). We will assume that the boundary conditions are $x(o) = 5$, and $\dot{x}(o) = 8$. The transient solution is given by

$$p^2 + 4p + 13 = 0$$

$$p = -2 \pm \sqrt{4 - 13} = -2 \pm j3$$

$$x_t = e^{-2t} (A \cos 3t + B \sin 3t)$$

We assume

$$x_p = D$$

$$\dot{x}_p = \frac{dx_p}{dt} = 0$$

$$\ddot{x}_p = 0$$

Substituting into (5.46), we get

$$D = 3/13$$

for a complete solution

$$x(t) = e^{-2t} (A \cos 3t + B \sin 3t) + 3/13$$

To solve for A and B, we will use the initial conditions specified above.

$$x(0) = 5 = A + 3/13$$

or

$$A = 62/13$$

Differentiating the complete solution, we get

$$\dot{x}(t) = e^{-2t} (3B \cos 3t - 3A \sin 3t)$$
$$-2e^{-2t} (A \cos 3t + B \sin 3t)$$

Substituting the second initial condition

$$\dot{x}(0) = 8 = 3B - 2A$$

$$B = \frac{228}{13}$$

Therefore, the complete solution to (5.46) with the given initial conditions is

$$x(t) = e^{-2t} \left[(62/13) \cos 3t + (228/13) \sin 3t\right] + 3/13$$

....

We have discussed the 1st and 2nd order differential equation in some detail. It is of great importance to note that many higher order systems quite naturally decompose into 1st and 2nd order systems. For example, the study of a 3rd order equation (or system) may be conducted by examining a 1st and a 2nd order system, a 4th order system analyzed by examining two 2nd order systems, etc...

All these cases are handled by solving the characteristic equation to get a transient solution and then obtaining the particular solution by any convenient method.

5.4 APPLICATIONS

Up to this point, we have considered differential equations in general and linear differential equations with constant coefficents in greater detail. We have developed methods for solving first and second order equations of the following type:

$$a\frac{dx}{dt} + bx = f(t) \qquad (5.47a)$$

$$a\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = f(t) \qquad (5.47b)$$

These two equations are mathematical models or forms. These same forms may be used to describe diverse physical systems. In this section we shall concentrate on the transient response of the systems under investigation, since this area is of primary interest in future studies.

<u>First Order Equation</u>:

Consider the following example:

EXAMPLE:

$$4\dot{x} + x = 3 \qquad (5.48)$$

where

$$\dot{x} = \frac{dx}{dt}$$

Physically, we can let x represent distance or displacement, and t represent time. To solve this equation, we find the transient solution by utilizing the homogeneous equation

$$4\dot{x} + x = 0$$

$$(4p + 1)x = 0$$

$$4p + 1 = 0$$

$$p = -1/4$$

Thus

$$x_t = ce^{-t/4}$$

The particular solution is found by assuming

$$x_p = A$$

$$\frac{dx_p}{dt} = 0$$

Substituting,

$$A = 3$$

or

$$x_p = 3$$

The complete solution is then

$$x = ce^{-t/4} + 3 \qquad (5.49)$$

....

The first term on the right of (5.49) represents the transient response of the physical system described by equation (5.48), and the second term represents the steady state response if the transient decays. A term useful in describing the physical effect of a negative exponential term is <u>time constant</u>

which is denoted by τ. We shall define τ as

$$\tau \triangleq \frac{-1}{p}$$

Thus, equation (5.49) could be rewritten as

$$x = ce^{-t/\tau} + 3 \qquad (5.50)$$

where $\tau = 4$.

Note the following:

1. We only discuss time constants if p is negative. If p is positive, the exponent of e is positive, and the transient solution will not decay.

2. If p is negative, τ is positive.

3. τ is the negative reciprocal of p, so that small numerical values of p give large numerical values of τ (and vice versa).

4. The value of τ is the time, in seconds, required for the displacement to decay to 1/e of its original displacement from equilibrium or steady value. To get a better understanding of this statement, let's look at (5.50).

$$x = ce^{-t/\tau} + 3 \qquad (5.50)$$

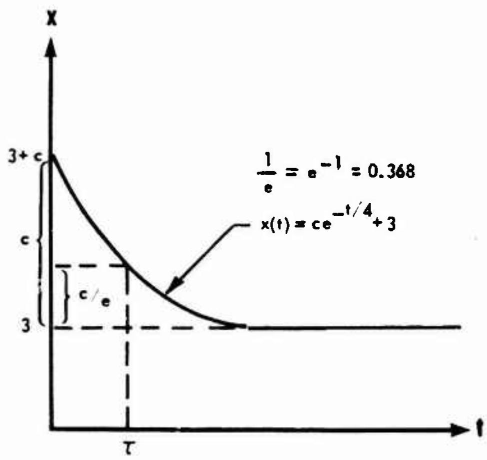
and let $t = \tau$. Then

$$x = ce^{-1} + 3$$

$$= c\left(\frac{1}{e}\right) + 3$$

Thus, when $t = \tau$, the exponential portion of the solution has decayed to 1/e of its original displacement (see figure 5.3).

FIGURE 5.3



Other measures of time are sometimes used to describe the decay of the exponential of a solution. If we let T_1 denote the time it takes for the transient to decay to one-half it's original amplitude, then

$$T_1 = 0.693\ \tau \qquad (5.51)$$

This relationship can be easily shown by investigating

$$x = c_1 e^{-at} + c_2 \qquad (5.52)$$

From our definition, $\tau = 1/a$. For the transient portion of (5.52), at $t = 0$, $x_t(0) = c_1$. We are looking for T_1, the value of t at which $x_t = 1/2\ x_t(0)$. Solving

$$x_t = c_1 e^{-at}$$

$$\frac{1}{2} c_1 = c_1 e^{-aT_1}$$

$$T_1 = \frac{-\ln 1/2}{a} = \frac{.693}{a} = .693\ \tau$$

Let's complete our solution of (5.48) by specifying a boundary condition and evaluating the arbitrary constant. Let x = 0 at t = 0.

$$x = ce^{-t/4} + 3$$

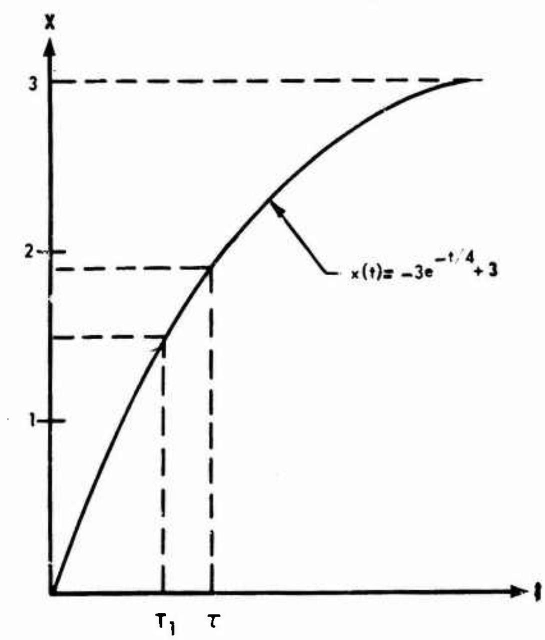
$$x(0) = 0 = c + 3$$

$$c = -3$$

Our complete solution for this boundary condition is

$$x = -3e^{-t/4} + 3$$

FIGURE 5.4



Second Order Equation:

Consider an equation of the form (5.47b). The characteristic equation (operator equation) can be written:

$$a p^2 + b p + c = 0 \qquad (5.53)$$

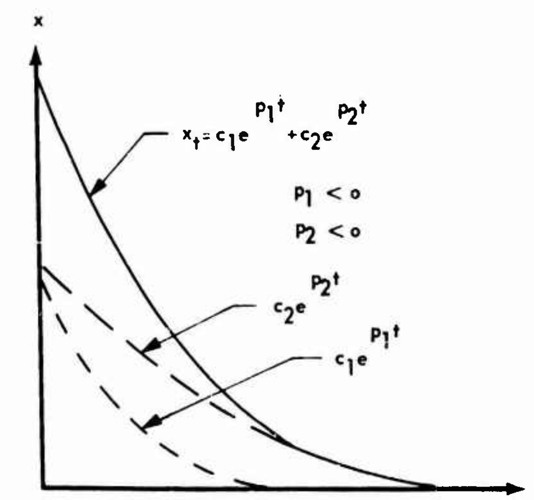
The roots of this quadratic equation determine the form of the transient solution as we have seen in section 5.3. We will now discuss physical implications of the algebraic property of the roots.

1. Roots real and unequal: When the roots are real and unequal, the transient solution has the form:

$$x_t = c_1 e^{p_1 t} + c_2 e^{p_2 t} \qquad (5.54)$$

Case 1: When p_1 and p_2 are both negative, the system decays and there will be a time constant associated with each exponential.

FIGURE 5.5



Case 2: When p_1 or p_2 (or both) is positive, the system will generally diverge.

FIGURE 5.6

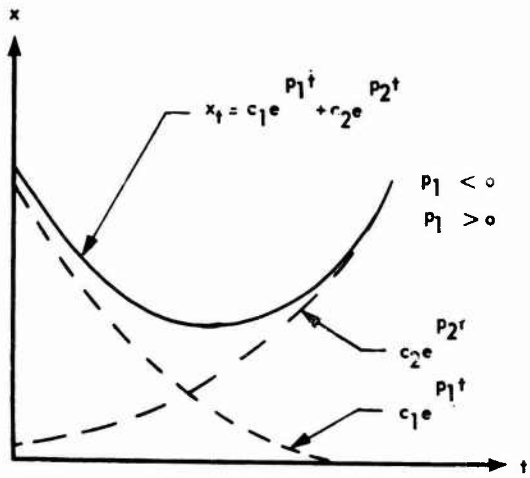
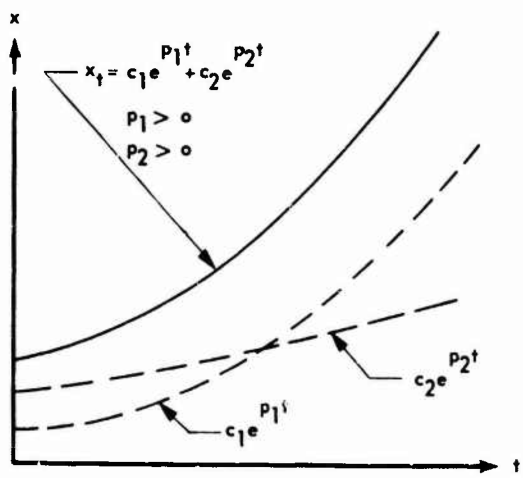


FIGURE 5.7



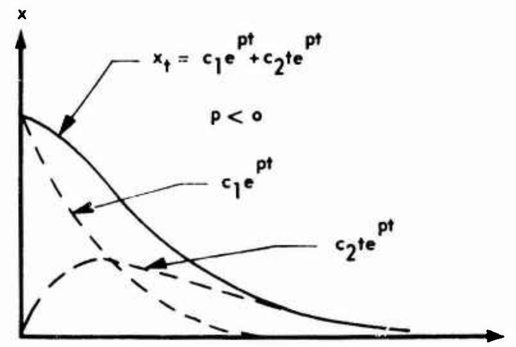
Case 3: Examples where p_1 or p_2 (or both) are zero, are usually not observed in practical cases.

2. Roots real and equal: When $p_1 = p_2$, the transient solution has the form

$$x_t = c_1e^{pt} + c_2te^{pt} \qquad (5.55)$$

Case 1: When p is negative, the system will usually decay. (If p is very small, the system may initially exhibit divergence.)

FIGURE 5.8



Case 2: When p is positive, the system will diverge.

3. Roots pure imaginary: When $p = \pm jk$, the transient solution has the form

$$x_t = c_1 \sin kt + c_2 \cos kt \qquad (5.56a)$$

or

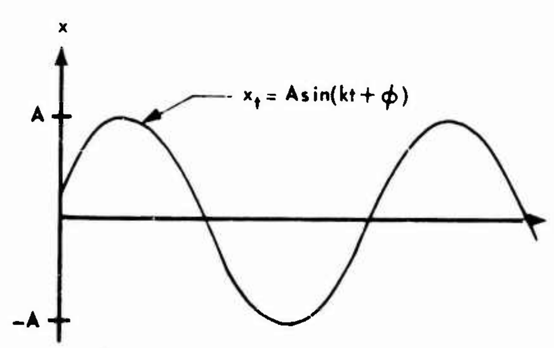
$$x_t = A \sin (kt + \phi) \qquad (5.56b)$$

or

$$x_t = A \cos (kt + \theta) \qquad (5.56c)$$

The system executes oscillations of constant amplitude with a frequency, k.

FIGURE 5.9



4. Roots complex conjugates: When the roots are given by $p = k_1 \pm jk_2$, the form of the transient solution is

$$x_t = e^{k_1 t} (c_1 \cos k_2 t + c_2 \sin k_2 t) \tag{5.57a}$$

or

$$x_t = A e^{k_1 t} \sin (k_2 t + \emptyset) \tag{5.57b}$$

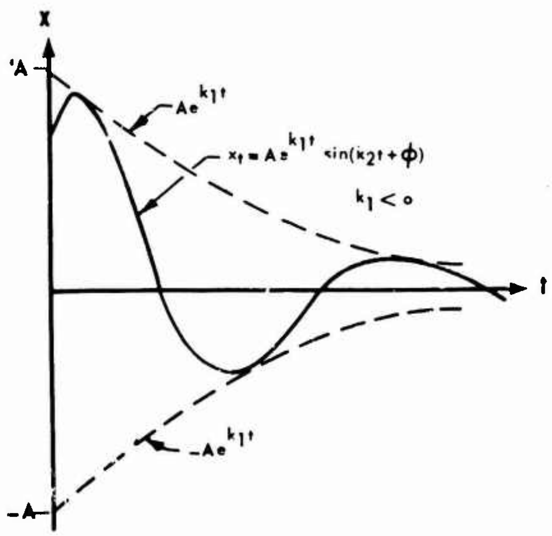
or

$$x_t = A e^{k_1 t} \cos (k_2 t + \theta) \tag{5.57c}$$

The system executes periodic oscillations contained in an envelope given by $x = \pm A e^{k_1 t}$

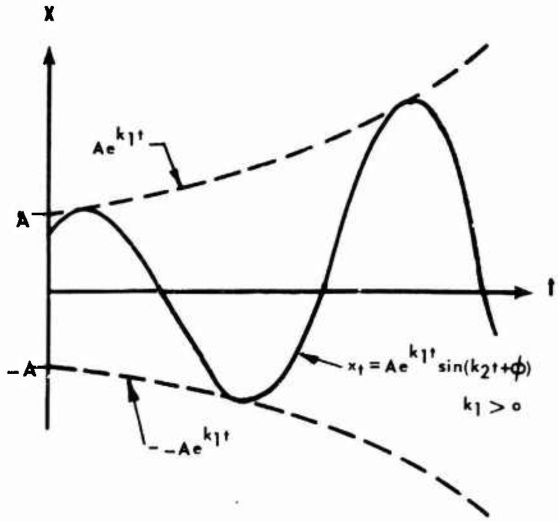
Case 1: When k_1 is negative, the system decays.

FIGURE 5.10



Case 2: When k_1 is positive, the system diverges.

FIGURE 5.11

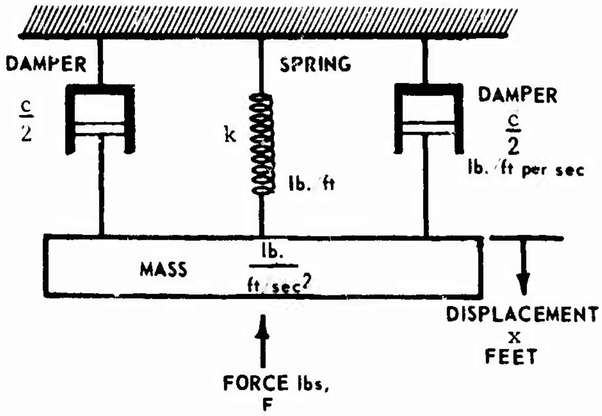


The discussion of transient solutions above reveals only part of the picture presented by equation (5.47b). We still have the input or forcing function to consider, i.e., f(x). In practice, a linear system that possesses a divergence (without input) may be changed to a damped system by carefully selecting or controlling input. Conversely, a nondivergent linear system with weak damping may be made divergent by certain types of inputs.

Second Order Linear Systems:

Consider the physical model shown in figure 5.12. The system consists of an object suspended by a spring, with a spring constant of k. The mass may move vertically and is subject to gravity, input and damping, with the total viscous damping constant equal to c.

FIGURE 5.12



The equation for this vibrating system is given by

$$m\ddot{x} + c\dot{x} + kx = f(t) \qquad (5.58)$$

The characteristic equation is given by

$$mp^2 + cp + k = 0 \qquad (5.59)$$

and the roots of this equation are

$$p_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$$= -\frac{c}{2m} \pm \sqrt{\frac{k}{m}}\sqrt{\frac{c^2}{4km} - 1} \qquad (5.60)$$

Let us, for simplicity, and for reasons that will be obvious later, define three constants

$$\zeta \triangleq \frac{c}{2\sqrt{mk}} \qquad (5.61)$$

The term ζ is called the damping ratio, and is a value which indicates the damping strength in the system.

$$\omega_n \triangleq \sqrt{\frac{k}{m}} \qquad (5.62)$$

ω_n is the undamped natural frequency of the system. This is the frequency at which the system would oscillate if there were no damping present.

$$\omega_d \triangleq \omega_n\sqrt{1 - \zeta^2} \qquad (5.63)$$

ω_d is the damped frequency of the system. It is the frequency at which the system oscillates when a damping ratio of ζ is present.

Substituting ζ and ω_n into (5.60) now gives

$$p_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2} \qquad (5.64)$$

With these roots, the transient solution becomes

$$x_t = c_1 e^{p_1 t} + c_2 e^{p_2 t}$$

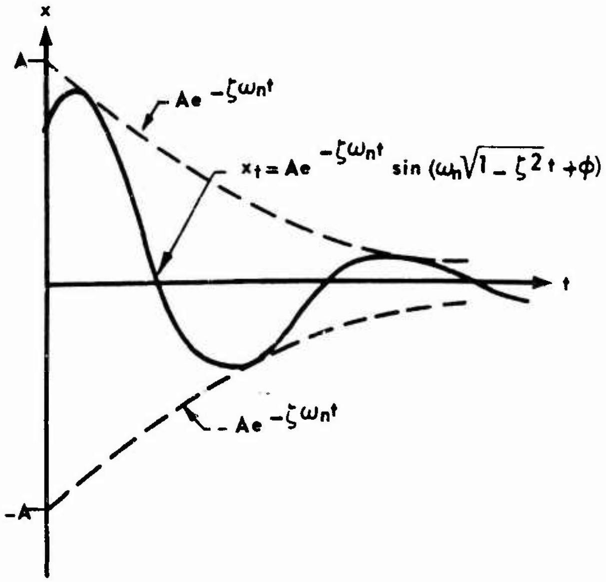
$$= e^{-\zeta\omega_n t}\left[c_1 \cos \omega_n\sqrt{1-\zeta^2}\, t + c_2 \sin \omega_n\sqrt{1-\zeta^2}\, t\right] \qquad (5.65a)$$

or

$$x_t = A e^{-\zeta\omega_n t} \sin(\omega_n\sqrt{1-\zeta^2}\, t + \emptyset) \qquad (5.65b)$$

Note that the solution will lie within an exponentially decreasing envelope which has a time constant of $1/(\zeta\omega_n)$. This damped oscillation is shown in figure 5.13.

FIGURE 5.13



If we rewrite equation (5.58) using the constants ω_n and ζ defined by equations (5.61) and (5.62) we have

$$\frac{1}{\omega_n^2}\ddot{x} + \frac{2\zeta}{\omega_n}\dot{x} + x = \frac{1}{k} f(t)$$

or

$$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = \frac{\omega_n^2}{k} f(t) = \frac{f(t)}{m} \tag{5.66}$$

Equation (5.66) is a form of (5.58) that is most useful in analyzing the behavior of any linear system.

A general second order physical system can be compared with the mass-spring-damper system. The equation defining the system was

$$m\ddot{x} + c\dot{x} + kx = f(t) \tag{5.58}$$

where we defined the parameters

$$\omega_n = \sqrt{\frac{k}{m}}, \text{ undamped natural frequency}$$

$$\zeta = \frac{c}{2\sqrt{mk}}, \text{ damping ratio}$$

From equation (5.64) we see that the numerical value of ζ is a powerful factor in determining the type of response exhibited by the system.

Let us now consider the physical problem and analyze the various conditions possible. The magnitude and sign of ζ, the damping ratio, determine the response properties of the system.

There are five distinct cases which are given names descriptive of the response associated with each case.

1. $\zeta = 0$, undamped
2. $0 < \zeta < 1$, underdamped
3. $\zeta = 1$, critically damped
4. $\zeta > 1$, overdamped
5. $\zeta < 0$, unstable

We shall now examine each case, making use of equation (5.64)

$$p_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} \tag{5.64}$$

....

Case 1: $\zeta = 0$, undamped: For this condition, the roots of the characteristic equation are

$$p_{1,2} = \pm j\omega_n \tag{5.67}$$

giving a transient solution of the form

$$x_t = c_1\cos\omega_n t + c_2\sin\omega_n t \tag{5.68a}$$

or

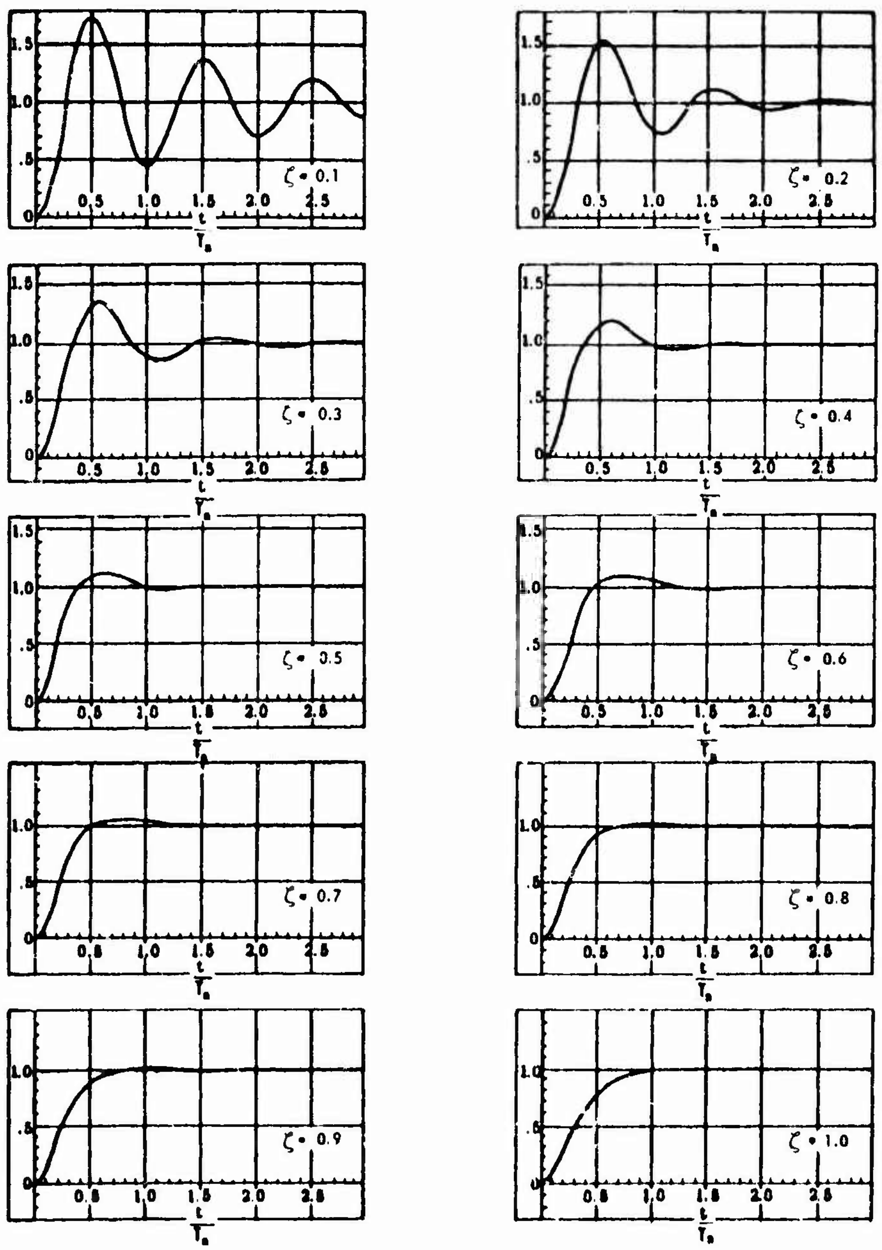
$$x_t = A\sin(\omega_n t + \emptyset) \tag{5.68b}$$

showing the system to have the transient response of an undamped sinusoidal oscillation with frequency ω_n. (Hence the designation of ω_n as the "undamped natural frequency.") Figure 5.9 shows an undamped system.

Case 2: $0 < \zeta < 1$, underdamped: For this case, p is given by equation (5.64) and the transient solution has the form

$$x_t = A e^{-\zeta \omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2}\, t + \phi) \tag{5.69}$$

FIGURE 5.14 *



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This solution shows that the system oscillates at the damped frequency, ω_d, and is bounded by an exponentially decreasing envelope with time constant $1/(\zeta \omega_n)$. Figure 5.14 shows the effect of increasing the damping ratio from 0.1 to 1.0.

....

Case 3: $\zeta = 1$, critically damped: For this condition, the roots of the characteristic equation are

$$p_{1,2} = -\omega_n \tag{5.70}$$

which gives a transient solution of the form

$$x_t = c_1 e^{-\omega_n t} + c_2 t e^{-\omega_n t} \tag{5.71}$$

This is called the critically damped case and generally will not overshoot. It should be noted, however, that large initial values of $\dot{x}$ can cause one overshoot. Figure 5.14 above shows a response when $\zeta = 1$.

....

Case 4: $\zeta > 1$, overdamped: In this case, the characteristic roots are

$$p_{1,2} = -\zeta\omega_n \pm \omega_n \sqrt{\zeta^2 - 1} \tag{5.72}$$

which shows that both roots are real and negative. This tells us that the system will have a transient which has an exponential decay without sinusoidal motion. The transient response is given by

$$x_t = c_1 e^{-\omega_n\left[\zeta - \sqrt{(\zeta^2 - 1)}\right]t} + c_2 e^{-\omega_n\left[\zeta + \sqrt{(\zeta^2 - 1)}\right]t} \tag{5.73}$$

This response can also be written as

$$x_t = c_1 e^{-t/\tau_1} + c_2 e^{-t/\tau_2} \tag{5.74}$$

where τ_1 and τ_2 are time constants for each exponential term.

This solution is the sum of two decreasing exponentials, one with time constant τ_1 and the other with time constant τ_2. The smaller the value of τ, the quicker the transient decays. Usually the larger the value of ζ, the larger τ_1 is compared to τ_2. For the case $\zeta >> 1$, τ_2 is small in comparison to τ_1 and can be neglected. The system then behaves like a first order system (i.e., the effect of mass can be neglected). This can be seen most readily from equation (5.74). Figure 5.5 shows an overdamped system.

....

Case 5: $\zeta < 0$, unstable: For this case, the roots of the characteristic equation are

$$p_{1,2} = -\zeta\omega_n \pm j\omega_n \sqrt{1 - \zeta^2} \tag{5.75}$$

These roots are the same as for the underdamped case, except that the exponential term in the transient solution shows an exponential increase with time.

$$x_t = e^{-\zeta\omega_n t}\left[c_1 \cos \omega_n\sqrt{1 - \zeta^2}\, t + c_2 \sin \omega_n\sqrt{1 - \zeta^2}\, t\right] \tag{5.76}$$

Whenever a term appearing in the transient solution grows with time (and especially an exponential growth), the system is generally unstable. This means that whenever the system is disturbed from equi-

librium, the disturbance will increase with time. Figure 5.11 shows an unstable system.

....

EXAMPLE:

Given

$$\ddot{x} + 4x = 0$$

from (5.66)

$$\zeta = 0$$

and

$$\omega_n = 2$$

The system is undamped with a solution

$$x_t = A \sin (2t + \phi)$$

where A and ϕ are determined by substituting the boundary conditions into the <u>complete</u> solution.

....

EXAMPLE:

Given

$$\ddot{x} + \dot{x} + x = 0$$

from (5.66)

$$\omega_n = 1$$

and

$$\zeta = 0.5$$

we also know from (5.63) that

$$\omega_d = \omega_n\sqrt{1 - \zeta^2} = 0.87$$

The system is underdamped with a solution

$$x_t = Ae^{-0.5t} \sin (0.87t + \phi)$$

....

EXAMPLE:

Given

$$\frac{\ddot{x}}{4} + \dot{x} + x = 0$$

We multiply by 4 to get the equation in the form of (5.66).

Then

$$\ddot{x} + 4\dot{x} + 4x = 0$$

and

$$\omega_n = 2$$

$$\zeta = 1$$

The system is critically damped and has a solution given by

$$x_t = c_1e^{-2t} + c_2te^{-2t}$$

....

EXAMPLE:

Given

$$\ddot{x} + 8\dot{x} + 4x = 0$$

we get

$$\omega_n = 2$$

and

$$\zeta = 2$$

The system is overdamped and has a solution

$$x_t = c_1c^{-7.45t} + c_2e^{-0.55t}$$

....

EXAMPLE:

Given

$$\ddot{x} - 2\dot{x} + 4 = 0$$

From (5.66)

$$\omega_n = 2$$

and

$\zeta = -0.5$

From (5.63)

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} = 2.2$$

The solution is unstable (negative damping) and has the form

$$x_t = Ae^t \sin(2.2t + \phi)$$

....

Damping: (see figure 5.14)

The best damping ratio for a system is determined by the intended use of the system. If a fast response is desired, and the size and number of overshoots is inconsequential, then we would use a small value of ζ. If it is essential that the system not overshoot, and we are not too concerned about response time, we could attempt to use a critically damped (or even an overdamped) system. The value $\zeta = 0.7$ is often referred to as the optimum damping ratio since it gives a small overshoot and a relatively quick response.

Analogous Second Order Linear Systems:

1. Mechanical system: The second order equation we have been working with represents the mass-spring-damper system of figure 5.12 and has a differential equation given by

$$m\ddot{x} + c\dot{x} + kx = f(t) \tag{5.77}$$

where

m = mass

c = damping coefficient

k = spring constant

and we defined

$$\omega_n = \sqrt{\frac{k}{m}} \tag{5.78a}$$

$$\zeta = \frac{c}{2\sqrt{mk}} \tag{5.78b}$$

and thus

$$2\zeta\omega_n = \frac{c}{m}$$

Equation (5.77) may then be rewritten,

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = f_1(t) \tag{5.79}$$

where

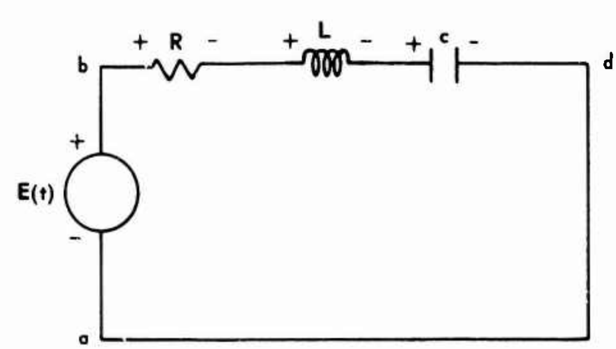
$$f_1(t) = \frac{f(t)}{m}$$

or

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = f_1(t) \tag{5.80}$$

2. Electrical System: The second order equation can also be applied to the series LRc circuit shown in figure 5.15.

FIGURE 5.15



where

L = inductance

R = resistance

c = capacitance

q = charge

i = current

Assume $q(0) = \dot{q}(0) = 0$, then Kirchhoff's voltage law gives

$$\Sigma V_{abd} = 0$$

or

$$E(t) - V_R - V_L - V_c = 0$$

$$E(t) - iR - L\frac{di}{dt} - \frac{1}{c}\int_0^t i\,dt = 0$$

Since

$$i = \frac{dq}{dt}$$

$$E(t) = L\ddot{q} + R\dot{q} + \frac{q}{c} \qquad (5.81)$$

We now define

$$\omega_n = \sqrt{\frac{1}{Lc}}$$

$$\zeta = \frac{R}{2\sqrt{L/c}}$$

$$2\zeta\omega_n = \frac{R}{L}$$

Using these parameters, equation (5.81) can be written

$$\ddot{q} + 2\zeta\omega_n\dot{q} + \omega_n^2 q = E_1(t) = \frac{E(t)}{L} \qquad (5.82)$$

3. Servomechanisms: For control systems work, the second order equation is

$$I\ddot{\theta}_o + f\dot{\theta}_o + \mu\theta_o = \mu\theta_i \qquad (5.83)$$

where

I = inertia

f = friction

μ = gain

θ_i = input

θ_o = output

Rearranging (5.83) we have

$$\ddot{\theta}_o + \frac{f}{I}\dot{\theta}_o + \frac{\mu}{I}\theta_o = \frac{\mu}{I}\theta_i$$
$$= \omega_n^2\theta_i \qquad (5.84a)$$

or

$$\ddot{\theta}_o + 2\zeta\omega_n\dot{\theta}_o + \omega_n^2\theta_o = \frac{\mu}{I}\theta_i \qquad (5.84b)$$

where we define

$$\omega_n = \sqrt{\frac{\mu}{I}}$$

$$\zeta = \frac{f}{2\sqrt{\mu I}}$$

Thus, we see that we can generally write any second order differential equation in the form.

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = f(t) \qquad (5.85)$$

where each term has the same qualitative significance, but different physical significance.

5.5 LAPLACE TRANSFORMS

We have developed a technique for solving linear differential equations with constant coefficients,

with and without inputs or forcing functions. We have admitted that our method has limitations. It is suited for differential equations with inputs of only certain forms. Further, the solution procedure requires that the student stay constantly alert for special cases that require careful handling. We accepted these "bookkeeping" chores because our solution procedure had the remarkable property of changing or "transforming" a problem of integration for a problem in algebra. (i.e., solving a quadratic equation in the case of second order differential equations.) This fortuitous turn of events was accomplished by making an assumption involving the number e, as follows:

Given:

$$a\ddot{x} + b\dot{x} + cx = 0 \qquad (5.86)$$

Assume:

$$x = e^{\lambda t} \qquad (5.87)$$

Substituting:

$$a\lambda^2 e^{\lambda t} + b\lambda e^{\lambda t} + ce^{\lambda t} = 0 \qquad (5.88a)$$

and

$$e^{\lambda t}(a\lambda^2 + b\lambda + c) = 0 \qquad (5.88b)$$

led us to assert that (5.87) would produce a solution if λ were a root of the characteristic equation

$$a\lambda^2 + b\lambda + c = 0 \qquad (5.88c)$$

We then introduced an operator, $p = d/dt$ and noted a short cut (bookkeeping coincidence) to writing the characteristic equation (5.88c) as

$$ap^2 + bp + c = 0 \qquad (5.89)$$

which we then solved for p to give solutions of the form

$$x = c_1e^{p_1t} + c_2e^{p_2t} \qquad (5.90)$$

Of course, the great shortcoming of this method was that it did not provide a solution to an equation of the form:

$$a\ddot{x} + b\dot{x} + cx = f(t) \qquad (5.91)$$

It only worked for the homogeneous equation. Still, we were able to patch together a solution by obtaining a particular solution (using still another technique) and adding it to the "transient" solution of the homogeneous equation. It should be appreciated that the method of undetermined coefficients also provided a solution by algebraic manipulation.

Suppose we were adventurous enough to inquire further. We ask, "Does there exist a technique which would exchange (transform) the whole differential equation, including the input, into an algebra problem?" The answer is a qualified "Yes." Fortunately, the "Yes" answer applies to the types of equations with which we have been working.

In equation (5.91), x is a function of t. To emphasize this we reqrite (5.91) as:

$$a\ddot{x}(t) + b\dot{x}(t) + cx(t) = f(t) \qquad (5.92)$$

Suppose we multiply each term of (5.92) by $e^{\lambda t}$, giving us:

$$a\ddot{x}(t)e^{\lambda t} + b\dot{x}(t)e^{\lambda t} + cx(t)e^{\lambda t} = f(t)e^{\lambda t} \qquad (5.93)$$

Now, a most remarkable feature begins to emerge. It so happens that (5.93) can be integrated term by term on both sides of the equation to produce an algebraic expression in λ. The algebraic expression can then be manipulated to eventually obtain the solution of (5.93).

The preceding statements have omitted many details, but express the method of solution we now seek to develop. Our new "fudge factor," $e^{\lambda t}$, should be distinguished from the previous technique for solving the homogeneous equation, so we shall replace the λ by the term, $-s$. The reason for the minus sign will become apparent later. If we are to integrate the terms of (5.93), we shall need limits of integration. In most physical problems we are interested in events that take place subsequent to a given starting time which we shall call $t = 0$. Since we are unsure of the duration of significant events, we shall sum up the composite of effects from time $t = 0$ to time $t = \infty$ (that should cover the field). So now equation (5.93) becomes

$$\int_0^\infty a\ddot{x}(t)\, e^{-st}\, dt + \int_0^\infty b\, \dot{x}(t)\, e^{-st}\, dt + \int_0^\infty c\, x(t)\, e^{-st}\, dt = \int_0^\infty f(t)\, e^{-st}\, dt \tag{5.94}$$

Equation (5.94) is called the Laplace Transform of equation (5.92).

There is one small problem. How do we integrate these terms? We now focus our attention upon this problem.

<u>Finding the Laplace Transform of a Differential Equation:</u>

We now attempt to find the integrals of the terms of the differential equation (5.94). The big unanswered question posed by (5.94) is, "What is x(t)?" (i.e., x(t) is an unknown.) Thus,

$$\int_0^\infty x(t)\, e^{-st}\, dt = L\left\{x(t)\right\} = X(s) \tag{5.95}$$

X(s) must, for the present, remain an unknown. (Remember that λ was carried along as an unknown until the characteristic equation evolved, at which time we solved for λ explicitly.) Since (5.95) transforms x(t) into a function of the variable, s, we shall say:

$$\int_0^\infty c\, x(t)\, e^{-st}\, dt = c\int_0^\infty x(t)\, e^{-st}\, dt = c\, X(s) \tag{5.96}$$

and be content to carry along X(s) until such time that we can solve for it.

Now consider the second term, $b\,\dot{x}(t)$. We want to find:

$$\int_0^\infty b\, \dot{x}(t)\, e^{-st}\, dt = b\int_0^\infty \dot{x}(t)\, e^{-st}\, dt \tag{5.97}$$

To solve (5.97) we call upon a useful tool known as integration by parts.

Recall:

$$d(uv) = udv + vdu$$

$$udv = d(uv) - vdu$$

$$\int udv = \int d(uv) - \int vdu$$

$$\int udv = uv - \int vdu$$

and if the integration is over limits,

$$\int_a^b udv = uv\Big]_a^b - \int_a^b vdu \tag{5.98}$$

Applying this tool to equation (5.97) we let

$$u = e^{-st}$$

and

$$dv = \dot{x}(t)dt$$

then

$$du = -se^{-st}dt$$

and

$$v = x(t)$$

Putting these values into (5.98) and integrating from t = 0 to t = ∞,

$$\int_0^\infty \dot{x}(t)e^{-st}dt = x(t)e^{-st}\Big]_0^\infty - \int_0^\infty x(t)\left[-se^{-st}dt\right]$$

$$= x(t)e^{-st}\Big]_\infty^0 + s\int_0^\infty x(t)e^{-st}dt$$

$$= x(t)e^{-st}\Big]_0^\infty + sX(s) \qquad (5.99)$$

Now,

$$x(t)e^{-st}\Big]_0^\infty = \lim_{t\to\infty} x(t)e^{-st} - x(0) \qquad (5.100)$$

and we shall assume that the term e^{-st} "dominates" the term x(t) as $t\to\infty$. Thus, $\lim_{t\to\infty} x(t)e^{-st} = 0$, and equation (5.99) becomes

$$\int_0^\infty \dot{x}(t)e^{-st}dt = 0 - x(0) + sX(s)$$

$$= {}_{\cdot}(s) - x(0) \qquad (5.101)$$

Equations (5.96) and (5.101) can be abbreviated by using the letter L to signify Laplace Transformations.

$$L\{x(t)\} = X(s)$$

$$L\{cx(t)\} = cX(s) \qquad (5.102a)$$

$$L\{\dot{x}(t)\} = sX(s) - x(0)$$

$$L\{b\dot{x}(t)\} = b\left[sX(s) - x(0)\right] \qquad (5.102b)$$

Equation (5.102b) can be extended to higher order derivatives. Such an extension gives

$$L\{a\ddot{x}(t)\} = a\left[s^2X(s) - sx(0) - \dot{x}(0)\right] \qquad (5.103)$$

Returning to equation (5.94), we note that we have found the Laplace Transforms of all the terms excepting the forcing function. To solve this transform, the forcing function must be specified. We shall consider a few typical input functions and illustrate, by example, the technique for finding the Laplace Transform.

EXAMPLE:

Let

$$f(t) = A, \text{ where } A = \text{constant}$$

Then

$$L\{A\} = \int_0^\infty Ae^{-st}dt$$

$$= \frac{A}{-s}\int_0^\infty e^{-st}(-sdt)$$

$$= -\frac{A}{s}\left[e^{-st}\right]_0^\infty$$

or

$$L\ \{A\} = \frac{A}{s} \tag{5.104}$$

....

EXAMPLE:

Let

$$f(t) = t$$

Then

$$L\ \{t\} = \int_0^{\infty} te^{-st}dt$$

To integrate by parts, we let

$$u = t$$

$$dv = e^{-st}dt$$

Then

$$du = dt$$

$$v = -\frac{1}{s}e^{-st}$$

Substituting into (5.98)

$$\int_0^{\infty} te^{-st}dt = \left.\frac{-te^{-st}}{s}\right]_0^{\infty} + \frac{1}{s}\int_0^{\infty} e^{-st}dt$$

$$= 0 - \frac{1}{s^2}\left[e^{-st}\right]_0^{\infty}$$

$$= 0 + \frac{1}{s^2}$$

or

$$L\ \{t\} = \frac{1}{s^2} \tag{5.105}$$

....

EXAMPLE:

Let

$$f(t) = e^{2t}$$

Then

$$L\ \left\{e^{2t}\right\} = \int_0^{\infty} e^{2t}e^{-st}dt$$

$$= \int_0^{\infty} e^{(2-s)t}dt$$

$$= \frac{1}{s-2}$$

or

$$L\ \left\{e^{2t}\right\} = \frac{1}{s-2} \tag{5.106}$$

....

EXAMPLE:

Let

$$f(t) = \sin t$$

Then

$$L\ \{\sin t\} = \int_0^{\infty} \sin t\ e^{-st}dt$$

Integrate by parts, letting

$$u = \sin t$$

$$dv = e^{-st}dt$$

Then

$$du = \cos t\ dt$$

$$v = -\frac{1}{s}e^{-st}$$

Substituting into (5.98)

$$\int_0^{\infty} \sin t\ e^{-st}dt = \left[\frac{-(\sin t)(e^{-st})}{s}\right]_0^{\infty}$$

$$+ \frac{1}{s}\int_0^{\infty} \cos t\ e^{-st}dt$$

or

$$\int_0^\infty \sin t \, e^{-st} dt = 0 + \frac{1}{s} \int_0^\infty \cos t \, e^{-st} dt \qquad (5.107)$$

The expression $\cos t \, e^{-st}dt$ can also be integrated by parts, letting

$$u = \cos t$$

$$dv = e^{-st}dt$$

and

$$du = -\sin t \, dt$$

$$v = -\frac{1}{s} e^{-st}$$

giving

$$\int_0^\infty \cos t \, e^{-st} dt = \left[\frac{-(\cos t)(e^{-st})}{s} \right]_0^\infty - \frac{1}{s} \int_0^\infty \sin t \, e^{-st} dt$$

or

$$\int_0^\infty \cos t \, e^{-st} dt = \frac{1}{s} - \frac{1}{s} L \{\sin t\} \qquad (5.108)$$

Substituting (5.108) into (5.107) gives

$$L\{\sin t\} = 0 + \frac{1}{s}\left[\frac{1}{s} - \frac{1}{s} L\{\sin t\}\right]$$

$$= \frac{1}{s^2} - \frac{1}{s^2} L \{\sin t\}$$

which can be manipulated to give

$$L \{\sin t\} = \frac{1}{s^2 + 1} \qquad (5.109)$$

....

The Laplace Transforms of more complicated functions may be quite tedious to derive, but the procedure is similar to that above. Fortunately, it is not necessary to derive Laplace Transforms each time we use them. Extensive tables of Transforms exist in most advanced mathematics and control system textbooks.

We originally asserted that the Laplace Transform was going to assist in the solution of a differential equation. The technique is best described by an example.

EXAMPLE:

Given

$$\ddot{x} + 4\dot{x} + 4x = 4e^{2t}$$

with conditions $x(0) = 1$, $\dot{x}(0) = -4$. Taking the Laplace Transform of the equation gives:

$$s^2X(s) - sx(0) - \dot{x}(0) + 4\left[sX(s) - x(0)\right] + 4X(s) = \frac{4}{s-2}$$

or

$$\left[s^2 + 4s + 4\right] X(s) + \left[-s + 4 - 4\right] = \frac{4}{s-2}$$

Solving for X(s),

$$X(s) = \frac{s^2 - 2s + 4}{(s+2)^2 (s-2)} \qquad (5.110)$$

....

In order to continue with our solution, it is necessary that we discuss partial fraction expansion.

<u>Partial Fractions</u>:

The method of partial fractions enables us to separate a complicated rational fraction into

a sum of simpler fractions. Suppose we are given a fraction of two polynomials in a variable, s. Suppose the fraction is proper (the degree of the numerator is less than the degree of the denominator). If it is not proper we make it proper by dividing the fraction and then consider the remainder expression. There occur several cases:

Case 1: Distinct Linear Factors: To each linear factor such as $(as + b)$, occurring once in the denominator, there corresponds a single partial fraction of the form, $A/(as + b)$, where A is a constant to be determined.

$$\text{Eg: } \frac{7s - 4}{s(s - 1)(s + 2)} = \frac{A}{s} + \frac{B}{s - 1} + \frac{C}{s + 2} \tag{5.111}$$

Case 2: Repeated Linear Factors: To each linear factor, $(as + b)$, occurring n times in the denominator there corresponds a set of n partial fractions.

$$\text{Eg: } \frac{s^2 - 9s + 17}{(s - 2)^2(s + 1)} = \frac{A}{(s + 1)} + \frac{B}{(s - 2)} + \frac{C}{(s - 2)^2} \tag{5.112}$$

(where A, B and C are constants to be determined)

Case 3: Distinct Quadratic Factors: To each irreducible quadratic factor, $as^2 + bs + c$, occurring once in the denominator, there corresponds a single partial fraction of the form, $(As + B)/(as^2 + bs + c)$, where A and B are constants to be determined.

$$\text{Eg: } \frac{3s^2 + 5s + 8}{(s + 2)(s^2 + 1)} = \frac{A}{s + 2} + \frac{Bs + C}{s^2 + 1} \tag{5.113}$$

Case 4: Repeated Quadratic Factors: To each irreducible quadratic factor, $as^2 + bs + c$, occurring n times in the denominator, there corresponds a set of n partial fractions.

$$\text{Eg: } \frac{10s^2 + s + 36}{(s - 4)(s^2 + 4)^2} = \frac{A}{s - 4} + \frac{Bs + C}{s^2 + 4} + \frac{Es + F}{(s^2 + 4)^2} \tag{5.114}$$

(where A, B, C, E, F are constants to be determined)

The solution technique for finding the constants will be illustrated by solving (5.114). Start by finding the common denominator on the right side of (5.114).

$$\frac{10s^2 + s + 36}{(s - 4)(s^2 + 4)^2} = \frac{A(s^2 + 4)^2 + (Bs + C)(s - 4)(s^2 + 4) + (Es + F)(s - 4)}{(s - 4)(s^2 + 4)^2} \tag{5.114a}$$

Then

$$10s^2 + s + 36 = A(s^2 + 4)^2 + (Bs + C) \cdot (s^2 + 4)(s - 4) + (Es + F)(s - 4) \tag{5.115}$$

and without justifying the statement we shall assert that (5.115) must hold for all values of s.

5.37

a. Suppose $s = 4$, then (5.115) becomes

$$(10)(16) + 4 + 36 = 400A$$

and

$$A = 1/2$$

....

b. Suppose $s = 2j$, then (5.115) becomes

$$-40 + 2j + 36 = -4E + 2jF - 8jE - 4F$$

$$-4 + 2j = -4(E + F) + 2j(F - 4E)$$

The real and imaginary parts must be equal to their counterparts on the opposite side of the equal sign, thus,

$$(E + F) = 1$$

and

$$F - 4E = 1$$

or

$$E = 0$$

and

$$F = 1$$

....

c. Now let $s = 0$, then (5.115) becomes

$$36 = 16A - 16(+C) - 4F$$

and from above

$$A = 1/2, \quad F = 1$$

hence

$$36 = 8 - 16C - 4$$

and

$$C = -2$$

....

d. Let $s = 1$, (5.115) becomes

$$47 = 25\left(\tfrac{1}{2}\right) + (B - 2)(-15) - 3$$

$$94 = 25 - 30B + 60 - 6,$$

or

$$B = -1/2$$

....

Then (5.114a) may be written:

$$\frac{10s^2 + s + 36}{(s - 4)(s^2 + 4)^2} = \frac{1}{2}\left(\frac{1}{s - 4}\right) - \frac{1}{2}\left(\frac{s + 4}{s^2 + 4}\right) + \frac{1}{(s^2 + 4)^2} \quad (5.116)$$

Let's continue with our attempt to solve the differential equation

$$\ddot{x} + 4\dot{x} + 4x = 4e^{2t}$$

We have Transformed the equation (and substituted initial conditions) to get

$$X(s) = \frac{s^2 - 2s + 4}{(s - 2)(s + 2)^2} \quad (5.110)$$

We now expand by partial fractions

$$\frac{s^2 - 2s + 4}{(s - 2)(s + 2)^2} = \frac{A}{s - 2} + \frac{B}{s + 2} + \frac{C}{(s + 2)^2} \quad (5.117)$$

Taking the common denominator, and setting numerators equal

$$s^2 - 2s + 4 = A(s + 2)^2 + B(s + 2)\cdot(s - 2) + C(s - 2) \quad (5.118)$$

We can now substitute different values of s into this equation and solve for the constants. An alternate method of solving for these constants exists, however, and we will demonstrate this new approach. If we multiply out the right side of (5.118) we get

$$s^2 - 2s + 4 = As^2 + 4As + 4A + Bs^2 - 4B + Cs - 2C$$

$$= (A + B)s^2 + (4A + C)s + (4A - 4B - 2C)$$

Now, the coefficients of like powers of s on both sides of the equation must be equal (i.e., the coefficient of s^2 on the left side equals the coefficient of s^2 on the right side, etc.) Equating gives

$$s^2 : \quad 1 = A + B$$

$$s^1 : \quad -2 = 4A + C$$

$$s^0 : \quad 4 = 4A - 4B - 2C$$

Solving, we get

$A = 1/4$

$B = 3/4$

$C = -3$

Substituting into (5.117), we get

$$X(s) = \frac{1}{4}\left(\frac{1}{s-2}\right) + \frac{3}{4}\left(\frac{1}{s+2}\right) - 3\left(\frac{1}{s+2}\right)^2 \qquad (5.119)$$

In order to complete our solution, we must convert (transform) back into the time domain. The operation which converts a function X(s) back to a function of time is called the inverse Laplace transformation.

$$L^{-1}\left\{L\{x(t)\}\right\} = L^{-1}\{X(s)\} = x(t) \qquad (5.120)$$

The inverse Laplace transformation can be solved directly

$$x(t) = \frac{1}{2\pi j}\int_{C-j\infty}^{C+j\infty} X(s)e^{st}ds \qquad (5.121)$$

(where C is an arbitrary constant)

This integral (5.121) is hardly ever used because the Laplace Transform is unique and therefore generally X(s) can be recognized as the Laplace Transform of some known x(t). In practice, tables of transform pairs as found in most mathematics texts will suffice to find the inverse of X(s).

Using a suitable transform table, the inverse of (5.119) can easily be found to give us a solution

$$x(t) = \frac{1}{4}e^{2t} + \frac{3}{4}e^{-2t} - 3te^{-2t} \qquad (5.122)$$

<u>Properties of Laplace Transforms</u>:

The Laplace transform of some f(t), a function of time, is defined as

$$L\{f(t)\} = F(s) = \int_0^\infty f(t)e^{-st}dt \qquad (5.123)$$

where

$$s = \sigma + j\omega \text{ (a complex number)}$$

5.39

This transform is defined only when[3]

1. f(t) is piecewise continuous
2. The integral (5.123) converges
3. $t \geq 0$

The strength of the Laplace Transform is that it converts linear differential equations with constant coefficients into algebraic equations in the s-domain. Algebraic manipulation of the resulting equation will produce an explicit solution for the variables in the s-domain. All that remains to do is to take the inverse transform of the explicit solutions to return to the time domain.

There are several important properties of the Laplace Transform which should be included in this discussion.

In the general case it can be shown that

$$L\left\{\frac{d^n f(t)}{dt^n}\right\} = s^n F(s) - \left[s^{n-1} f(o) + s^{n-2}\frac{df(o)}{dt} + \ldots + \frac{d^{n-1} f(o)}{dt^{n-1}}\right] \qquad (5.124)$$

It is obvious that for quiescent systems (i.e., initial conditions zero)

$$L\left\{\frac{d^n ft}{dt^n}\right\} = s^n F(s) \qquad (5.125)$$

This result enables us to write down transfer functions by inspection.

Another significant transform is that of an indefinite integral.

[3] See for example, C. R. Wylie, Chapter 7.

In the general case;

$$L\left\{\iiint\cdots f(t)dt^n\right\} = \frac{F(s)}{s^n} + \frac{\left.\int f(t)dt\right]_{t=0+}}{s^n} + \frac{\left.\iint f(t)dt\right]_{t=0+}}{s^{n-1}} + \ldots \qquad (5.126)$$

Equation (5.126) allows us to transform Integro-differential equations such as those arising in electrical engineering.

A third useful property of the Laplace transform arises if we consider the Laplace Transform of the product of some exponential and any other function of time.

$$L\left\{e^{-at} f(t)\right\} = \int_0^\infty \left[e^{-at} f(t)e^{-st}\right] dt$$

$$= \int_0^\infty f(t)\, e^{-(s+a)t}\, dt \qquad (5.127)$$

It is apparent that this is the same form as the transform of f(t), except that the transformed independent variable is (s + a) rather than s. We conclude therefore that

$$L\left\{e^{-at} f(t)\right\} = L\left.\{f(t)\}\right]_{(s \to s+a)}$$

$$= F(s+a) \qquad (5.128)$$

It is important to note at this point, that the transform of the product of two functions of time is not equal to the product of the individual transforms. In symbolic form,

$$L\{f(t)\,g(t)\} \neq F(s)\,G(s) \quad (5.129)$$

The $L\{f(t)\,g(t)\}$ can be solved for directly by the definition of the Laplace Transform.

The last property we will consider is the Laplace transform of a pure time delay. A pure time delay of the function f(t) can be represented mathematically as

$$f(t - a)\,u(t - a) \quad (5.130)$$

where a is the length of delay and u(t - a) is the unit step defined as

$$u(t - a) = \begin{cases} 1, & (t - a) > 0 \\ 0, & (t - a) < 0 \end{cases}$$

For such a time delay

$$L\{f(t - a)\,u(t - a)\} = e^{-as} L\{f(t)\} \quad (5.131)$$

We shall now demonstrate the usefulness of the Laplace Transform by solving several example problems.

EXAMPLE:

Solve the given equation for x(t),

$$\dot{x} + 2x = 1 \quad (5.132)$$

when x(o) = 1.

By Laplace

$$L\{\dot{x}\} = sX(s) - x(o)$$

$$L\{2x\} = 2X(s)$$

$$L\{1\} = \frac{1}{s}$$

Thus

$$(s + 2)\,X(s) = \frac{1}{s} + x(o)$$

$$X(s) = \frac{1 + sx(o)}{s(s + 2)}$$

$$= \frac{A}{s} + \frac{B}{s + 2}$$

Solving,

$$A = 1/2$$

and

$$B = x(o) - 1/2$$

$$X(s) = \frac{1/2}{s} + \frac{x(o) - 1/2}{s + 2}$$

Inverse transforming gives

$$x(t) = 1/2 + \left[x(o) - 1/2\right] e^{-2t} \quad (5.132a)$$

The initial condition, x(o), was purposely carried along to show that the Laplace technique will yield the general solution of the differential equation with arbitrary constants. We could have substituted the value for x(o) when we transformed $\dot{x}$. Substituting our initial condition in (5.132a) gives

$$x(t) = 1/2 + 1/2\,e^{-2t} \quad (5.132b)$$

....

EXAMPLE:

Given

$$\dot{x} + 2x = \sin t\,,\quad x(o) = 5 \quad (5.133)$$

solve for x(t).

Taking the transform of (5.133)

$$sX(s) - x(o) + 2X(s) = \frac{1}{s^2 + 1}$$

and

$$X(s) = \frac{1}{(s^2 + 1)(s + 2)} + \frac{5}{s + 2} \tag{5.134}$$

Expanding the first term on the right side of the equation gives

$$\frac{1}{(s^2 + 1)(s + 2)} = \frac{As + B}{s^2 + 1} + \frac{C}{s + 2}$$

Taking the common denominator, and equating numerators gives

$$1 = (As + B)(s + 2) + C(s^2 + 1)$$

Substituting values of s leads to

$A = -1/5$

$B = 2/5$

$C = 1/5$

and substituting back into (5.134) gives

$$X(s) = \frac{-1/5\ s}{s^2 + 1} + \frac{2/5}{s^2 + 1} + \frac{1/5}{s + 2} + \frac{5}{s + 2}$$

Inverse transforming gives our solution

$$x(t) = -\frac{1}{5} \cos t + \frac{2}{5} \sin t + 5\frac{1}{5} e^{-2t} \tag{5.135}$$

....

EXAMPLE:

Given

$$\ddot{x} + 5\dot{x} + 6x = 3e^{-3t}, \quad x(o) = \dot{x}(o) = 1 \tag{5.136}$$

solve for x(t).

Taking the transform of (5.136)

$$s^2X(s) - sx(o) - \dot{x}(o) + 5sX(s) - 5x(o) + 6X(s) = \frac{3}{s + 3}$$

or

$$X(s) = \frac{s^2 + 9s + 21}{(s + 3)(s^2 + 5s + 6)}$$

Factoring the denominator,

$$X(s) = \frac{s^2 + 9s + 21}{(s + 3)(s + 2)(s + 3)} \tag{5.137a}$$

$$= \frac{s^2 + 9s + 21}{(s + 3)^2(s + 2)} \tag{5.137b}$$

$$= \frac{A}{s + 3} + \frac{B}{(s + 3)^2} + \frac{C}{s + 2} \tag{5.137c}$$

Finding the common denominator of (5.137c), and setting the resultant numerator equal to the numerator of (5.137b)

$$s^2 + 9s + 21 = A(s + 3)(s + 2) + B(s + 2) + C(s + 3)^2$$

which can be solved easily for

$A = -6$

$B = -3$

$C = 7$

Now X(s) is given by

$$X(s) = \frac{-6}{s + 3} - \frac{3}{(s + 3)^2} + \frac{7}{s + 2}$$

which can be transformed to

$$x(t) = -6e^{-3t} - 3te^{-3t} + 7e^{-2t} \tag{5.138}$$

....

EXAMPLE:

Given

$$\ddot{x} + 2\dot{x} + 10x = 3t + \frac{6}{10} \qquad (5.139)$$

$$x(o) = 3$$

$$\dot{x}(o) = -\frac{27}{10}$$

solve for $x(t)$

Transforming (5.139) and solving for $X(s)$ gives

$$X(s) = \frac{3s^3 + 3.3s^2 + 0.6s + 3}{s^2(s^2 + 2s + 10)}$$

$$= \frac{A}{s} + \frac{B}{s^2} + \frac{Cs + D}{s^2 + 2s + 10}$$

where

$A = 0$

$B = 0.3$

$C = 3$

$D = 3$

Thus,

$$X(s) = \frac{0.3}{s^2} + \frac{3s + 3}{s^2 + 2s + 10} \qquad (5.140)$$

In order to make our inverse transforming a bit easier, let's rewrite (5.140) as

$$X(s) = \frac{0.3}{s^2} + 3\left[\frac{(s + 1)}{(s + 1)^2 + 3^2}\right] \qquad (5.141)$$

which is readily transformable to

$$x(t) = 0.3t + 3e^{-t} \cos 3t \qquad (5.142)$$

....

Transfer Function:

Before beginning simultaneous differential equations, we shall define the transfer function of a system. Consider the following equation with initial conditions as shown.

$$a\ddot{x} + b\dot{x} + cx = f(t) \qquad (5.143)$$

$$x(o) = \dot{x}(o) = 0$$

If we take the Laplace Transform of (5.143), we get

$$as^2X(s) + bsX(s) + cX(s) = F(s) \qquad (5.144)$$

or

$$\frac{X(s)}{F(s)} = \frac{1}{as^2 + bs + c} \qquad (5.145)$$

Since equation (5.143) represents a system whose input is $f(t)$ and whose output is $x(t)$, we shall define

$X(s) \triangleq$ output transform

$F(s) \triangleq$ input transform

We can then define the transfer function of the system, TF, as

$$TF \triangleq \frac{X(s)}{F(s)} \qquad (5.146)$$

For our example,

$$TF = \frac{1}{as^2 + bs + c} \qquad (5.147)$$

Note that the denominator of the transfer function is algebraically the same as the characteristic equation of (5.143). We have already seen in the section on Operator Notation that the characteristic equation completely defines the transient solution, and that the total solution is only altered by

the effect of the particular solution due to the input (or forcing function). Thus, from a physical standpoint, the transfer function completely characterizes a linear system.

The transfer function has several properties which we wish to exploit. Suppose that we have two systems characterized by the differential equations

$$a\ddot{x} + b\dot{x} + cx = f(t) \qquad (5.148)$$

and

$$d\ddot{y} + e\dot{y} + gy = x(t) \qquad (5.149)$$

From the equations it can be seen that the first system has an input f(t), and an output x(t). The second system has an input x(t), and an output y(t). If we take the Laplace Transforms of (5.148) and (5.149) we get (assuming all initial conditions are equal to zero)

$$(as^2 + bs + c)\,X(s) = F(s) \qquad (5.150)$$

and

$$(ds^2 + es + g)\,Y(s) = X(s) \qquad (5.151)$$

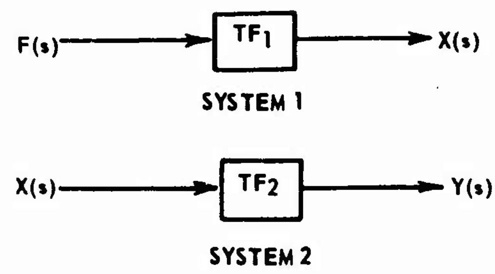
Finding the transfer functions,

$$TF_1 = \frac{X(s)}{F(s)} = \frac{1}{as^2 + bs + c} \qquad (5.152)$$

$$TF_2 = \frac{Y(s)}{X(s)} = \frac{1}{ds^2 + es + g} \qquad (5.153)$$

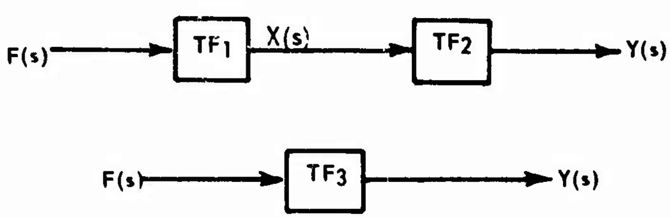
Now, both of these systems can be represented schematically as shown in figure 5.16.

FIGURE 5.16



Suppose that we now wish to find the output, y(t), of system 2 due to the input, f(t), of system 1. Our first inspiration might tell us that the logical thing to do is to find x(t), but this is not necessary. We can "link" the two systems using the transfer functions, as shown in figure 5.17.

FIGURE 5.17



$TF_3 = (TF_1)(TF_2)$

The solution we seek, y(t), is then given by the inverse transform of Y(s), or

$$Y(s) = \left[TF_3\right] F(s) \qquad (5.154a)$$

or

$$Y(s) = \left[TF_1\right]\left[TF_2\right] F(s) \qquad (5.154b)$$

This method of solution can be logically extended to include any number of systems we desire.

5.6 SIMULTANEOUS LINEAR DIFFERENTIAL EQUATIONS

In many physical problems, the mathematical description of

the system can most conveniently be written as simultaneous differential equations with constant coefficients. The basic procedure for solving a system of n ordinary differential equations in n dependent variables consists in obtaining a set of equations from which all but one of the dependent variables, say x, can be eliminated. The equation resulting from the elimination is then solved for the variable x. Each of the other dependent variables is then obtained in a similar manner.

We shall consider three basic procedures for solution of simultaneous linear differential equations, and then demonstrate how a solution is effected using Laplace Transforms.

1. Simultaneous solution of the basic differential equation: Basically, this method consists of eliminating one of the dependent variables by substitution into the other equation.

Consider the system

$$\frac{2dx}{dt} + \frac{dy}{dt} - 4x - y = e^t \qquad (5.155)$$

$$\frac{dx}{dt} + 3x + y = 0 \qquad (5.156)$$

Taking d/dt of equation (5.156) yields

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + \frac{dy}{dt} = 0 \qquad (5.157)$$

Now, multiply (5.155) by -1, (5.156) by -1, and (5.157) by 1, and add to obtain

$$\frac{d^2x}{dt^2} + x = -e^t \qquad (5.158)$$

This latter differential equation is free of y and its derivatives and may be readily solved. Thus,

$$x(t) = c_1 \cos t + c_2 \sin t - \frac{1}{2}e^t \qquad (5.159)$$

To find y, the easiest solution is obtained from equation (5.156). Substituting the value of x(t) obtained in (5.159) into equation (5.156) yields

$$y(t) = (c_1 - 3c_2)\sin t + 2e^t - (3c_1 + c_2)\cos t \qquad (5.160)$$

The above equation lends itself nicely to this method of solution. It is unusual, however, to have all the constants immediately determined. Usually the solutions must be substituted into the basic differential equations, and the resulting system must be solved simultaneously for the correct determination of the constants of integration. The number of arbitrary constants is determined by the order of the system. This system is second order, since the single combined equation is of second order.

2. Simultaneous solution using operator notation: Again we consider equations (5.155) and (5.156). Using operator notation, they become

$$2(p - 2)x + (p - 1)y = e^t \qquad (5.161)$$

$$(p + 3)x + y = 0 \qquad (5.162)$$

We attempt to eliminate one of the variable by "operating" on these equations. Multiplying (5.162) by (p - 1) gives

$$(p - 1)(p + 3)x + (p - 1)y = 0 \qquad (5.163)$$

Subtracting (5.161) from (5.163) yields

$$\left[(p - 1)(p + 3) - 2(p - 2)\right] x = -e^t \qquad (5.164)$$

or

$$(p^2 + 1)x = -e^t \qquad (5.165)$$

The general solution of this system is obtained as shown in the previous method.

....

3. Solution by means of determinants: Recall that for a determinant of second order, the value of the determinant is given by

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb \qquad (5.166)$$

Suppose that we rewrite equations (5.161) and (5.162) in the following form

$$P_1 x + P_2 y = f_1(t) \qquad (5.167)$$

$$P_3 x + P_4 y = f_2(t) \qquad (5.168)$$

where the P's denote the polynomial operators which act on x and y. If we treat (5.167) and (5.168) as algebraic equations, we can multiply (5.167) by P_4 to get

$$P_1P_4 x + P_2P_4 y = P_4 f_1(t) \qquad (5.169)$$

We can also multiply (5.168) by P_2 to get

$$P_2P_3 x + P_2P_4 y = P_2 f_2(t) \qquad (5.170)$$

Subtracting (5.170) from (5.169) yields

$$(P_1P_4 - P_2P_3)x = P_4 f_1(t) - P_2 f_2(t) \qquad (5.171)$$

We can note, however, that the operational coefficient of x in equation (5.171) is merely the expanded form of the determinant

$$\begin{vmatrix} P_1 & P_2 \\ P_3 & P_4 \end{vmatrix} = P_1P_4 - P_3P_2 \qquad (5.172)$$

Furthermore, the right side of (5.171) can be interpreted as the expanded form of

$$\begin{vmatrix} f_1(t) & P_2 \\ f_2(t) & P_4 \end{vmatrix}$$

provided we keep in mind that the operator polynomials must operate on the forcing functions (i.e., we must remember that this determinent gives an expanded form of

$$\begin{vmatrix} f_1(t) & P_2 \\ f_2(t) & P_4 \end{vmatrix} = P_4 f_1(t) - P_2 f_2(t)$$

and not

$$f_1(t) P_4 - f_2(t) P_2$$

as might be construed).

Thus, our solution for x can be given by

$$\begin{vmatrix} P_1 & P_2 \\ P_3 & P_4 \end{vmatrix} x = \begin{vmatrix} f_1(t) & P_2 \\ f_2(t) & P_4 \end{vmatrix} \qquad (5.173)$$

and our solution for y can be expressed as

$$\begin{vmatrix} P_1 & P_2 \\ P_3 & P_4 \end{vmatrix} y = \begin{vmatrix} P_1 & f_1(t) \\ P_3 & f_2(t) \end{vmatrix} \quad (5.174)$$

To solve the system given by equations (5.161) and (5.162), we write these equations in determinant form

$$\begin{vmatrix} 2(p - 2) & (p - 1) \\ (p + 3) & 1 \end{vmatrix} x = \begin{vmatrix} e^t & (p - 1) \\ 0 & 1 \end{vmatrix} \quad (5.175)$$

which is expanded to

$$(p^2 + 1) x = -e^t \quad (5.176)$$

giving a solution

$$x(t) = c_1 \cos t + c_2 \sin t - 1/2 e^t \quad (5.177)$$

Solving for y,

$$\begin{vmatrix} 2(p - 2) & (p - 1) \\ (p + 3) & 1 \end{vmatrix} y = \begin{vmatrix} 2(p - 2) & e^t \\ (p + 3) & 0 \end{vmatrix} \quad (5.178)$$

which can be expanded to

$$(p^2 + 1)y = 4e^t \quad (5.179)$$

giving a solution

$$y(t) = c_3 \cos t + c_4 \sin t + 2e^t \quad (5.180)$$

We know by examining (5.177) and (5.180) that extraneous constants are present, and to eliminate them we substitute back into equation (5.156) and see that

$$(c_2 + 3c_1 + c_3) \cos t + (3c_2 - c_1 + c_4) \cdot \sin t = 0 \quad (5.181)$$

Since (5.181) must hold for all values of t, the terms in parenthesis must vanish, giving

$$c_3 = -(3c_1 + c_2)$$

and

$$c_4 = c_1 - 3c_2$$

When these values are substituted in (5.180), we obtain the general solution found in the previous methods.

....

4. Solution by means of Laplace Transforms: A very effective means of handling simultaneous linear differential equation is to take the Laplace Transform of the set of equations and reduce the problem to a set of algebraic equations which can then be solved explicitly for the dependent variable in s. This method is demonstrated below.

Given the set of equations

$$3 \frac{d^2x}{dt^2} + x + \frac{d^2y}{dt^2} + 3y = f(t) \quad (5.182)$$

$$2 \frac{d^2x}{dt^2} + x + \frac{d^2y}{dt^2} + 2y = g(t) \quad (5.183)$$

where $x(0) = \dot{x}(0) = y(0) = \dot{y}(0) = 0$, find $x(t)$ and $y(t)$. Taking

the Laplace Transform of this system yields

$$(3s^2 + 1) X (s) + (s^2 + 3) Y (s) = F(s) \tag{5.184}$$

$$(2s^2 + 1) X (s) + (s^2 + 2) Y (s) = G (s) \tag{5.185}$$

From the previous section, we can solve for X(s) by rewriting these equations in determinant form.

$$\begin{vmatrix} (3s^2 + 1) & (s^2 + 3) \\ (2s^2 + 1) & (s^2 + 2) \end{vmatrix} X(s) = \begin{vmatrix} F(s) & (s^2 + 3) \\ G(s) & (s^2 + 2) \end{vmatrix} \tag{5.186}$$

Since we are using Laplace Transforms instead of operators, however, we can take this equation one step further. We can now solve explicity for X(s), giving us

$$X(s) = \frac{\begin{vmatrix} F(s) & (s^2 + 3) \\ G(s) & (s^2 + 2) \end{vmatrix}}{\begin{vmatrix} (3s^2 + 1) & (s^2 + 3) \\ (2s^2 + 1) & (s^2 + 2) \end{vmatrix}} \tag{5.187}$$

In a similar manner,

$$Y(s) = \frac{\begin{vmatrix} (3s^2 + 1) & F(s) \\ (2s^2 + 1) & G(s) \end{vmatrix}}{\begin{vmatrix} (3s^2 + 1) & (s^2 + 3) \\ (2s^2 + 1) & (s^2 + 2) \end{vmatrix}} \tag{5.188}$$

For the particular inputs f(t) = t and g(t) = 1,

$$X(s) = \frac{\begin{vmatrix} \frac{1}{s^2} & (s^2 + 3) \\ \frac{1}{s} & (s^2 + 2) \end{vmatrix}}{(s^4 - 1)} = \frac{-s^3 + s^2 - 3s + 2}{s^2(s^4 - 1)} \tag{5.189}$$

Expanded as a partial fraction

$$X(s) = \frac{A}{s^2} + \frac{B}{s} + \frac{Cs + D}{(s^2 + 1)} + \frac{E}{(s - 1)} + \frac{F}{(s + 1)} = \frac{-s^3 + s^2 - 3s + 2}{s^2(s^4 - 1)} \tag{5.190}$$

Solving for A, B, etc., we have

$$X(s) = \frac{-2}{s^2} + \frac{3}{s} + \frac{\frac{1}{2} - s}{s^2 + 1} - \frac{\frac{7}{4}}{s + 1} - \frac{\frac{1}{4}}{s - 1} \tag{5.191}$$

which yields a solution

$$x(t) = -2t + 3 - \frac{7}{4} e^{-t} - \frac{1}{4} e^{t} + \frac{1}{2} \sin t - \cos t \tag{5.192}$$

A similar approach will obtain the solution for y(t).

In the case of three simultaneous differential equations, the application of Laplace will yield the proper solutions.

$$P_1(s)\,X(s) \;+\; P_2(s)\,Y(s) \;+\; P_3(s)\,Z(s) \;=\; F_1(s) \qquad (5.193)$$

$$Q_1(s)\,X(s) \;+\; Q_2(s)\,Y(s) \;+\; Q_3(s)\,Z(s) \;=\; F_2(s) \qquad (5.194)$$

$$R_1(s)\,X(s) \;+\; R_2(s)\,Y(s) \;+\; R_3(s)\,Z(s) \;=\; F_3(s) \qquad (5.195)$$

where

$$X(s) \;=\; \frac{\begin{vmatrix} F_1 & P_2 & P_3 \\ F_2 & Q_2 & Q_3 \\ F_3 & R_2 & R_3 \end{vmatrix}}{\begin{vmatrix} P_1 & P_2 & P_3 \\ Q_1 & Q_2 & Q_3 \\ R_1 & R_3 & R_3 \end{vmatrix}} \qquad (5.196)$$

....

Y(s) and Z(s) will have similar
forms.

CHAPTER 6

DYNAMICS

6.1 DYNAMIC STABILITY

While static stability is concerned with the tendency of a displaced body to return to equilibrium, dynamic stability deals with the resulting motion as a function of time. Airplane dynamics is a study of the motion that results when an airplane is disturbed from steady flight. The initial tendency of an airplane to return to equilibrium after a disturbance is its inherent static stability. In any system, the existence of static stability does not necessarily guarantee the existence of dynamic stability. However, the existence of dynamic stability implies the existence of static stability. The degree of dynamic stability is determined by the character of the motion after the aircraft is disturbed. If the aircraft motion decays with time, the aircraft is dynamically stable; if the motion increases, the aircraft is unstable. Neutral stability results if the motion neither increases nor decreases.

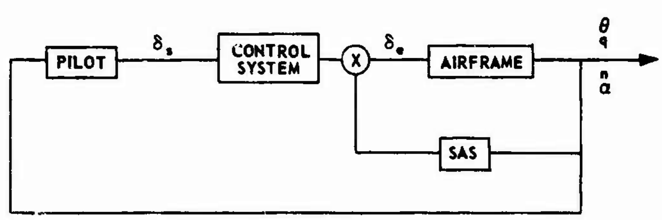
The early development of modern aircraft was a trial and error routine with the unsuccessful designs dropping by the wayside. The modern approach is to write the equations of motion for a given design, solve the equations, and then examine the resultant modes of motion with respect to the handling qualities desired for the aircraft's intended mission. By selective adjustment of stability derivatives, the aircraft's motion may be modified until it is satisfactory. The prototype can then be constructed.

This chapter will consider the equations of motion for an aircraft and will derive the equations for longitudinal and lateral-directional motions. Solutions to the equations will be assumed and the possible modes of motion discussed.

The effect that the major stability derivatives have will then be examined. And finally, flying qualities, as related to the dynamic characteristics observed by a pilot in flight, will be discussed.

Future courses at the Aerospace Research Pilot School will discuss aircraft control systems and transfer functions in detail; however, a look at a simplified block diagram will aid in understanding dynamic stability.

Figure 6.1



This course will study the response of the airframe as predicted by the equations of motion.

6.2 EQUATION OF MOTION

The following restrictions to the aircraft equations of motion were made in Chapter I:

1. The aircraft is a rigid-body - this allows aircraft motion to be described completely by translation of the center of gravity and by rotation about this point. Aeroelastic effects will be considered separately.

2. Mass of the aircraft is constant.

3. The xz plane is a plane of symmetry.

The following equations were derived in Chapter I:

<u>Inertial Forces</u>

DRAG $F_x = m(\dot{u} + qw - rv)$ (6.1)

SIDE $F_y = m(\dot{v} + ru - pw)$ (6.2)

LIFT $F_z = m(\dot{w} + pv - qu)$ (613)

<u>Moments</u>

ROLL $\mathcal{L} = \dot{p}I_x + qr(I_z - I_y) - (\dot{r} + pq)I_{xz}$ (6.4)

PITCH $\mathcal{M} = \dot{q}I_y - pr(I_z - I_x) + (p^2 - r^2)I_{xz}$ (6.5)

YAW $\mathcal{N} = \dot{r}I_z + pq(I_y - I_x) + (qr - \dot{p})I_{xz}$ (6.6)

6.3 PERTURBATIONS

The aircraft motion under study will be considered the result of some small disturbance, or perturbation, from some steady flight condition. Accordingly, each of the linear velocity components which describe the aircraft's motion at a given instant can be written as the sum of a steady flight condition and some slight perturbation.

$x = x_O + \bar{x}$ where x_O is the steady state value and $\bar{x}$ is the disturbance measured from the steady state value. Thus:

$$u = u_o + \bar{u}$$

$$v = v_o + \bar{v}$$

$$w = w_o + \bar{w}$$

$$p = p_o + \bar{p}$$

$$q = q_o + \bar{q}$$

$$r = r_o + \bar{r}$$

$$\alpha = \alpha_o + \bar{\alpha}$$

$$\beta = \beta_o + \bar{\beta}$$

The zero subscripts indicate the steady flight conditions and the bar represents the perturbation.

Note that the angle of attack (α) and sideslip angle (β) are functions of w and v, respectively. Assuming that $u_O = V_T$,

$$\bar{\alpha} = \arctan \frac{\bar{w}}{V_T} = \frac{\bar{w}}{u_o}$$

$$\dot{\alpha} = \frac{\dot{\bar{w}}}{u_o} = \frac{\dot{w}}{u_o} = \dot{\bar{\alpha}}$$

similarly

$$\bar{\beta} = \arctan \frac{\bar{v}}{V_T} = \frac{\bar{v}}{u_o}$$

$$\dot{\beta} = \frac{\dot{\bar{v}}}{u_o} = \frac{\dot{v}}{u_o} = \dot{\bar{\beta}}$$

6.4 AERODYNAMIC FORCES AND MOMENTS

The external forces and moments caused by the aerodynamic load on the aircraft in flight in terms of stability parameters are:

DRAG

$$\frac{D}{m} = D_o^{(*)} + D_\alpha\bar{\alpha} + D_{\dot{\alpha}}\dot{\alpha} + D_u\bar{u} + D_q\bar{q} + D_{\delta e}\delta e \quad (6.7)$$

LIFT

$$\frac{L}{mV_T} = L_o + L_\alpha\bar{\alpha} + L_{\dot{\alpha}}\dot{\alpha} + L_u\bar{u} + L_q\bar{q} + L_{\delta e}\delta e \quad (6.8)$$

SIDE

$$\frac{Y}{mV_T} = Y_o + Y_\beta\bar{\beta} + Y_{\dot{\beta}}\dot{\beta} + Y_p\bar{p} + Y_r\bar{r} + Y_{\delta a}\delta a + Y_{\delta r}\delta r \quad (6.9)$$

ROLL

$$\frac{\mathcal{L}}{I_x} = \mathcal{L}_o + \mathcal{L}_\beta\bar{\beta} + \mathcal{L}_{\dot{\beta}}\dot{\beta} + \mathcal{L}_p\bar{p} + \mathcal{L}_r\bar{r} + \mathcal{L}_{\delta a}\delta a + \mathcal{L}_{\delta r}\delta r \quad (6.10)$$

PITCH

$$\frac{M}{I_y} = M_o + M_u\bar{u} + M_\alpha\bar{\alpha} + M_{\dot{\alpha}}\dot{\alpha} + M_q\bar{q} + M_{\delta e}\delta e \quad (6.11)$$

YAW

$$\frac{N}{I_z} = N_o + N_\beta\bar{\beta} + N_{\dot{\beta}}\dot{\beta} + N_p\bar{p} + N_r\bar{r} + N_{\delta a}\delta a + N_{\delta r}\delta r \quad (6.12)$$

The side, roll and yaw equations show the interrelationship of the lateral and directional controls of the aircraft. This is one of the reasons the lateral motion is inseparable from the directional motion. When the aerodynamic portion of the equations of motion are expanded and the stability parameters linearized, certain restrictions are placed upon the motion and values in the equations. These restrictions are:

1. The moments of inertia are measured with respect to the Body Axis System.

2. Each aerodynamic equation is written so that the plane to which it applies is variable and the others are held constant. For example, the pitch equation is written with the longitudinal motion variable and lateral or directional motion excluded.

3. The x body axis and the X stability axis are the same. ($\alpha_o = 0$) Refer to Chapter I for a complete discussion of this assumption. The axis system used was aligned with the stability axis system so that the drag, side and lift equations would have values that could be determined by an instrumentation system installed in the aircraft.

* D_α, L_α, etc., includes the normalizing factors $1/m$, $1/I_x$, $1/I_y$ and $1/I_z$, as applicable.

6.5 OTHER EXTERNAL FORCES

There are external forces on the aircraft that must be considered other than aerodynamic forces. They are the weight or gravity force and the thrust force.

The weight force is considered to act through the cg and is always directed towards the center of the earth. The thrust force does not necessarily pass through the cg nor is it always straight along the aircraft longitudinal axis.

Drag due to the weight of the aircraft can be a significant factor when large pitch angles are encountered. Consider the longitudinal effect of weight and thrust on an aircraft in flight (figure 6.2).

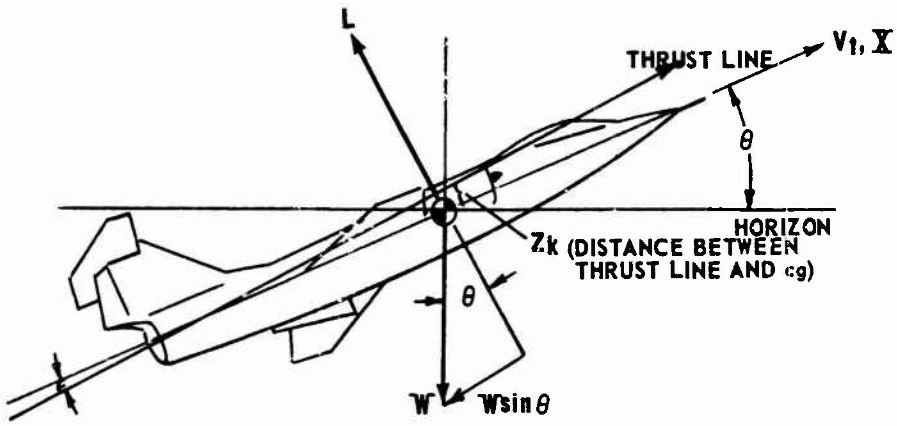


Figure 6.2

The weight force acts through the cg and makes no contribution to the aircraft moments. It does, however, affect the force equations. Considering weight only,

$$F_x = W \sin \theta \qquad (6.13)$$

assuming small angles,

$$F_x = W\theta \qquad (6.14)$$

$$\frac{dFx}{d\theta} = W \qquad (6.15)$$

$$\frac{dFx}{md\theta} = \frac{W}{m} = g = D_\theta \qquad (6.16)$$

By definition, the drag due to weight = D_θ. (Only if the wings are level.)

Thus:

$$\frac{D_{wt}}{m} = D_{o_{wt}} + D_\theta \bar{\theta} \qquad (6.17)$$

Similarly

$$\frac{L_{wt}}{m} = L_{o_{wt}} + L_\theta \bar{\theta} \qquad (6.18)$$

The thrust vector can be considered in the same way. There is a moment created if the thrust vector does not pass through the cg. (Figure 6.2)

The lift component would be

$-T \sin \varepsilon$

The drag component,

$T \cos \varepsilon$

Pitching moment component,

TZ_K

Z_K is the perpendicular distance from the thrust line to the cg. For small disturbances of the aircraft, the air density can be considered constant and the changes in thrust dependent only upon the change in forward speed and engine rpm.

$$\Delta T = \frac{\partial T}{\partial u} \bar{u} + \frac{\partial T}{\partial \delta_{rpm}} \delta_{rpm} \qquad (6.19)$$

$$T = T_o + \frac{\partial T}{\partial u} \bar{u} + \frac{\partial T}{\partial \delta_{rpm}} \delta_{rpm} \qquad (6.20)$$

Thus:

$$D_{THRUST} = \left| T_o + \frac{\partial T}{\partial u} \bar{u} + \frac{\partial T}{\partial \delta_{rpm}} \delta_{rpm} \right| \left| \cos \varepsilon \right| \qquad (6.21)$$

$$L_{THRUST} = \left(T_o + \frac{\partial T}{\partial u}\bar{u} + \frac{\partial T}{\partial \delta_{rpm}} \delta_{rpm}\right)\left(-\sin\varepsilon\right) \qquad (6.22)$$

$$M_{THRUST} = \left(T_o + \frac{\partial T}{\partial u}\bar{u} + \frac{\partial T}{\partial \delta_{rpm}} \delta_{rpm}\right)\left(z_K\right) \qquad (6.23)$$

6.6 LONGITUDINAL DYNAMIC STABILITY

The longitudinal motion of an aircraft can be considered independent of the motion not in the plane of symmetry (i.e., lateral-directional motion). The motion described in this section will be pure longitudinal motion which eliminates all roll, yaw or side force components. This will greatly simplify the analysis.

The complete drag equation,

$$\frac{Fx}{m} = \dot{u} + qw - rv = D_o + D_\alpha\bar{\alpha} + D_{\dot\alpha}\dot{\alpha} + D_u\bar{u} + D_q\bar{q} + D_{\delta e}\delta e + D_\theta\bar{\theta} + \frac{1}{m}\cdot \left(T_o + \frac{\partial T}{\partial u}\bar{u} + \frac{\partial T}{\partial \delta_{rpm}} \delta_{rpm}\right)\cos\varepsilon \qquad (6.24)$$

can be simplified by assumming:

$$D_o + \frac{To}{m}\cos\varepsilon = 0 \qquad \text{(steady state)}$$

$$\left(\frac{\partial T}{\partial u}\bar{u} + \frac{\partial T}{\partial \delta_{rpm}}\delta_{rpm}\right)\cos\varepsilon = 0 \qquad (\varepsilon \text{ normally small, constant rpm})$$

$$rv = 0 \qquad \text{(no lat-dir motion)}$$

$$qw = 0 \qquad \text{(order of magnitude)}$$

An order of magnitude check of a typical airplane would allow us to drop the terms $D_{\dot\alpha}$, D_q and D_{δ_e}. Thus

$$\text{DRAG} - \dot{u} = D_\alpha\bar{\alpha} + D_u\bar{u} + D_\theta\bar{\theta} \qquad (6.25)$$

The standard arrangement for a motion equation is to group the variables and set them equal to any forcing function that might exist.

$$\text{DRAG} \quad \dot{u} + D_\alpha\bar{\alpha} + D_u\bar{u} + D_\theta\bar{\theta} = 0 \qquad (6.26)$$

The lift equation,

$$-\left(\frac{Fz}{mV_T}\right) = -\left(\frac{\dot{w} + pv - qu}{V_T}\right) = L_o + L_\alpha\bar{\alpha} + L_{\dot\alpha}\dot{\alpha} + L_u\bar{u} + L_q\bar{q} + L_{\delta e}\delta e + \frac{L_\theta\bar{\theta}}{V_T} + \text{THRUST TERMS} \qquad (6.27)$$

can be simplified as before

$$L_o + \text{THRUST TERMS} = 0 \qquad \text{(steady state)}$$

$$L_\theta\frac{\bar{\theta}}{V_T} = 0 \qquad \text{(magnitude)}$$

$$\frac{\dot{w}}{V_T} = \dot{\alpha}$$

$$pv = 0$$

$$\frac{qu}{V_T} = q \qquad (u = V_T)$$

Thus:

$$-\dot{\alpha} + q = L_\alpha\bar{\alpha} + L_{\dot\alpha}\dot{\alpha} + L_u\bar{u} + L_q\bar{q} + L_{\delta e}\delta e \qquad (6.28)$$

or

LIFT

$$-\dot{\alpha} + q - L_\alpha\bar{\alpha} - L_{\dot\alpha}\dot{\alpha} - L_u\bar{u} - L_q\bar{q} = L_{\delta e}\delta e \qquad (6.29)$$

The pitch equation,

$$\frac{M}{I_y} = \dot{q} - pr\,(I_z - I_x) + (p^2 - r^2)\, I_{xz}$$

$$= M_o + M_\alpha \bar{\alpha} + M_{\dot{\alpha}} \dot{\alpha}$$

$$+ M_u \bar{u} + M_q \bar{q} + M_{\delta e} \delta e \quad + \text{THRUST TERMS} \tag{6.30}$$

can be simplified as before. Thus:

$$\dot{q} = M_\alpha \bar{\alpha} + M_{\dot{\alpha}} \dot{\alpha} + M_u \bar{u} + M_q \bar{q} + M_{\delta e} \delta e \tag{6.31}$$

or

PITCH

$$\dot{q} - M_\alpha \bar{\alpha} - M_{\dot{\alpha}} \dot{\alpha} - M_u \bar{u} - M_q \bar{q} = M_{\delta e} \delta e \tag{6.32}$$

The three equations [(6.26), (6.29), and (6.32)] have been and will be further simplified to allow an approximate solution. Computer solutions would be used in any actual aircraft analysis.

DRAG

$$\dot{u} + D_\alpha \bar{\alpha} + D_u \bar{u} + D_\theta \bar{\theta} = 0 \tag{6.26}$$

LIFT

$$-\dot{\alpha} + q - L_\alpha \bar{\alpha} - L_{\dot{\alpha}} \dot{\alpha} - L_u \bar{u} - L_q \bar{q} = L_{\delta e} \delta e \tag{6.29}$$

PITCH

$$\dot{q} - M_\alpha \bar{\alpha} - M_{\dot{\alpha}} \dot{\alpha} - M_u \bar{u} - M_q \bar{q} = M_{\delta e} \delta e \tag{6.32}$$

Note that we have three equations and four unknowns (u, α, q, θ). However, for a constant or small angle of attack,

$$q = \dot{\theta} \quad \text{and} \quad \dot{q} = \ddot{\theta}$$

Rearranging and substituting $q = \dot{\theta}$,

$$\begin{array}{llllll}
 & (\theta) & (u) & (\alpha) & & \\
\text{DRAG} & D_\theta \bar{\theta} & + \dot{u} + D_u \bar{u} & + D_\alpha \bar{\alpha} & = 0 & (6.33) \\
\text{LIFT} & \dot{\theta} - L_q \dot{\theta} & - L_u \bar{u} & - \dot{\alpha} - L_{\dot{\alpha}} \dot{\alpha} - L_\alpha \bar{\alpha} & = L_{\delta e} \delta e & (6.34) \\
\text{PITCH} & \ddot{\theta} - M_q \dot{\theta} & - M_u \bar{u} & - M_{\dot{\alpha}} \dot{\alpha} - M_\alpha \bar{\alpha} & = M_{\delta e} \delta e & (6.35)
\end{array}$$

If the Laplace Transform of equations 6.33, 6.34, and 6.35 is performed, the three equations can be written as

$$\begin{array}{lll}
\text{DRAG} & D_\theta \bar{\theta}(s) + (s + D_u)\bar{u}(s) + D_\alpha \bar{\alpha}(s) & = 0 \\
\text{LIFT} & (s - L_q s)\bar{\theta}(s) - L_u \bar{u}(s) - [(1 + L_{\dot{\alpha}})s + L_\alpha]\bar{\alpha}(s) & = L_{\delta e} \delta e(s) \\
\text{PITCH} & (s^2 - sM_q)\bar{\theta}(s) - M_u \bar{u}(s) - (sM_{\dot{\alpha}} + M_\alpha)\bar{\alpha}(s) & = M_{\delta e} \delta e(s)
\end{array}$$

and putting in matrix form,

$$\begin{bmatrix} D_\theta & s + D_u & D_\alpha \\ s(1 - L_q) & -L_u & -s(1 + L_{\dot{\alpha}}) - L_\alpha \\ s^2 - sM_q & -M_u & -sM_{\dot{\alpha}} - M_\alpha \end{bmatrix} \begin{bmatrix} \bar{\theta}(s) \\ \bar{u}(s) \\ \bar{\alpha}(s) \end{bmatrix} = \begin{bmatrix} 0 \\ L_{\delta e} \delta e(s) \\ M_{\delta e} \delta e(s) \end{bmatrix} \tag{6.36}$$

The solution for any of the three variables θ, u or α will have the same denominator or characteristic equation of the form

$$s^4 + K_3 s^3 + K_2 s^2 + K_1 s + K_o$$

To find the transient solution, we set the characteristic equation equal to zero.

$$s^4 + K_3 s^3 + K_2 s^2 + K_1 s + K_o = 0$$

The roots of the equation may be positive or negative real numbers or complex pairs with either positive or negative parts.[1]

From experience, we know the roots are usually two sets of complex pairs which result in two distinct oscillations.

1. Short period, heavily damped
2. Long period, lightly damped (Phugoid).

[1] Section 5.5 – Modern Flight Dynamics, W. Richard Kolk.

By selecting the proper terms in the determinant, solutions for these two motions can be found without solving the complete determinant.

SHORT PERIOD MODE

The short period is essentially a constant airspeed, varying angle of attack motion. (Figure 6.3)

Neglecting the drag equation and the $\bar{u}(s)$ column in the matrix (6.36) leaves

$$\begin{bmatrix} s(1 - L_q) & -s(1 + L_{\dot{\alpha}}) - L_\alpha \\ s^2 - sM_q & -sM_{\dot{\alpha}} - M_\alpha \end{bmatrix} \begin{bmatrix} \bar{\theta}(s) \\ \bar{\alpha}(s) \end{bmatrix} = \begin{bmatrix} L_{\delta e}\delta e(s) \\ M_{\delta e}\delta e(s) \end{bmatrix} \qquad (6.37)$$

The solution for $\alpha(s)$ would be

$$\bar{\alpha}(s) = \frac{\begin{vmatrix} s(1 - L_q) & L_{\delta e}\delta e(s) \\ s^2 = sM_q & M_{\delta e}\delta e(s) \end{vmatrix}}{\begin{vmatrix} s(1 - L_q) & -s(1 + L_{\dot{\alpha}}) - L_\alpha \\ s^2 - sM_q & -sM_{\dot{\alpha}} - M_\alpha \end{vmatrix}} \qquad (6.38)$$

$$\bar{\alpha}(s) = \frac{s(1 - L_q)\, M_{\delta e}\delta e(s) - (s^2 - sM_q)L_{\delta e}\delta e(s)}{s(1 - L_q)(-sM_{\dot{\alpha}} - M_\alpha) + [s(1 + L_{\dot{\alpha}}) + L_\alpha](s^2 - sM_q)} \qquad (6.39)$$

Neglecting L_q, $L_{\delta e}$, and $L_{\dot{\alpha}}$ which are frequently quite small,

$$\bar{\alpha}(s) = \frac{M_{\delta e}\delta e(s)}{s^2 + (-M_{\dot{\alpha}} + L_\alpha - M_q)s + (-M_\alpha - L_\alpha M_q)} \qquad (6.40)$$

and the transfer function is

$$\frac{\bar{\alpha}(s)}{\delta_e(s)} = \frac{M_{\delta e}}{s^2 + (-M_{\dot{\alpha}} + L_\alpha - M_q)s + (-M_\alpha - L_\alpha M_q)} \qquad (6.41)$$

The other transfer functions will not be derived, but can be found in many texts. All, however, have the same denominator.

SHORT PERIOD FREQUENCY DAMPING RATIO

The denominator of (6.41) is in the familiar form

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

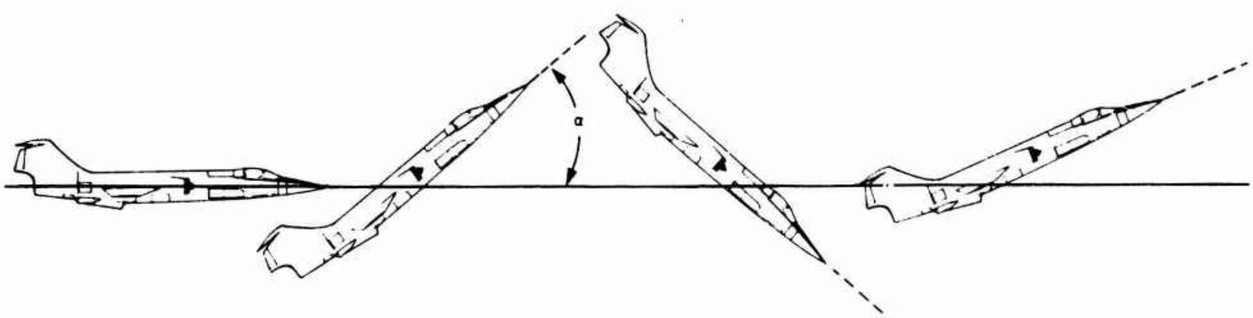
where $2\zeta\omega_n = -M_{\dot{\alpha}} + L_\alpha - M_q$ (6.42)

and

$$\omega_n^2 = -M_\alpha - L_\alpha M_q \qquad (6.43)$$

Thus the natural frequency (ω_n) of the short period is

Figure 6.3



$$\omega_n = \sqrt{-M_\alpha - L_\alpha M_q} \approx \sqrt{-M_\alpha} \tag{6.44}$$

and the damping ratio (ζ) of the short period is

$$\zeta = \frac{L_\alpha - M_q - M_{\dot\alpha}}{2\sqrt{-M_\alpha - L_\alpha M_q}} \approx \frac{-M_q}{2\sqrt{-M_\alpha}} \tag{6.45}$$

Equations (6.44) and (6.45) show that the natural frequency and the damping ratio of the short period are heavily dependent upon the longitudinal static stability of the aircraft.

ROOT LOCUS

A brief look at a basic root locus plot will aid in understanding the important stability parameters involved in the short period mode.

Figure 6.4

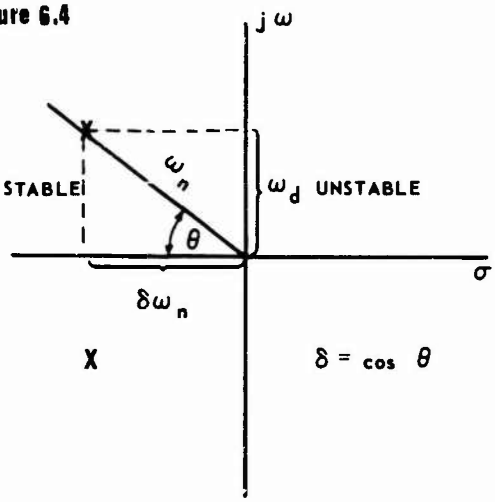
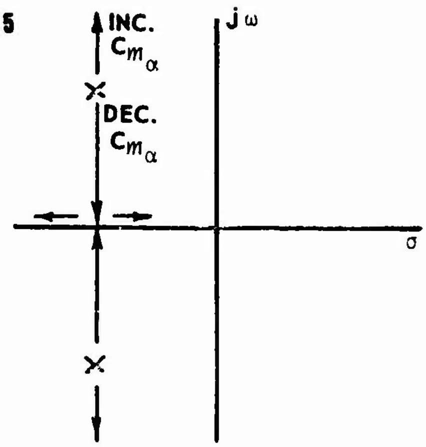


Figure 6.5 shows the effect of changing the stability derivative Cm_α.

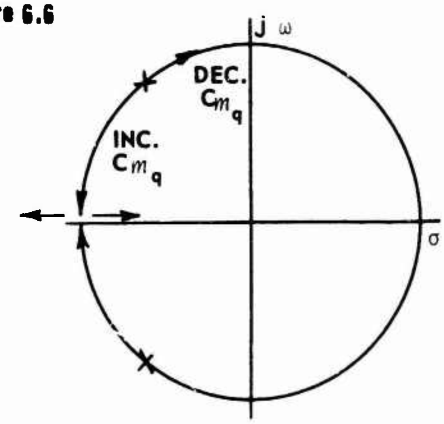
Figure 6.5



A change in Cm_α also changes the natural frequency (ω_n), the damped frequency (ω_d) and the damping ratio (ζ).

Figure 6.6 shows the effect of changing Cm_q (pitch damping). Note that ω_n does not change while ω_d and ζ do change.

Figure 6.6



PHUGOID MODE

The phugoid mode is essentially a constant angle of attack, varying airspeed motion which has a long period (20 sec. - 2 min.). The aircraft may be thought of as flying at constant energy and oscillating in such a way as to give a periodic trade-off of potential energy (altitude) for kinetic energy (airspeed).

Eliminating the $\bar{\alpha}(s)$ terms and the pitching moment equation in matrix (6.36) leaves

$$\begin{bmatrix} D_\theta & s + D_u \\ s(1 - L_q) & -L_u \end{bmatrix} \begin{bmatrix} \bar{\theta}(s) \\ \bar{u}(s) \end{bmatrix} = \begin{bmatrix} 0 \\ L_{\delta e}\delta e(s) \end{bmatrix} \quad (6.46)$$

$$\bar{\theta}(s) = \frac{\begin{vmatrix} 0 & s + D_u \\ L_{\delta e}\delta e(s) & -L_u \end{vmatrix}}{\begin{vmatrix} D_\theta & s + D_u \\ s(1 - L_q) & -L_u \end{vmatrix}}$$

$$= \frac{-L_{\delta e}\delta e\,(s)\,(s + D_u)}{-D_\theta L_u - s(s + D_u)(1 - L_q)} \quad (6.47)$$

Neglecting L_q as was done in the short period, the characteristic equation is

$$-D_\theta L_u - s^2 - sD_u = 0 \quad (6.48)$$

rearranging,

$$s^2 + D_u s + D_\theta L_u = 0 \quad (6.49)$$

using the standard form, $s^2 + 2\zeta\omega_n s + \omega_n^2$,

$$\omega_n = \sqrt{D_\theta L_u} \quad (6.50)$$

$$\zeta = \frac{D_u}{2\sqrt{D_\theta L_u}} \quad (6.51)$$

D_u, the phugoid damping term, is generally small so the natural frequency (ω_n) and the damped frequency (ω_d) are indistinguishable. The phugoid frequency is closely approximated in unaccelerated flight as[2]

$$\omega_n = \frac{g}{u_o}\sqrt{2} \quad (6.52)$$

where u_o = TAS (ft/sec). The period is

$$P = \frac{2\pi u_o}{\sqrt{2}\;g} = 0.138 u_o \quad (6.53)$$

Flight testing has confirmed the proportionality between phugoid period and airspeed, but the proportionality constant should exceed the theoretical value of 0.138 by about 20 percent. Knots being the most common measure of airspeed, the phugoid period in seconds can be quickly estimated as

$$P = \frac{u_o \text{ (Knots TAS)}}{9} \quad (6.54)$$

6.7 EFFECT OF LONGITUDINAL STABILITY DERIVATIVES

The principal stability parameters that govern longitudinal motion are D_u, D_θ, L_u, L_α, M_α, $M_{\dot{\alpha}}$ and M_q. The effect of the stability derivatives (C_D, C_{D_u}, C_{D_α}, C_{L_α}, C_{m_α}, $C_{m_{\dot{\alpha}}}$ and C_{m_q}) that correspond to each of the parameters can be considered to be the same as that of the parameters. Therefore a discussion of the stability derivatives follows:

$\underline{C_D}$

The equilibrium drag coefficient,[3] C_D, is the main contributor to the dimensional stability derivative parameter D_u, the change in

[2]Section 5.8– Modern Flight Dynamics, W. Richard Kolk.

[3]Dynamics of the Airframe, BuAir Report, AE–61–411.

fore and aft force with forward velocity, and a minor contributor to the dimensional stability derivative parameter L_α, the change in lift force with angle of attack. In general, any portion of the airframe in contact with the external airstream contributes to the airframe drag. The fuselage, engine nacelles, external stores, tail surfaces, and internal engine ducts all contribute relatively small increments in comparison with the wing which contributes the major portion of the drag, especially at high angles of attack or high Mach numbers. As far as the performance of an airframe is concerned - range, speed, rate of climb, etc., - drag coefficient is one of the most important parameters. It is apparent, then, that a desirable value of C_D is one which is as small as possible. On the other hand, when airframe dynamics are considered, C_D is the main contributor to the damping of the phugoid mode, and the larger the value of C_D, the better the damping. However, flight experience has shown that the damping of the phugoid is of little importance in determining satisfactory flying qualities of an airframe as far as the pilot is concerned. Clearly then, performance requirements rather than flying qualities should dictate the design value of C_D.

$\underline{C_{D_u}}$

C_{D_u} can arise from two sources: Mach number effects and aeroelastic effects. In most cases, C_{D_u} arising from the latter of these is zero or very small and can be neglected. C_{D_u} arising from Mach number effects is very small for low subsonic Mach numbers but sometimes reaches a considerable positive value near the critical Mach number of an airframe ($0.8 < M < 1.0$), where a large increase in drag occurs. The effect of a positive value of C_{D_u} on longitudinal dynamics is an increase in the damping of the phugoid mode.

$\underline{C_{D_\alpha}}$

The stability derivative C_{D_α} is the change in drag coefficient with varying angle of attack. When the angle of attack of an airframe increases from the equilibrium condition, the total drag will increase; hence C_{D_α} will normally be positive in sign. By far the largest contribution to C_{D_α} comes from the wing, but there are small contributions from the horizontal tail and the fuselage. The derivative C_{D_α} is usually unimportant in airframe dynamics. It mainly affects the phugoid mode, where a decrease in C_{D_α} increases stability; however, this effect is slight, mainly because the changes in angle of attack occurring in phugoid motion are small or zero.

$\underline{C_L}$

Although not referred to as a stability derivative in the usual sense, the equilibrium lift coefficient C_L, contributes the major portion of the dimensional stability derivative parameter L_V, the change in vertical force with varying forward velocity, and D_α, the change in fore and aft force with varying angle of attack. In longitudinal dynamics, variations in the equilibrium lift coefficient principally affect the phugoid mode, with both the damping and the period decreasing with an increase in C_L. In addition, because many of the lateral derivatives are functions of C_L, the lateral dynamics are also affected; the main effect is a decrease in Dutch roll damping with an increase in C_L. Although low values for C_L are therefore preferable for stability, more important performance considerations usually take precedence in determining the desirable values of this derivative. As far as performance is concerned, a large range of equilibrium lift coefficients is desirable because a low value of

C_L and its associated low drag is desirable to attain high cruising speeds, and yet a high value of C_L is desirable to permit low landing speeds.

$\underline{C_{L_u}}$

C_{L_u} arises from two sources: Mach number effects and aeroelastic effects. The magnitude of the total C_{L_u} can vary considerably and its sign can change, depending not only on the airframe geometry and its elastic properties, but also upon the Mach number and dynamic pressure at which it is flying. The magnitude of C_{L_u} is negligibly small for low speed flight, but it may reach a considerable value near the critical Mach number of the airframe.

$\underline{C_{L_\alpha}}$

The stability derivative C_{L_α} is the change in lift coefficient with varying angle of attack. C_{L_α} is commonly known as the "lift curve slope." When the angle of attack of the airframe is increased, the lift force will increase more or less linearly until the wing stalls. The derivative C_{L_α} is therefore always positive in sign at angles of attack below the stall. The total airframe C_{L_α} is made up of contributions from the wing, the fuselage, and the horizontal tail. Ordinarily the wing accounts for about 80 percent to 90 percent of the total C_{L_α}, although the wing contribution becomes less if the size of the fuselage is large in comparison with the size of the wing. The derivative C_{L_α} is very important to the equilibrium flight condition of an airframe; it is also important in dynamic considerations. In the equilibrium flight condition, a high value of C_{L_α} is desirable because, for a given angle of attack, the airframe with the higher value of C_{L_α} will usually have a lower drag, and therefore better performance. C_{L_α} is also important in establishing the attitude of the airframe at landing and takeoff; when the value of C_{L_α} is low, the airframe must land and takeoff at a relatively high angle of attack. If this has to be done, pilot visibility is impaired, and difficulty in designing the landing gear may be encountered. As far as dynamic stability is concerned, this derivative makes an important contribution to the damping of the longitudinal short period mode for all aircraft and especially for tailless aircraft because in this case almost all the damping of the short period mode comes from C_{L_α}. A high value of C_{L_α} would therefore be desirable on all counts were it not for the fact that such values of C_{L_α} are necessarily associated with a high aspect-ratio, unswept-wing configuration which is contrary to present design trands; consequently this high C_{L_α} is not always realized in practice.

$\underline{C_{L_{\dot\alpha}}}$

The stability derivative $C_{L_{\dot\alpha}}$ is the change in lift coefficient with variation in rate of change of angle of attack. The derivative $C_{L_{\dot\alpha}}$ arises essentially from two independent sources: An aerodynamic time lag effect and various "dead-weight" aeroelastic effects. For low speed flight, $C_{L_{\dot\alpha}}$ arises mostly from the aerodynamic lag effect, and its sign is positive. For high speed flight the sign of $C_{L_{\dot\alpha}}$ can be positive or negative, depending on the nature of the aeroelastic effects. The horizontal tail of a conventional airframe is immersed in the downwash field of the wing and is mounted some distance aft of the wing. Whenever the wing undergoes a change in angle of attack, the downwash field is altered; and since it takes a finite length of time

before this downwash alteration arrives at the tail, the resulting lift on the tail lags the motion of the aircraft and creates the derivative $C_{L_{\dot\alpha}}$. Even for tailless aircraft, $C_{L_{\dot\alpha}}$ apparently has a value due to the fact that the wing must accelerate the air mass in its path as it accelerates (apparent mass effect). Since the type of motion under consideration is an acceleration, $C_{L_{\dot\alpha}}$ can also arise from aeroelastic effects such as wing twisting due to the dead weight moment caused by nacelles projecting in front of the wing, and from fuselage bending caused by the dead weight of the aft fuselage and empennage section, both of which change the effective angle of attack of the horizontal tail.

C_{L_q}

The stability derivative C_{L_q} is the change in lift coefficient with varying pitching velocity and with no change of angle of attack of the airplane as a whole. As the airframe pitches about its center of gravity, the angle of attack of the horizontal tail changes, and a lift force is developed on the horizontal tail producing a contribution to the derivative C_{L_q}. The sign of this contribution is positive. There is also a contribution to C_{L_q} because of various "deadweight" aeroelastic effects. Since the airframe is moving in a curved flight path due to its pitching, a centrifugal force is developed on all the components of the airframe. This force can cause the wing to twist as a result of the deadweight moment of overhanging nacelles, and can cause the horizontal tail angle of attack to change as a result of fuselage bending due to the weight of the tail section.

In low speed flight, C_{L_q} comes mostly from the effect of the curved flight path on the horizontal tail and its sign is positive. In high speed flight the sign of C_{L_q} can be positive or negative, depending on the nature of the aeroelastic effects. In past experience, the effect of C_{L_q} on longitudinal stability has usually been very small and it has therefore been neglected in dynamic analyses, but because of the possibility of great aeroelastic effects, especially at high Mach number flight, the magnitude of C_{L_q} may be increased considerably.

$C_{L_{\delta_e}}$

The stability derivative $C_{L_{\delta_e}}$ is the change in lift coefficient with changes in elevator deflection. On conventional aircraft with the horizontal tail mounted at an appreciable distance aft of the center of gravity, $C_{L_{\delta_e}}$ is usually very small and its effect is unimportant; however, on tailless aircraft, $C_{L_{\delta_e}}$ is comparatively large, and cannot be neglected.

C_{m_u}

The stability derivative C_{m_u} is the change in pitching moment coefficient with variation in forward velocity, angle of attack and altitude remaining constant. The magnitude of C_{m_u} can vary considerably and the sign can change, depending upon such factors as the airframe's geometry and its elastic properties, and the Mach number and dynamic pressure at which it is flying. This derivative can arise from three sources: thrust or power effects, Mach number effects and aeroelastic effects. In the past, the literature has treated C_{m_u} only as a power effect arising from the propwash of propeller-driven aircraft. Today, however, because of the use of jet engines and the associated alleviation of power effects on dynamic stability, the C_{m_u} from thrust effects is probably small. On the other hand, the contributions

to C_{m_u} due to Mach number and aeroelastic effects are becoming more and more important, and it is believed that these effects should no longer be neglected when evaluating C_{m_u}. It principally affects the longitudinal phugoid mode, where positive values of C_{m_u} will tend to decrease both the period and the damping of the oscillation. The effect can become quite objectionable, especially when the phugoid motion is lightly damped. It appears therefore that zero, or at most, very small values of C_{m_u} are desirable.

$\underline{C_{m_\alpha}}$

The stability derivative C_{m_α} is the change in pitching moment coefficient with varying angle of attack and is commonly referred to as the longitudinal static stability derivative. When the angle of attack of the airframe increases from the equilibrium condition, the increased lift on the horizontal tail causes a negative pitching moment about the center of gravity of the airframe. Simultaneously, the increased lift of the wing causes a positive or negative pitching moment, depending on the fore and aft location of the lift vector with respect to the center of gravity. These contributions together with the pitching moment contribution of the fuselage are combined to establish the derivative C_{m_α}. The magnitude and sign of the total C_{m_α} for a particular airframe configuration are thus a function of the center of gravity position, and this fact is very important in longitudinal stability and control. If the center of gravity is ahead of the neutral point, the value of C_{m_α} is negative, and the airframe is said to possess static longitudinal stability. Conversely, if the center of gravity is aft of the neutral point, the value of C_{m_α} is positive, and the airframe is then statically unstable.

C_{m_α} is perhaps the most important derivative as far as longitudinal stability and control are concerned. C_{m_α} primarily establishes the natural frequency of the short period mode, and is a major factor in determining the response of the airframe to elevator motions and to gusts. In general, a large negative value of C_{m_α} (i.e., large static stability) is desirable for good flying qualities. However, if C_{m_α} is too large, the required elevator effectiveness for satisfactory control may become unreasonably high. A compromise is thus necessary in selecting a design range for C_{m_α}. Design values of static stability are usually expressed not in terms of C_{m_α} but rather in terms of the derivative $C_{m_{C_L}}$, where the relation is: $C_{m_\alpha} = C_{m_{C_L}} C_{L_\alpha}$. It should be pointed out that $C_{m_{C_L}}$ in the above expression is actually a partial derivative for which the forward speed remains constant.

$\underline{C_{m_{\dot\alpha}}}$

The stability derivative $C_{m_{\dot\alpha}}$ is the change in pitching moment coefficient with variation in rate of change of angle of attack. The derivative $C_{m_{\dot\alpha}}$ arises essentially from two independent sources: an aerodynamic time lag effect and various "deadweight" aeroelastic effects. For low speed flight $C_{m_{\dot\alpha}}$ will come mostly from the aeroelastic lag effect and its sign will be negative. For high speed flight the sign of $C_{m_{\dot\alpha}}$ can be positive or negative depending on the nature of the aeroelastic effects. The horizontal tail of a conventional aircraft is mounted some distance aft of the wing. Whenever the wing undergoes a change in angle of attack, the downwash field is altered, and since it takes a finite length of time before this downwash alteration arrives at the tail, the resulting lift on the tail, and consequently the resulting pitching

moment on the airframe, lags the motion and creates the derivative $C_{m_{\dot{\alpha}}}$. Even for tailless aircraft there apparently exists a value for $C_{m_{\dot{\alpha}}}$ due to the fact that the wing must accelerate the air mass in its path as it accelerates. $C_{m_{\dot{\alpha}}}$ can also arise from aeroelastic effects such as wing twisting due to the deadweight moment caused by the projection of the nacelles in front of the wing, and bending of the fuselage caused by the dead weight of the aft fuselage and empennage section. This twisting and bending changes the effective angle of attack of the horizontal tail. The derivative $C_{m_{\dot{\alpha}}}$ is quite important in longitudinal dynamics because it is involved in the damping of the short period mode. A negative value of $C_{m_{\dot{\alpha}}}$ increases the damping of this mode; consequently, high negative values of this derivative are desirable.

$\underline{C_{m_q}}$

The stability derivative C_{m_q} is the change in pitching moment coefficient with varying pitch velocity and is commonly referred to as the pitch damping derivative. As the airframe pitches about its center of gravity path, the angle of attack of the horizontal tail changes, and a lift force is developed on the horizontal tail producing a negative pitching moment on the airframe and hence a contribution to the derivative C_{m_q}. There is also a contribution to C_{m_q} because of various "deadweight" aeroelastic effects. Since the airframe is moving in a curved flight path due to its pitching, a centrifugal force is developed on all the components of the airframe. The force can cause the wing to twist as a result of the dead weight moment of overhanging nacelles, and can cause the horizontal tail angle of attack to change as a result of fuselage bending due to the weight of the tail section. In low speed flight, C_{m_q} comes mostly from the effect of the curved flight path on the horizontal tail and its sign is negative. In high speed flight the sign of C_{m_q} can be positive or negative, depending on the nature of the aeroelastic effects. The derivative C_{m_q} is very important in longitudinal dynamics because it contributes a major portion of the damping of the short period mode for conventional aircraft. As pointed out, this damping effect comes mostly from the horizontal tail. For tailless aircraft, the magnitude of C_{m_q} is consequently small; this is the main reason for the usually poor damping of this type of configuration. C_{m_q} is also involved to a certain extent in the damping of the phugoid mode. In almost all cases, high negative values of C_{m_q} are desired. In the light of the present design trend toward larger radii of gyration in pitch and high altitude flight, it is believed that consideration of C_{m_q} is necessary in the preliminary design stage.

$\underline{C_{m_{\delta_e}}}$

The stability derivative $C_{m_{\delta_e}}$ is the change in pitching moment coefficient with change in elevator deflection and is commonly referred to as the elevator power. When the elevator is deflected upward, the resultant increment in lift on the horizontal tail creates a positive pitching moment about the center of gravity of the airframe; hence the derivative $C_{m_{\delta_e}}$ is normally positive in sign.

The primary function of the elevator is to control the angle of attack of the airframe both in equilibrium flight and in maneuvers. This function is usually considered to be the most important of all the control functions about the three axes, and so the elevator control effectiveness, $C_{m_{\delta_e}}$ is of great importance in airframe design. The

design value of $C_{m\delta_e}$ is essentially determined by the anticipated fore and aft center of gravity travel of an airframe. The larger the center of gravity range, the larger the required value of $C_{m\delta_e}$ will be. To keep the size of the elevator within practical bounds, the center of gravity range must be held as small as possible. Thus in many cases of design, the maximum practical $C_{m\delta_e}$ determines the allowable center of gravity range, and in other cases the center of gravity range determines the value of $C_{m\delta_e}$. A desirable value of $C_{m\delta_e}$ cannot be stated generally, for each case must be analyzed separately.

6.8 LATERAL-DIRECTIONAL DYNAMIC STABILITY

The lateral-directional motion will be considered independent of the longitudinal motion. This is very difficult to achieve in actual practice since any change in lift due to roll or sideslip will cause longitudinal disturbances. In the following analysis, any change in pitching moment, lift or drag will be considered zero and the aircraft will remain stabilized longitudinally.

Definitions and identities:

$$w = 0, \quad q = 0$$

$$\beta = \frac{v}{u_o}, \quad \dot{\beta} = \frac{\dot{v}}{u_o}, \quad u_o = V_T$$

LATERAL-DIRECTIONAL EQUATIONS OF MOTION

The complete side force equation is

$$\frac{F_x}{mV_T} = \frac{\dot{v} + ru - pw}{V_T} = Y_o + Y_\beta\bar{\beta} + Y_{\dot{\beta}}\dot{\beta} + Y_p\bar{p} + Y_r\bar{r} + Y_{\delta a}\delta a + Y_{\delta r}\delta r + Y_\phi\bar{\phi} \tag{6.55}$$

where $Y_\phi = \frac{1}{mV_T}\frac{\partial Y}{\partial \phi}$ (side force due to weight)

The rolling moment equation is

$$\frac{G_x}{I_x} = \dot{p} - \dot{r}\frac{I_{xz}}{I_x} + qr\left(\frac{I_z - I_y}{I_x}\right) - pq\frac{I_{xz}}{I_x} = L_o + L_\beta\bar{\beta} + L_{\dot{\beta}}\dot{\beta} + L_p\bar{p} + L_r\bar{r} + L_{\delta a}\delta a + L_{\delta r}\delta r \tag{6.56}$$

and the yawing moment equation is

$$\frac{G_z}{I_z} = \dot{r} + pq\left(\frac{I_y - I_x}{I_z}\right) + (rq - \dot{p})\frac{I_{xz}}{I_z} = N_o + N_{\dot{\beta}}\dot{\beta} + N_\beta\bar{\beta} + N_p\bar{p} + N_r\bar{r} + N_{\delta a}\delta a + N_{\delta r}\delta r \tag{6.57}$$

In order to simplify the equations, the following assumptions are made:

1. $I_{xz} \ll I_z$ Thus the I_{xz}/I_z term can be neglected.
2. $p = \dot{\phi}$ (no pitch rate)
3. $r = -\dot{\beta}$ ("cg" travels in a straight flight path)
4. The terms $Y_{\dot{\beta}}\dot{\beta}$, $Y_p\bar{p}$, $Y_r\bar{r}$, and $Y_{\delta a}\delta a$ are very small for a typical airplane.

Thus the equations reduce to

SIDE $\quad \dot{\beta} - \dot{\beta} = Y_\beta\bar{\beta} + Y_{\delta r}\delta r + Y_\phi\bar{\phi}$ (6.58)

ROLL $\quad \ddot{\phi} + \ddot{\beta}\frac{I_{xz}}{I_x} = L_\beta\bar{\beta} + L_{\dot{\beta}}\dot{\beta} - L_r\dot{\beta} + L_p\dot{\phi} + L_{\delta r}\delta r + L_{\delta a}\delta a$

(6.59)

YAW

$$-\ddot{\beta} = N_{\beta}\bar{\beta} + N_{\dot{\beta}}\dot{\beta} - N_{r}\dot{\beta} + N_{p}\dot{\phi} + N_{\delta r}\delta r + N_{\delta a}\delta a \tag{6.60}$$

grouping variables and equating them to the forcing functions,

$$\text{SIDE} \quad Y_{\beta}\bar{\beta} + Y_{\phi}\bar{\phi} = -Y_{\delta r}\delta r \tag{6.61}$$

$$\text{ROLL} \quad \frac{I_{xz}}{I_x}\ddot{\beta} - L_{\dot{\beta}}\dot{\beta} + L_{r}\dot{\beta} - L_{\beta}\bar{\beta} + \ddot{\phi} - L_{p}\dot{\phi} = L_{\delta r}\delta r + L_{\delta a}\delta a \tag{6.62}$$

YAW

$$-\dot{\beta} - N_{\dot{\beta}}\dot{\beta} + N_{r}\dot{\beta} - N_{\beta}\bar{\beta} - N_{p}\dot{\phi} = N_{\delta r}\delta r + N_{\delta a}\delta a \tag{6.63}$$

Using the Laplace operator(s) to replace the derivatives of the dependent variables,

$$\text{SIDE} \quad Y_{\beta}\bar{\beta}(s) + Y_{\phi}\bar{\phi}(s) = -Y_{\delta r}\delta r(s)$$

$$\text{ROLL} \quad \left(\frac{I_{xz}}{I_x}s^2 - L_{\dot{\beta}}s + L_{r}s - L_{\beta}\right) \cdot \bar{\beta}(s) + (s^2 - L_{p}s)\bar{\phi}(s) = L_{\delta r}\delta r(s) + L_{\delta a}\delta a(s)$$

YAW

$$(-s^2 - N_{\dot{\beta}}s + N_{r}s - N_{\beta})\bar{\beta}(s) - (N_{p}s)\bar{\phi}(s) = N_{\delta r}\delta r(s) + N_{\delta a}\delta a(s)$$

Ignoring the side force equation, and putting in matrix form,

$$\begin{bmatrix} \frac{I_{xz}}{I_x}s^2 + (-L_{\dot{\beta}} + L_{r})s - L_{\beta} & s^2 - L_{p}s \\ -s^2 - (N_{\dot{\beta}} - N_{r})s - N_{\beta} & -N_{p}s \end{bmatrix}$$

$$\begin{bmatrix} \bar{\beta}(s) \\ \bar{\phi}(s) \end{bmatrix} = \begin{bmatrix} L_{\delta r}\delta r(s) + L_{\delta a}\delta a(s) \\ N_{\delta r}\delta r(s) + N_{\delta a}\delta a(s) \end{bmatrix}$$

and the determinant of the coefficient matrix would be

$$\begin{vmatrix} \frac{I_{xz}}{I_x}s^2 + (-L_{\dot{\beta}} + L_{r})s - L_{\beta} & s^2 - L_{p}s \\ -s^2 - (N_{\dot{\beta}} - N_{r})s - N_{\beta} & -N_{p}s \end{vmatrix} = 0 \tag{6.64}$$

Expansion gives a fourth order equation of the form

$$s^4 + K_3s^3 + K_2s^2 + K_1s + K_o$$

To find the transient solution, the characteristic equation is set equal to zero.

$$s^4 + K_3s^3 + K_2s^2 + K_1s + K_o = 0 \tag{6.65}$$

Experience gained from extensive engineering studies and flight tests have shown that the roots are usually a large negative real one, a complex pair, and a very small one, either positive or negative. These correspond to the roll mode, the Dutch roll, and the spiral mode.

The roll mode is a heavily damped convergence, so heavily damped in fact that it is not readily discernible to the pilot. This root will be shown to be numerically very close to the roll damping derivative, L_p.

The oscillation called Dutch roll was so named because of its similarity to the motion of a Dutchman happily ice skating down a canal. The motion is a complex variation of sideslip, yaw and bank angle and is dependent upon the terms N_β, L_β, N_r and I_{xz}. It is often noticeably oscillatory, and when lightly damped, it is annoying and uncomfortable.

The spiral mode can be seen as the tendency of the aircraft to recover slowly, or diverge slowly, from an initial bank angle. If the characteristic is divergence, an ever-steepening spiral occurs, which is responsible for the name.

In order to gain some knowledge of the effects of the stability parameters, the terms N_p and $L_{\dot\beta}$ will be neglected. These terms are small in magnitude and will not detract from an understanding of lateral-directional dynamics. Thus the determinant (6.64) reduces to

$$\begin{vmatrix} \dfrac{I_{xz}}{I_x} s^2 + L_r s - L_\beta & s^2 - L_p s \\ -s^2 - (N_{\dot\beta} - N_r)s - N_\beta & 0 \end{vmatrix} = 0 \tag{6.66}$$

Expansion results in the following equation:

$$s(s - L_p)\,[s^2 + (N_{\dot\beta} - N_r)s + N_\beta] = 0 \tag{6.67}$$

The first two roots are:

$$\begin{aligned} s_1 &= 0 \qquad \text{(spiral mode)} \\ s_2 &= L_p \qquad \text{(roll mode)} \end{aligned} \tag{6.68}$$

The second order equation is in the familiar form

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

where

$$2\zeta\omega_n s = N_{\dot\beta} - N_r \tag{6.69}$$

and

$$\omega_n^2 = N_\beta$$

Thus the natural frequency of the Dutch roll is

$$\omega_n = \sqrt{N_\beta} \tag{6.70}$$

and the damping ratio of the Dutch roll is

$$\zeta = \frac{N_{\dot\beta} - N_r}{2\sqrt{N_\beta}} \tag{6.71}$$

Recalling that the damped frequency of an oscillation is given by:

$$\omega_d = \omega_n\sqrt{1 - \zeta^2}$$

the damped frequency of the Dutch roll is

$$\omega_d = \sqrt{N_\beta - \frac{(N_{\dot\beta} - N_r)^2}{4}} \tag{6.72}$$

Detailed analysis of the Dutch roll oscillation is much more complex than the simplified equations (6.71) and (6.72) would lead us to believe. The preceeding derivatives ignored the effect of X_{xz} and L_β, which reduce their validity to essentially a pure yawing or "snake" motion. In reality, the Dutch roll is a complex yawing and rolling motion. It is very difficult to iso-

late the terms having a primary effect on the motion, and thus there are no hard and fast rules that can be made regarding the effect of a single term on the frequency or damping.[4]

It has been found by a trial and error process of elimination that three terms exert a very profound effect on the motion, these being the lateral and directional stabilities and the product of inertia, C_{ℓ_β}, C_{n_β} and I_{xz} respectively.

There are some empirical rules that usually indicate conditions of low Dutch roll stability. The signs (±) of the stability parameters which lead to desired flying qualities are listed below:

Term	Usual Sign
C_{n_β}	positive
C_{ℓ_β}	negative
C_{n_r}	negative
I_{xz}	zero

Without regard to the nature of the stabilizing or destabilizing effect, an increase in the magnitudes of the above terms on the Dutch roll is:

Term	Effect on Stability
C_{n_β}	stabilizing
C_{ℓ_β}	destabilizing
C_{n_r}	stabilizing
I_{xz}	destabilizing

Both the directional stability, C_{n_β}, and yaw damping, C_{n_r}, are functions of the vertical tail lift curve slope, which decreases supersonically indicating a possible area of low Dutch roll stability above Mach 2.

Likewise, the lateral stability, C_{ℓ_β}, is heavily influenced by the vertical tail lift curve slope, and is usually a maximum near sonic velocity. Therefore this too is an area to be wary of.

The nose-down attitude associated with flaps extended gives a positive product of inertia and hence is a third area in which to anticipate poor Dutch roll stability.

Flight experience with the Dutch roll has shown that the pilot senses the ratio of roll to sideslip as well as the cyclic damping. This ratio is commonly called the ϕ/β (phi to beta) ratio and can be expressed by the following equation[5]

$$\left|\frac{\phi}{\beta}\right| = -\left(\frac{C_{\ell_\beta}}{C_{n_\beta}}\right)\left(\frac{I_z}{I_x}\right)\frac{1}{\sqrt{1+\frac{L_p^{\,2}}{N_\beta}}} \qquad (6.73)$$

6.9 EFFECT OF LATERAL-DIRECTIONAL STABILITY DERIVATIVES

C_{y_β}

The stability derivative C_{y_β} is the change in side force coefficient with changing sideslip angle. It can be referred to as the "side force damping derivative." When the airframe sideslips, the relative wind strikes the airframe obliquely, creating a side force, Y, on the vertical tail, the fuselage, and the wing. It must be remembered that this side force is measured along the y-axis, which is fixed to the airframe during the steady flight condition and moves with the airframe during a disturbance.

[4] Section 6.10, Modern Flight Dynamics, Kolk.

[5] Section 8.5, Modern Flight Dynamics, Kolk.

The major portion of C_{y_β} usually comes from the vertical tail, with smaller contributions from the fuselage and wing.

The derivative C_{y_β} is fairly important in lateral dynamics. Because it contributes to the damping of the Dutch roll mode, a large negative value of this derivative would appear desirable; however, a large negative value of C_{y_β} may create an undesirable lag effect in the airplane's response when an attempt is made to hold the wings level in rough air, or to perform roll maneuvers.

C_{ℓ_β}

The stability derivative C_{ℓ_β} is the change in rolling moment coefficient with variation in sideslip angle and is usually referred to as the "effective dihedral derivative." When the airframe sideslips, a rolling moment is developed because of the dihedral effect of the wing and because of the usual high position of the vertical tail relative to the equilibrium x-axis. No general statements can be made concerning the relative magnitudes of the contributions to C_{ℓ_β} from the vertical tail and from the wing since these contributions vary considerably from airframe to airframe and for different angles of attack of the same airframe. C_{ℓ_β} is nearly always negative in sign, signifying a negative rolling moment for a positive sideslip.

The derivative C_{ℓ_β} is very important in lateral stability and control, and it is therefore usually considered in the preliminary design of an airframe. It is involved in damping both the Dutch roll mode and the spiral mode. It is also involved in the maneuvering characteristics of an airframe, especially with regard to lateral control with the rudder alone near stall.

C_{ℓ_r}

The stability derivative C_{ℓ_r} is the change in rolling moment coefficient with change in yawing velocity. If the airframe is yawing at the rate r about the vertical axis, the left wing panel will move faster than the right, producing more lift on the left panel and consequently a positive rolling moment. In addition to this major wing contribution, the vertical tail will also contribute to C_{ℓ_r}.

The derivative C_{ℓ_r} is of secondary importance in lateral dynamics, but it should not be neglected in lateral dynamic calculations. For a conventional airframe configuration, changes in C_{ℓ_r} of reasonable magnitude show only slight effect on the Dutch roll damping characteristics. In the spiral mode, however, C_{ℓ_r} has a considerable effect.

C_{ℓ_p}

The stability derivative C_{ℓ_p} is the change in rolling moment coefficient with change in rolling velocity and is usually known as the roll damping derivative. When the airframe rolls at an angular velocity p, a rolling moment is produced as a result of this velocity; this moment opposes the rotation. C_{ℓ_p} is composed of contributions, negative in sign, from the wing and the horizontal and vertical tails. However, unless the size of the tails in unusually large in comparison with the size of the wing, the major portion of the total C_{ℓ_p} comes from the wing.

The derivative C_{ℓ_p} is quite important in lateral dynamics because essentially C_{ℓ_p} alone determines the damping in roll characteristics of the aircraft. Normally, it appears that small negative values of C_{ℓ_p} are more desirable than large ones because the airframe will respond better to a

given aileron input and will suffer fewer flight perturbations due to gust inputs.

The value of $C\ell_p$ does directly affect the design of the ailerons, since $C\ell_p$ in conjunction with $C\ell_{\delta a}$ establishes the airframe's maximum available rolling velocity; this is an important consideration in determining the aircraft's handling qualities.

$C\ell_{\delta_r}$

The stability derivative $C\ell_{\delta_r}$ is the change in rolling moment coefficient with variation in rudder deflection. Because the rudder is usually located above the x-axis, a positive rudder deflection will create a negative rolling moment. $C\ell_{\delta_r}$ is therefore usually negative in sign, however, it can be positive, depending on the particular airframe configuration and the angle of attack at which it is flying.

$C\ell_{\delta a}$

The stability derivative $C\ell_{\delta a}$ is the change in rolling moment coefficient with change in aileron deflection. It is commonly referred to as the aileron power.

As far as lateral dynamics are concerned, the derivative $C\ell_{\delta a}$ is the most important of the control surface derivatives. The aileron effectiveness in conjunction with the damping in roll ($C\ell_p$) establishes the maximum available rate of roll of an airframe, which is a very important consideration in fighter combat tactics. The aileron power is also very important in low speed flight during takeoffs and landings where adequate lateral control is necessary to counteract asymmetric gusts tending to roll the aircraft.

$C n_\beta$

The stability derivative $C n_\beta$ is the change in yawing moment coefficient with variation in sideslip angle. It is usually referred to as the static directional derivative or the "weathercock" derivative. When the airframe sideslips, the relative wind strikes the airframe obliquely, creating a yawing moment, N, about the center of gravity. The major portion of $C n_\beta$ comes from the vertical tail, which stabilizes the body of the airframe just as the tail feathers of an arrow stabilize the arrow shaft. The $C n_\beta$ contribution due to the vertical tail is positive, signifying static directional stability, whereas the $C n_\beta$ due to the body is negative, signifying static directional instability. There is also a contribution to $C n_\beta$ from the wing, the value of which is usually positive but very small compared to the body and vertical tail contributions.

The derivative $C n_\beta$ is very important in determining the dynamic lateral stability and control characteristics. Most of the references concerning full-scale flight tests and free-flight wind tunnel model tests agree that $C n_\beta$ should be as high as possible for good flying qualities. A high value of $C n_\beta$ aids the pilot in effecting coordinated turns and prevents excessive sideslip and yawing motions in extreme flight maneuvers and in rough air. $C n_\beta$ primarily determines the natural frequency of the Dutch roll oscillatory mode of the airframe, and it is also a factor in determining the spiral stability characteristics.

$C n_{\dot\beta}$

The stability derivative $C n_{\dot\beta}$ is the change in yawing moment coefficient with variations in rate of change of sideslip angle. If the airframe is undergoing a rate

of change of sideslip angle, $\dot{\beta}$, a yawing moment can be produced on the airframe by the vertical tail because of the sidewash time lag effects from the wing and fuselage.

The derivative $C_{n_{\dot{\beta}}}$ must be distinguished from the derivative C_{n_r}. All stability derivatives are partial derivatives; that is, they are taken with respect to one independent variable at a time, the rest of the independent variables remaining fixed. Thus, $C_{n_{\dot{\beta}}}$ arises from a transient motion in which the sideslip angle is increasing with time but the rate of yaw remains zero, whereas C_{n_r} arises from a motion where yaw angle is increasing with time but the change in sideslip angle remains zero.

When $C_{n_{\dot{\beta}}}$ cannot be neglected for a particular configuration, its effect on lateral dynamics will appear mainly in the Dutch roll damping characteristics. To increase this damping, positive values of $C_{n_{\dot{\beta}}}$ are desired.

C_{n_r}

The stability derivative C_{n_r} is the change in yawing moment coefficient with change of yawing velocity. It is known as the yaw damping derivative. When the airframe is yawing at an angular velocity r, a yawing moment is produced which opposes the rotation. C_{n_r} is made up of contributions from the wing, the fuselage, and the vertical tail, all of which are negative in sign. The contribution from the vertical tail is by far the largest, usually amounting to about 80 or 90 percent of the total C_{n_r} of the airframe.

The derivative C_{n_r} is very important in lateral dynamics because it is the main contributor to the damping of the Dutch roll oscillatory mode. It also is important to the spiral mode. For each mode, large negative values of C_{n_r} are desired.

C_{n_p}

The stability derivative C_{n_p} is the change in yawing moment coefficient with varying rolling velocity. It arises from two main sources: The wing and the vertical tail. A negative yawing moment is developed on the airframe because of the unsymmetrical lift distribution causing a difference between the drag on the right wing and that on the left wing when the airframe is rolling. The contribution from the vertical tail can be either positive or negative depending on the vertical tail geometry, the sidewash from the wing, and the equilibrium angle of attack of the airframe.

The derivative C_{n_p} is fairly important in lateral dynamics because of its influence on Dutch roll damping. It is usually negative in sign, and for most airframe configurations, the larger its negative value, the greater the reduction in Dutch roll damping.

$C_{n_{\delta_r}}$

The stability derivative $C_{n_{\delta_r}}$ is the change in yawing moment coefficient with variation in rudder deflection. This derivative is commonly referred to as the rudder power.

The importance of $C_{n_{\delta_r}}$ in determining lateral and directional control qualities varies considerably with different types of airframes.

The design value of $C_{n_{\delta_r}}$ for a jet-powered airframe is usually determined by considering such requirements as counteracting adverse yaw in rolling maneuvers, directional control in crosswind takeoffs and landings, antisymmetric power, and spin recovery control.

$\underline{Cn_{\delta a}}$

The stability derivative $Cn_{\delta a}$ is the change in yawing moment coefficient with change of aileron deflection. This derivative arises from the difference in drag due to the down aileron compared to the drag of the up aileron. The sign of $Cn_{\delta a}$ depends mainly upon the rigging of the aileron and the angle of attack of the airframe. If negative, as it usually is, $Cn_{\delta a}$ is called the "adverse yaw coefficient due to ailerons" because it causes the airframe to yaw initially in a direction opposite to that desired by the pilot when he deflects the ailerons for a turn. If positive, it produces favorable yaw in the turning maneuver.

The derivative $Cn_{\delta a}$ is quite important in determining the lateral control qualities of an airframe. For good response to aileron deflection, $Cn_{\delta a}$ should be zero or of a very small positive value.

6.10 DYNAMIC MOTION

Period of an Oscillation:

The period is the time required to complete one cycle and is given by:

$$P = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}} \quad \text{(sec.)} \qquad (6.74)$$

Time to Dampen to 1/nth Amplitude:

The amplitude at any time is given by

$$A = A_o e^{-\zeta\omega_n t}$$

Consider the amplitude of a dynamic response at time t_1 and t_2:

$$A_1 = A_o e^{-\zeta\omega_n t_1}$$

$$A_2 = A_o e^{-\zeta\omega_n t_2}$$

The time required for the amplitude to dampen from A_1 to A_2 is:

$$\Delta t = t_2 - t_1 = \frac{1}{-\zeta\omega_n} \ln\left(\frac{A_2}{A_1}\right)$$

The time to dampen to 1/n amplitude is important in dynamics and is given by

$$\Delta t_{1/n} = \frac{1}{-\zeta\omega_n} \ln\left(\frac{1}{n}\right) \qquad (6.75)$$

The time to dampen to 1/2 and 1/10 amplitude is:

$$\Delta t_{1/2} = \frac{0.693}{\zeta\omega_n} \qquad (6.76)$$

$$\Delta t_{1/10} = \frac{2.30}{\zeta\omega_n} \qquad (6.77)$$

Cycles to Dampen to 1/n Amplitude:

The number of cycles to dampen to a given fraction of the original amplitude is important in any dynamic stability investigation. The values of particular interest are the number of cycles to dampen to 1/2 amplitude and 1/10 amplitude.

$$C_{1/n} = \frac{\Delta t_{1/n}}{P} = -\frac{\ln\left(\frac{1}{n}\right)}{2\pi} \frac{\sqrt{1-\zeta^2}}{\zeta} \qquad (6.78)$$

and for 1/2 and 1/10 amplitude.

$$C_{1/2} = 0.1104 \frac{\sqrt{1-\zeta^2}}{\zeta} \qquad (6.79)$$

$$C_{1/10} = 0.367 \frac{\sqrt{1-\zeta^2}}{\zeta} \qquad (6.80)$$

It is apparent that the cyclic damping is only a function of the damping ratio. For small damping ratios, the cyclic damping is closely approximated by:

$$C_{1/2} \approx \frac{0.11}{\zeta} \qquad (6.81)$$

$$C_{1/10} \approx \frac{0.37}{\zeta} \qquad (6.82)$$

For the Dutch roll, the number of cycles to dampen to 1/2 amplitude is approximately

$$C_{1/2} \approx 0.22 \frac{\sqrt{N_\beta}}{N_{\dot\beta} - N_r} \qquad (6.83)$$

which shows that the number of cycles to dampen is primarily a function of yaw stability and yaw damping and is somewhat insensitive to stability or damping in roll.

6.11 HANDLING QUALITIES

The characteristic modes of motion of the airplane as previously discussed are the airplane's response to some random disturbance, or perturbation. The motions (short period, phugoid, Dutch roll and spiral) are stable if they do not persist but rather decay with time. This stability is called controls - fixed stability because there are no control inputs by the pilot. The pilot does have an inherent desire to maintain equilibrium flight, however, and the amount of effort and concentration necessary to do so is an indication to him of the handling qualities of his aircraft. Furthermore, the ease with which he can maneuver gives him a feeling of either confidence or apprehension about the airplane.

The pilot issues his commands to the airplane through the flight controls and therefore his primary judgment of the handling qualities is his assessment of what he must do to the controls to execute a particular maneuver, or conversely, how the aircraft responds to a given control input. He is, therefore, aware of two things; his control input and the eventual aircraft response. Thus the distinction must be made between the control system dynamics and the dynamics of the airframe. This chapter will not include the effects of control system dynamics (a subject in itself) but will be limited to discussing the effects of airframe dynamics on handling qualities.

Handling qualities can only be defined when related to a particular pilot task or aircraft mission. First we must relate the characteristic modes of motion to flight vehicle handling qualities; then through the proper selection of control systems or artificial stabilization devices, the overall handling qualities can be changed.

LONGITUDINAL SHORT PERIOD MOTION

The short period mode is essentially a constant speed oscillation with the aircraft experiencing both normal and pitching accelerations. The frequency and damping vary with center of gravity position and dynamic pressure. The pilot evaluates the frequency and damping characteristics of the type aircraft under consideration. If the frequency of motion is low, the response is usually termed "sluggish." As the frequency increases, it can move into a region where the pilot cannot keep in phase with the motion. When he does attempt to damp the oscillation, he in fact reinforces it with a "pilot induced oscillation" (PIO). As the frequency further increases, the pilot recognizes his inability to damp the oscillation and does not attempt to

do so. Previous discussion has shown that the number of overshoots the oscillation has is a function of the damping ratio and not the frequency of oscillation.

Figure 6.7 shows the results of a pilot opinion study for the combination of natural frequency and damping ratio which produces the best aircraft handling qualities. Generally, the pilot is aware of an oscillatory tendency if ζ is less than 0.5, and regards the aircraft as being "sluggish" when the natural frequency is 0.5 cps or less.

If the airplane short period response to an ideal elevator input is considered, the best ratio of frequency to damping can be determined (figure 6.8).

Case (a) is low frequency that has a heavily damped response, (b) is high frequency with a lightly damped response. Case (c) is low frequency, lightly damped, and (d) is high frequency which is about 50 percent damped.

A low frequency short period is typical of a large subsonic bomber-transport aircraft that has

Figure 6.7

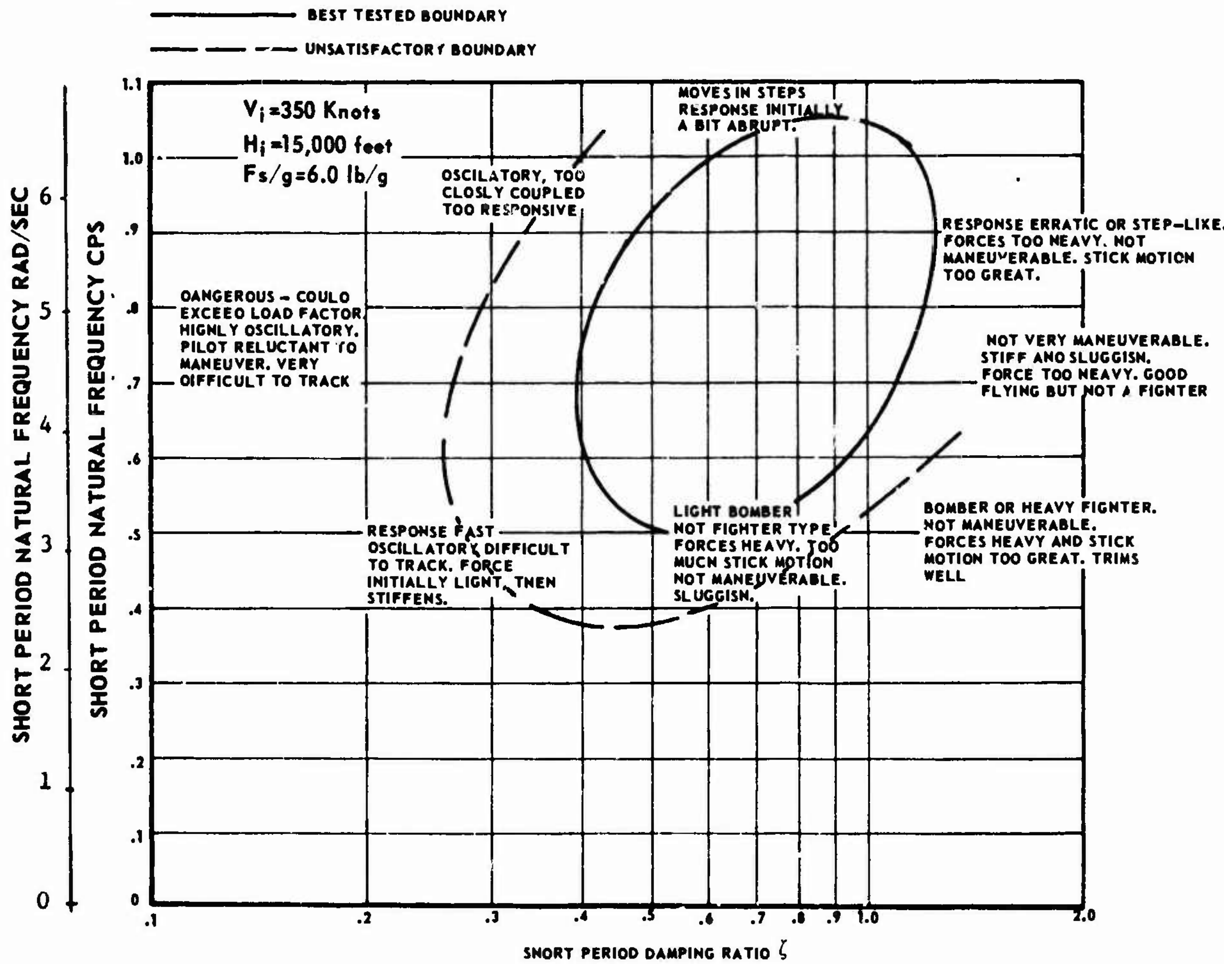
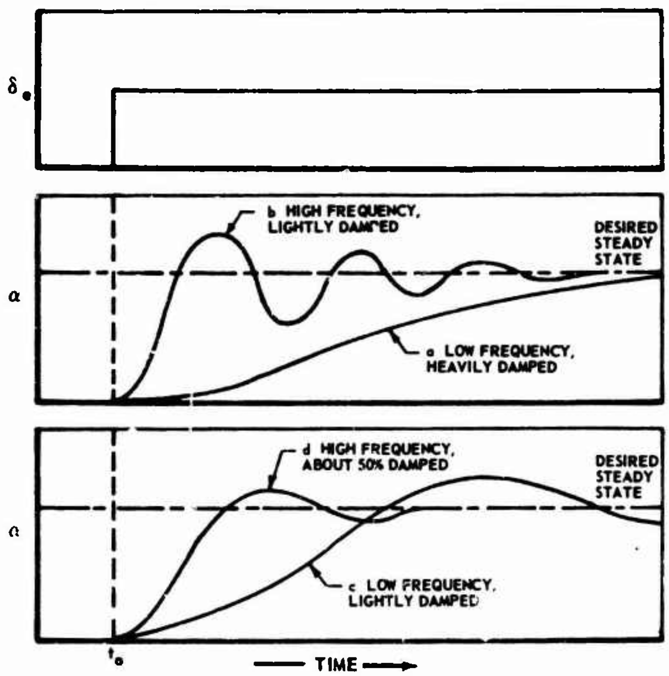


Figure 6.8



a nonmaneuvering mission. If this motion is lightly damped, the pilot observes a quick response but it overshoots and his attempts to correct this results in overcontrolling. A high frequency short period is typical of a subsonic fighter-type aircraft that has a maneuvering mission. If this motion is lightly damped the pilot again sees a quick response and an oscillation. The motion steadies out faster than the low frequency heavily damped motion but the pilot doesn't like the "bouncing" of the nose and the difficulty it imposes on a tracking task. If the high frequency motion is about 50 percent damped the pilot observes a quick response and the overshoot is past so soon he hardly notices it.

In both cases a heavily damped short period is desired. If the short period frequency is too low, the response can be corrected by a control system that puts in a large deflection and then automatically takes it out as the desired angle of attack is reached. If the frequency is too high, the response can be corrected by the use of a pitch damper. Thus the short period characteristics can be modified and on occasion brought from unacceptable to acceptable.

LONGITUDINAL LONG PERIOD MOTION (PHUGOID)

The phugoid mode (essentially a constant angle of attack, varying airspeed and altitude oscillation) has such a long period that even large changes in frequency make little difference to the pilot. Even the highest phugoid frequency has a period several times in excess of the pilot's normal response time.

Section 6.6 mentioned the possibility of a divergent phugoid in the power approach configuration. There are, in addition, some objectionable features of the phugoid, even though it may not be divergent. Studies have shown that pilots do not object to the phugoid over a wide variation of frequency and damping as long as they have a visual horizon. Poor handling qualities do result when the pilot uses an artificial horizon coupled with low damping ratios, regardless of frequency. The problem stems from the increased pilot attention required to hold airspeed and altitude.

Equation (6.51) shows that the damping of the phugoid is heavily dependent on the stability parameter D_u. Thus the minimum damping occurs when the aircraft is cruising at the speed for (L/D) max (maximum range for propeller aircraft and maximum endurance for jet aircraft). The damping ratio for the phugoid can also be expressed as:[6]

$$\zeta = \frac{1}{\sqrt{2}\,(L/D)} \qquad (6.84)$$

[6] Section 8.4, Modern Flight Dynamics, Kolk.

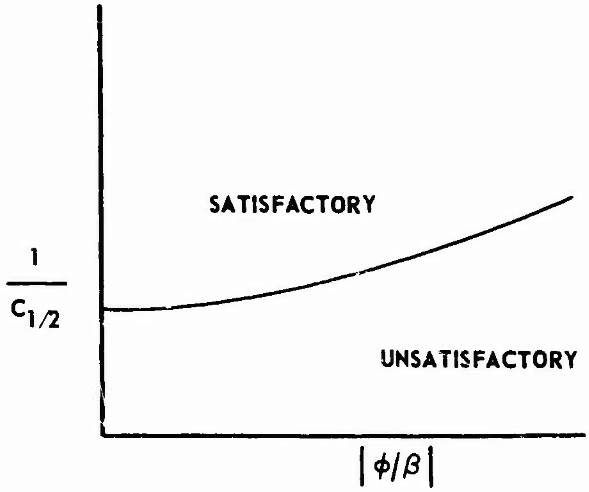
Configurations designed specifically for high lift-to-drag ratios are most susceptible to unsatisfactory damping. The designer's best efforts to reduce drag will ironically not be in the best interests of phugoid handling qualities. The features of an aircraft with low phugoid damping describe the subsonic long range transport, and every effort to maximize its performance undermines its inherent cruising ability. With rapidly expanding air traffic, the threat of mid-air collision emphasizes the need for aircraft to maintain prescribed altitudes; yet their capacity to do so is deteriorating. Clearly, automatic stabilization will become commonplace to correct poor phugoid characteristics.

LATERAL-DIRECTIONAL OSCILLATORY MOTION (DUTCH ROLL)

Substantial effort has been expended to define the limits of satisfactory and unsatisfactory Dutch roll handling qualities. Frequency and damping considerations alone do not give the true picture. The pilot is not concerned with how long the motion persists, but rather with the number of cycles he senses. Early conclusions were that cyclic damping could serve as a guide, with the Dutch roll becoming more objectionable as the cycles to damp to one-half amplitude increased.

World War II found aircraft flying at higher altitudes than before. Combat ceilings rose, but so did increasing complaints of a rolling tendency that was not present at medium or low altitudes. Jet aircraft pushed the ceilings higher and found more acute rolling tendencies. The problem was to provide better cyclic damping at high altitude, but a parameter indicative of such a requirement was not obvious. The parameter had to be one that reflected the transition of the Dutch roll from a yawing motion at low altitude to a rolling motion at high altitude. The ratio of roll to sideslip seemed to describe the motion. It was hoped that a line delineating between satisfactory and unsatisfactory Dutch roll could be empirically found on a plot of inverse cyclic damping versus the $|\phi/\beta|$ ratio as shown in figure 6.9.

Figure 6.9



This was found to be the case, and better cyclic damping was needed as the $|\phi/\beta|$ ratio increased. Unfortunately, the plot is valid only for one airspeed at one altitude. In order to make the plot valid for all flight conditions, the Dutch roll is analyzed in terms of the roll parameter $|\phi/v_e|$ where

$$\left|\frac{\phi}{v_e}\right| = \left|\frac{\phi}{\beta}\right|\left(\frac{57.3}{v_e}\right) \qquad (6.85)$$

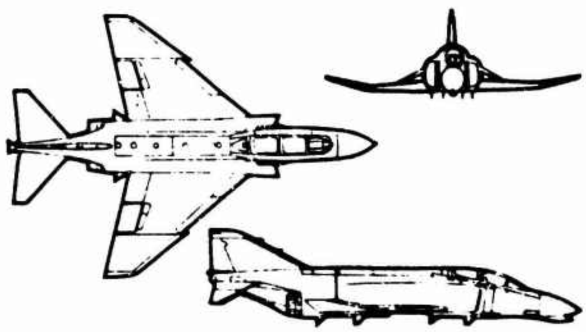
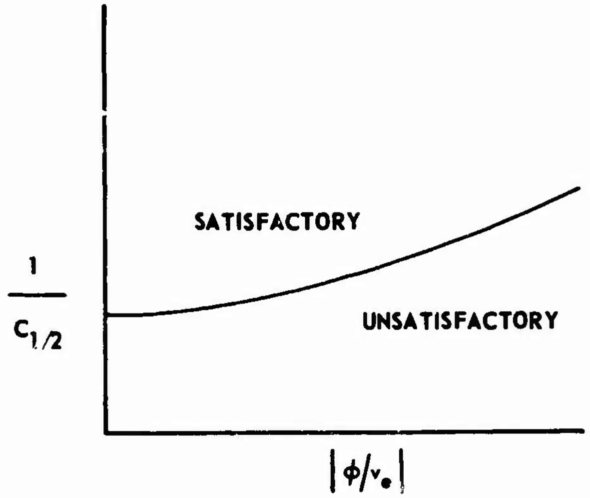


Figure 6.18



Equation (6.73) is repeated here to facilitate discussion of the $|\phi/\beta|$ ratio - the primary consideration in the acceptability of the Dutch roll mode.

$$\left|\frac{\phi}{\beta}\right| = -\left(\frac{C_{\ell_\beta}}{C_{n_\beta}}\right)\left(\frac{I_z}{I_x}\right)\frac{1}{\sqrt{1+\frac{L_p^{\,2}}{N_\beta}}} \qquad (6.73)$$

Reducing C_{ℓ_β} and increasing C_{n_β} reduces the rolling tendency. C_{ℓ_β} has little effect on cyclic damping, but C_{n_β} does improve it. A decrease in yaw inertia (I_z) and an increase in roll inertia (I_x) also reduces the rolling tendency. However, the emphasis on speed has dictated thin, low aspect ratio wings and high concentration of mass in the fuselage. Thus fuselage loaded aircraft with their higher ratios of yaw to roll inertia are characterized by their high $|\phi/\beta|$ ratios. The denominator of equation (6.73) includes the term $L_p{}^2/N_\beta$, which can be shown to be proportional to the air density ratio, σ. As an increase in altitude reduces σ, the $|\phi/\beta|$ ratio will increase. Modern aircraft all use artificial damping in the form of yaw dampers to improve the Dutch roll handling qualities.

LATERAL-DIRECTIONAL NONOSCILLATORY MOTION (SPIRAL MODE)

Flight investigations have shown that pilots tolerate some spiral instability, but the severity of acceptable divergence varies with the flight conditions. Under VFR conditions, the divergence is acceptable even if it doubles in amplitude in 5 seconds. Under IFR conditions, however, where the pilot's workload is increased with enroute charts, letdown books, and radio transmissions coupled with precision instrument flying, the acceptable time to double the amplitude is about 20 seconds.

The spiral motion is nonoscillatory and is usually divergent. Either a gust or control movement causing the aircraft to enter a bank is sufficient to initiate the motion. Once the bank is attained, the aircraft sideslips toward the low wing. If the aircraft has more lateral stability (C_{ℓ_β}) than directional stability (C_{n_β}), the rolling moment due to sideslip will tend to roll the aircraft out of the bank. In this case, the spiral mode is convergent. If the directional stability is dominant, the aircraft will weathervane into a spiral.

In level flight, the spiral root to the characteristic equation (6.67) is closely approximated by

$$s = \frac{g}{u_o}\left(-L_\beta N_r + L_r N_\beta\right) \qquad (6.86)$$

The roll due to yaw rate, L_r, is usually positive, and with the remaining parameters taking their conventional signs, the first term of (6.86) is negative and therefore stabilizing whereas the second term is positive and destabilizing. The most important parameters are L_β and N_β.

To correct for a severe spiral divergence, any of the four parameters can be altered, but only L_r can be changed without modifying the Dutch roll. The only independent correction that can be made to L_r is by artificial means, such as aileron deflection proportional to yaw rate.

LATERAL NONOSCILLATORY MOTION (ROLL MODE)

An airplane's ability to roll is properly a characteristic of its maneuverability, but it is also part of its handling qualities, due to the possibility of encountering roll coupling instability at high roll rates. This is the upper limit on usable roll rate.

Maneuverability makes two demands of roll performance: ability to lift a wing and ability to change direction. During landing, the pilot seeks to quickly raise a wing that dropped due to crosswinds or a gust. The faster his airspeed, the higher the roll rate must be for a given level of roll performance. Takeoff and landing thus provide natural design limits on minimum roll rate.

The ability to change direction quickly is contingent upon roll performance. The pilot, through his lateral control, rolls into a bank and pulls back on the stick making the plane fly a curved path. The time to reach a desired bank angle, Φ, is the important consideration.

6.12 PILOT RATING SCALES[7]

The pilot, being the final judge of an airplane's handling qualities, is ideally the most qualified individual to lead the design effort; but this presupposes a thorough engineering background. It is usually necessary for the engineer to accept the design burden and determine which aerodynamic qualities the pilot interprets as desirable and undesirable. Recourse is made to an evaluation of pilot opinion, a complicated process because each pilot expects something different from an airplane.

The Cornell Aeronautical Laboratory's Flight Research Department has made notable contributions to the use and understanding of pilot rating scales and pilot opinion surveys. Except for minor variations between pilots, which sometimes prevent a sharp delineation between acceptable and unacceptable flight characteristics, there is very definite consistency and reliability in pilot opinion. In addition, the opinions of well qualified test pilots can be exploited because it has been found that other pilots are invariably satisfied with the characteristics the test pilots conclude to be acceptable.

The stability and control characteristics of airplanes are generally established by wind tunnel measurement and by technical analysis as part of the airplane design process. The handling qualities of a particular airplane are realted to the stability and control characteristics. The relationship is a complex one which involves the combination of the airplane and its human pilot in the accomplishment of the intended use, or mission. It is important that the effects of specific stability and control characteristics be evaluated in terms of their ultimate effects on the suitability of the pilot-vehicle combination for the mission. On the basis of this information, intelligent decisions can be made during the airplane design phase which will lead to the desired handling qualities of the final product.

There are three general ways in which the relationship between stability and control parameters

[7] A Revised Pilot Rating Scale for the Evaluation of Handling Qualities, CAL Report No. 153, Robert P. Harper and George E. Cooper.

and the degree of suitability of the airplane for the mission may be examined:

1. Theoretical analysis,

2. Experimental performance measurement,

3. Pilot evaluation.

Each of the three approaches has an important role in the complete evaluation. One might ask, however, why is the pilot assessment necessary? At present the mathematical analysis including representation of the human operator best lends itself to analysis of specific simple tasks. Since the intended use (mission) is made up of several tasks and several modes of pilot-vehicle behavior, difficulty is experienced first in accurately describing all modes analytically, and second in integrating the quality of the subordinate parts into a measure of overall quality for the intended use. In spite of these difficulties, theoretical analysis is fundamental to understanding pilot-vehicle difficulties, and pilot evaluation without it remains a purely experimental process.

The attainment of satisfactory performance in fulfillment of a designated mission is, of course, a fundamental reason for our concern with handling qualities. Why cannot the experimental measurement of performance replace pilot evaluation? Why not measure pilot-vehicle performance in the intended use - isn't good performance consonant with good quality? A significant difficulty arises here in that the performance measurement tasks may not demand of the pilot all that the real mission demands. The pilot is an adaptive controller whose goal (when so instructed) is to achieve good performance. In a specific task, he is capable of attaining essentially the same performance for a wide range of vehicle characteristics, at the expense of significant reductions in his capacity to assume other duties and planning operations. Significant differences in task performance may not be measured where very real differences in mission suitability do exist.

The questions which arise in using performance measurements may be summarized as follows: (1) For what maneuvers and tasks should measurements be made to define the mission suitability? (2) How do we integrate and weigh the performance in several tasks to give an overall measure of quality if measurable differences do exist? (3) Is it necessary to measure or evaluate pilot workload and attention factors for performance to be meaningful? If so, how are these factors weighed with those in (2)? (4) What disturbances and distractions are necessary to provide a realistic workload for the pilot during the measurement of his performance in the specified task?

Pilot evaluation still remains the only method of assessing the interactions between pilot performance and workload in determining suitability of the airplane for the mission. It is required in order to provide a basic measure of quality and to serve as a standard against which pilot-airplane system theory may be developed, against which performance measurements may be correlated, and with which significant airplane design parameters may be determined and correlated.

The technical content of the pilot evaluation generally falls into two categories: one, the identification of characteristics which interfere with the intended use, and two, the determination of the extent to which these characteristics affect mission accomplishment. The latter judgment may be formalized as a pilot rating.

In 1956, the newly formed Society of Experimental Test Pilots

accepted responsibility for one program session at the annual meeting of the Institute of Aeronautical Sciences. For this purpose, a paper, entitled "Understanding and Interpreting Pilot Opinion" was prepared, which represented an attempt to create better understanding and utilization of pilot opinion and evaluation in the field of aeronautical research and development. At that time, the pilot rating system shown in figure 6.11 was introduced. In recent years, this scale, developed at the National Advisory Committee for Aeronautics (NACA), Ames Aeronautical Laboratory, has come into rather extensive use, not only in aeronautical, but space flight work as well. The widespread acceptance of such a rating system has indicated a general need for some uniform method of assessing aircraft handling qualities through pilot opinion. The Cooper rating system has received widespread use and in some cases, misuse.

One other scale in use when the Cooper scale was introduced was developed at Cornell Aeronautical Laboratory, Inc. (CAL), and is shown in figure 6.12. Both rating scales were independently developed during the early use of variable stability aircraft. These vehicles, as well as the use of ground simulation, made possible systematic studies of aircraft handling qualities through pilot evaluation, and rating of the effects of specific stability and control parameters. During subsequent evaluation programs, CAL made use of the 10-point Cooper scale. CAL developed a revised 10-point rating scale for later evaluations and is shown in figure 6.13

It appeared logical therefore, for Ames and CAL to prepare a joint report for the purpose of proposing an improved scale as well as clarifying the use of rating scales. A joint venture was undertaken in an attempt to use the experience gained at Ames and at CAL to stimulate a critical review of the existing methods of handling qualities assessment, to review the use of rating scales in general, and to clarify the concepts and procedures used in any pilot evaluation. Constructive criticism of the original scales was made by many individual test pilots and engineers, and to the extent possible, was considered in the preparation of the report. It was hoped that a single improved scale might result from the coordinated effort.

Figure 6.11 ORIGINAL COOPER SCALE

	ADJECTIVE RATING	NUMERICAL RATING	DESCRIPTION	PRIMARY MISSION ACCOMPLISHED?	CAN BE LANDED
NORMAL OPERATION	SATISFACTORY	1	EXCELLENT, INCLUDES OPTIMUM	YES	YES
		2	GOOD, PLEASANT TO FLY	YES	YES
		3	SATISFACTORY, BUT WITH SOME MILDLY UNPLEASANT CHARACTERISTICS	YES	YES
EMERGENCY OPERATION	UNSATISFACTORY	4	ACCEPTABLE, BUT WITH UNPLEASANT CHARACTERISTICS	YES	YES
		5	UNACCEPTABLE FOR NORMAL OPERATION	DOUBTFUL	YES
		6	ACCEPTABLE FOR EMERGENCY CONDITION ONLY*	DOUBTFUL	YES
NO OPERATION	UNACCEPTABLE	7	UNACCEPTABLE EVEN FOR EMERGENCY CONDITION*	NO	DOUBTFUL
		8	UNACCEPTABLE - DANGEROUS	NO	NO
		9	UNACCEPTABLE - UNCONTROLLABLE	NO	NO
	UNPRINTABLE	10	"MOTIONS POSSIBLY VIOLENT ENOUGH TO PREVENT PILOT ESCAPE"		

* (Failure of a stability augmenter)

The following paragraphs take us through the development of a revised rating scale. This analysis was performed by the Cornell Aeronautical Laboratory, and is presented here to give the reader an insight into the considerations involved.

To examine these considerations let us first review the present Cooper scale, shown in figure 6.11. The Cooper scale first proposed the basic framework that is still the foundation of pilot rating scales and, as will be noted later, is fundamental to the revised scale. This framework involves several grades of quality as related to the intended use of the vehicle: acceptable and satisfactory - and therefore sufficiently good; acceptable but unsatisfactory - not sufficient, but still usable; unacceptable - and therefore not adequate for the mission; and uncontrollable.

Figure 6.12 EARLY CAL PILOT RATING SCALE

OPTIMUM	THIS CONFIGURATION IS THE BEST ALL AROUND. IT COMBINES BEST PRECISION OF CONTROL WITH MOST COMFORTABLE CONTROL.
ACCEPTABLE GOOD	NOTICEABLY BETTER THAN ACCEPTABLE BUT STILL COULD BE IMPROVED. FOR EXAMPLE, VERY COMFORTABLE TO FLY BUT NOT THE BEST CONTROL PRECISION.
ACCEPTABLE	IN THIS CONFIGURATION, THE AIRPLANE'S MISSION COULD BE ACCOMPLISHED REASONABLY WELL, BUT WITH CONSIDERABLE PILOT EFFORT OR ATTENTION REQUIRED DIRECTLY FOR FLYING THE AIRPLANE.
ACCEPTABLE POOR	AIRPLANE SAFE TO FLY, BUT PILOT EFFORT OR ATTENTION REQUIRED IS SUCH AS TO REDUCE SERIOUSLY THE EFFECTIVENESS OF THE AIRPLANE IN ACCOMPLISHING ITS MISSION.
UNACCEPTABLE	PILOT EFFORT OR ATTENTION REQUIRED TO THE EXTENT THAT THE AIRPLANE'S ABILITY TO ACCOMPLISH ITS MISSION IS DOUBTFUL. OR, AIRPLANE WOULD BE UNSAFE TO FLY IF PILOT'S ATTENTION IS REQUIRED FOR NAVIGATION, RADIO, COMBAT, ETC.

THE PILOT IS PERMITTED TO ATTACH A PLUS OR MINUS TO THE RATINGS GIVEN ABOVE IF HE FEELS A FINER BREAKDOWN IS NECESSARY.

Figure 6.13 LATER CAL PILOT RATING SCALE

CATEGORY	ADJECTIVE DESCRIPTION WITHIN CATEGORY	NUMERICAL RATING
ACCEPTABLE AND SATISFACTORY	EXCELLENT	1
	GOOD	2
	FAIR	3
ACCEPTABLE BUT UNSATISFACTORY	FAIR	4
	POOR	5
	BAD	6
UNACCEPTABLE	BAD*	7
	VERY BAD**	8
	DANGEROUS'	9
UNFLYABLE	UNFLYABLE	10

*REQUIRES MAJOR PORTION OF PILOT'S ATTENTION
**CONTROLLABLE ONLY WITH A MINIMUM OF COCKPIT DUTIES
'AIRCRAFT JUST CONTROLLABLE WITH COMPLETE ATTENTION

The CAL scale of figure 6.13 was developed primarily because the Cooper scale was confusing to some in that it could be interpreted as introducing an alternate mission concept. Separate boundaries were shown for normal operation and for an undefined emergency condition. By removing this doubt of mission completion in the adjective descriptions in the acceptable range, as well as removing all consideration of an alternate mission from the scale itself, the CAL scale clarified this situation. However, the very simple descriptions of the CAL scale are not considered particularly helpful by many pilots.

Aside from these points, the two scales reflect a basic similarity which is indicative of a common desire to define mission suitability by:

1. Ensuring that investigators adequately define the mission and program objectives.
2. Firmly establishing within the scale the acceptable - unacceptable, and satisfactory - unsatisfactory boundaries.
3. Creating a logical basis to enable the pilot to express his assessment accurately and repeatably.

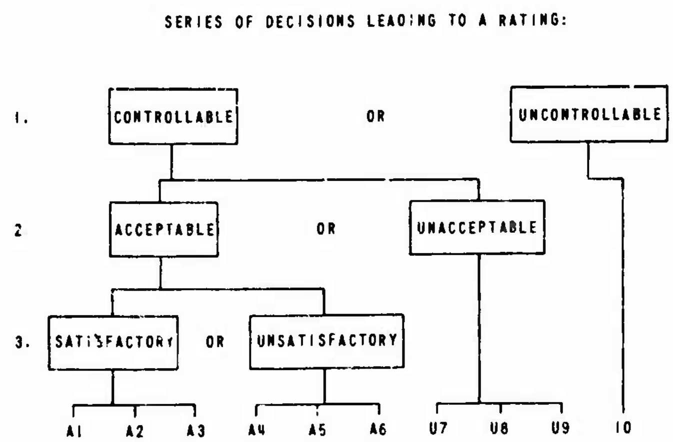
DEVELOPMENT OF A REVISED SCALE

Having now indicated certain of the objections and recognized deficiencies of two rating scales, we are in a position to construct a revised scale by starting with those fundamental areas of common agreement just mentioned. As previously noted, a pilot rating is a portion of the technical report of the evaluator, and is the overall summation of the suitability of the vehicle for the specified use. The pilot rating scale is then a systematic means of denoting the quality of the pilot-vehicle combination in the accomplishment of its intended purpose.

MAJOR CATEGORIES

The basic structure of the Cooper and CAL rating scales is retained in the proposed scale. Similar major categories of quality are proposed which stress the relationship of quality relative to the intended use. However, the meanings of the major categories are explicitly defined. Furthermore, the pilot's consideration of the categories is systematized by arranging a sequential series of selections between two alternatives that lead to the proper category selection.

Figure 6.14 SEQUENTIAL PILOT RATING DECISIONS



This structure is shown in figure 6.14, where a flow chart is presented to enable tracing the series of dichotomous decisions which the pilot makes in arriving at the final rating. As a rule, the first decision may be fairly obvious. Is the configuration controllable or uncontrollable? To determine whether this decision applies throughout the mission may not be so obvious.

If the airplane is uncontrollable in the mission, it is rated

10. If it is controllable, the second decision examines whether it is acceptable or unacceptable. If unacceptable, the ratings U7, U8, and U9 are to be considered (rating 10 has been excluded by the "controllable" answer to the first decision). If it is acceptable, the third decision must examine whether it is satisfactory or unsatisfactory. If unsatisfactory, the ratings A4, A5 and A6 are to be considered; if satisfactory, the ratings A1, A2, and A3 are to be considered.

The basic categories must be described in carefully selected terms to clarify and standardize the boundaries desired. Following a careful review of dictionary definitions and consideration of the pilot's requirement for clear, concise descriptions, the category definitions shown in figure 6.15 were selected. When considered in conjunction with the structural outline presented in figure 6.14, a clearer picture of the series of decisions which the pilot must make is obtained.

Figure 6.15 MAJOR CATEGORY DEFINITIONS

CATEGORY	DEFINITION
CONTROLLABLE	CAPABLE OF BEING CONTROLLED OR MANAGED IN CONTEXT OF MISSION, WITH AVAILABLE PILOT ATTENTION.
UNCONTROLLABLE	CONTROL WILL BE LOST DURING SOME PORTION OF MISSION.
ACCEPTABLE	MAY HAVE DEFICIENCIES WHICH WARRANT IMPROVEMENT BUT ADEQUATE FOR MISSION. PILOT COMPENSATION, IF REQUIRED TO ACHIEVE ACCEPTABLE PERFORMANCE, IS FEASIBLE.
UNACCEPTABLE	DEFICIENCIES WHICH REQUIRE MANDATORY IMPROVEMENT. INADEQUATE PERFORMANCE FOR MISSION, EVEN WITH MAXIMUM FEASIBLE PILOT COMPENSATION.
SATISFACTORY	MEETS ALL REQUIREMENTS AND EXPECTATIONS; GOOD ENOUGH WITHOUT IMPROVEMENT. CLEARLY ADEQUATE FOR MISSION.
UNSATISFACTORY	RELUCTANTLY ACCEPTABLE. DEFICIENCIES WHICH WARRANT IMPROVEMENT. PERFORMANCE ADEQUATE FOR MISSION WITH FEASIBLE PILOT COMPENSATION.

Let us examine what is meant by controllable. To control is to exercise direction of, or to command. Control also means to regulate. The determination as to whether the airplane is controllable or not must be made within the framework of the defined mission or intended use. An example of the considerations of this decision would be the evaluation of fighter handling qualities during which the evaluation pilot encounters a configuration over which he can maintain control only with his complete and undivided attention. The configuration is "controllable" in the sense that the pilot can maintain control by restricting the tasks and maneuvers which he is called upon to perform, and by giving the configuration his undivided attention. However, for him to answer "Yes, it is controllable in the mission," he must be able to retain control in the mission tasks with whatever effort and attention are available from the totality of his mission duties.

The dictionary shows acceptable to mean that a thing offered is received with a consenting mind, unacceptable to mean that is is refused or rejected. Acceptable means that the mission can be accomplished; it means that the evaluation pilot would agree to buy it for the mission: for him to fly, for his son to fly, or for either to ride in as a passenger. "Acceptable" in the rating scale doesn't say how good it is for the mission, but it does say it is good enough. With these characteristics, the mission can be accomplished. It may be accomplished with considerable expenditure of effort and concentration on the part of the pilot, but the levels of effort and concentration required in order to achieve this acceptable performance are feasible in the intended use. By the same token, unacceptable does not necessarily mean that the mission cannot be accomplished; it does mean that the effort, concentration, and workload

necessary to accomplish the mission are of such a magnitude that the evaluation pilot rejects that airplane for the mission.

Consider now a definition of satisfactory. The dictionary defines this as adequate for the purpose, of a kind to meet all requirements or expectations. A pilot's definition of satisfactory might be that it isn't necessarily perfect or even good, but it is good enough that he wouldn't ask that it be fixed. It meets a standard, it has sufficient goodness; it's of a kind to meet all requirements of a mission. Unsatisfactory, though acceptable, implies that the objectionable characteristics should be improved if possible, that it is defective or deficient in a limited sense, that there is insufficient goodness, that it is reluctantly acceptable.

Thus, the quality is either:

a. completely acceptable (satisfactory) and therefore of the best category, or

b. reluctantly acceptable (unsatisfactory) and of the next best category, or

c. unacceptable, not suitable for the mission but still controllable, and in the third category, or

d. unacceptable for the mission and uncontrollable, and of the poorest quality.

INDIVIDUAL RATING DESCRIPTIONS

The complete rating scale is shown in figure 6.16.

Figure 6.16 REVISED PILOT RATING SCALE

CONTROLLABLE CAPABLE OF BEING CONTROLLED OR MANAGED IN CONTEXT OF MISSION, WITH AVAILABLE PILOT ATTENTION	ACCEPTABLE MAY HAVE DEFICIENCIES WHICH WARRANT IMPROVEMENT, BUT ADEQUATE FOR MISSION. PILOT COMPENSATION, IF REQUIRED TO ACHIEVE ACCEPTABLE PERFORMANCE, IS FEASIBLE.	SATISFACTORY MEETS ALL REQUIREMENTS AND EXPECTATIONS, GOOD ENOUGH WITHOUT IMPROVEMENT CLEARLY ADEQUATE FOR MISSION.	EXCELLENT, HIGHLY DESIRABLE	A1
			GOOD, PLEASANT, WELL BEHAVED	A2
			FAIR. SOME MILDLY UNPLEASANT CHARACTERISTICS. GOOD ENOUGH FOR MISSION WITHOUT IMPROVEMENT.	A3
		UNSATISFACTORY RELUCTANTLY ACCEPTABLE. DEFICIENCIES WHICH WARRANT IMPROVEMENT. PERFORMANCE ADEQUATE FOR MISSION WITH FEASIBLE PILOT COMPENSATION.	SOME MINOR BUT ANNOYING DEFICIENCIES. IMPROVEMENT IS REQUESTED. EFFECT ON PERFORMANCE IS EASILY COMPENSATED FOR BY PILOT.	A4
			MODERATELY OBJECTIONABLE DEFICIENCIES. IMPROVEMENT IS NEEDED. REASONABLE PERFORMANCE REQUIRES CONSIDERABLE PILOT COMPENSATION.	A5
			VERY OBJECTIONABLE DEFICIENCIES. MAJOR IMPROVEMENTS ARE NEEDED. REQUIRES BEST AVAILABLE PILOT COMPENSATION TO ACHIEVE ACCEPTABLE PERFORMANCE.	A6
	UNACCEPTABLE DEFICIENCIES WHICH REQUIRE MANDATORY IMPROVEMENT. INADEQUATE PERFORMANCE FOR MISSION EVEN WITH MAXIMUM FEASIBLE PILOT COMPENSATION.		MAJOR DEFICIENCIES WHICH REQUIRE MANDATORY IMPROVEMENT FOR ACCEPTANCE. CONTROLLABLE. PERFORMANCE INADEQUATE FOR MISSION, OR PILOT COMPENSATION REQUIRED FOR MINIMUM ACCEPTABLE PERFORMANCE IN MISSION IS TOO HIGH.	U7
			CONTROLLABLE WITH DIFFICULTY. REQUIRES SUBSTANTIAL PILOT SKILL AND ATTENTION TO RETAIN CONTROL AND CONTINUE MISSION.	U8
			MARGINALLY CONTROLLABLE IN MISSION. REQUIRES MAXIMUM AVAILABLE PILOT SKILL AND ATTENTION TO RETAIN CONTROL.	U9
UNCONTROLLABLE CONTROL WILL BE LOST DURING SOME PORTION OF MISSION.			UNCONTROLLABLE IN MISSION.	10

It is seen that the complete scale includes further subdivisions of quality within each of the four foregoing categories, and these subdivisions incorporate descriptions to define quality differences separating each numerical rating. It is emphasized, however, that these descriptions supplement the sequential decisions which lead the evaluation pilot to the particular category within which the descriptions of the individual ratings are given. That is to say, the pilot should not make his rating decision based upon the individual descriptions alone. These are most meaningful when used in conjunction with the category decisions.

The essentially new features to be found in the revised rating scale are the descriptions applied to the individual ratings. In this regard, it might be well to reveiw the guide lines which were applied in arriving at the new descriptive words and phrases. It was considered fundamental to a good, easily applied scale that they be both brief and general in nature. Key words and phrases were sought which could be easily understood and yet sufficiently definitive so that each rating would be clearly separated from every other rating.

It is an interesting exercise to attempt to devise a pilot rating scale without the structure of the proposed scale. First of all, one finds it very difficult to write down ten distinctly different, and we might add, repeatable, brief adjective descriptions of quality. One is led to descriptions like bad, very bad, and very, very bad. If such a scale is used in pilot evaluations, it is extremely difficult to retain from one evaluation to the next a consistent definition of the differences between these adjectives. Using the proposed scale structure and making the sequential decisions previously noted, the evaluation pilot is led to a quality decision within a category which involves at most only three degrees of quality. Since only three quality descriptions appear in a category, it has been possible to select descriptions which are clearly separable and definitive.

SOME QUESTIONS DISCUSSED

Why a 10-Point Scale:

In discussing the revised scale, one question which might be anticipated concerns the number of individual ratings which the scale defines. Most simply, this is dictated by the four categories already selected. Separation of each of the upper three categories into three subratings appears to provide an adequate spread for pilot use. Additional ratings in the uncontrollable category would not appear to be of general value.

Identifying the Revised Rating:

The oft-proclaimed criticism that the scale should start with 10 and progress to 1 instead of from 1 to 10 may be valid but there are also examples which support the 1 to 10 logic. We are reluctant at this point to suggest a change simply because of the widespread use of 3.5 to 6.5 boundaries. To now reverse these would likely introduce more confusion than the situation warrants. In proposing a revised scale, it was recognized that some confusion might result from continued use of the same numerical scale which has been identified with both the Cooper and CAL scales. A second objection to these scales has been the lack of clear identification regarding whether 1 or 10 represents the most favorable rating. It is therefore proposed that an identifying letter prefix be used with each numerical rating to eliminate both of these possibilities. By using the letter "A" with numerical ratings from 1

to 6 and the letter "U" with ratings 7 to 9, the 1 to 6 ratings are easily identified as "acceptable" and 7 through 9 as "unacceptable." In addition, it is generally understood that A-1 represents the best. It is recommended that a standard method of plotting pilot rating be adopted with "best" (or A1) at the top and "worst" (or 10) at the bottom of the ordinate scale in order to minimize confusion.

Linearity:

This is a desirable characteristic of any scale. A temperature scale is linear with heat added for a material with constant specific heat, in that the temperature rise per unit quantity of heat added is the same throughout the scale. Temperature is a normal - and useful - scale associated with comfort. Even though temperature may be quite linear with heat added, it is readily apparent that comfort is not linear with temperature. With what should the pilot rating scale be linear? Since it is purported to measure quality, it should then be linear with the added quality of the pilot-vehicle combination in that the change in pilot rating per unit quality addition should be the same throughout the rating scale. The rating scale may possibly have this characteristic, but to demonstrate that the scale is indeed linear would require an independent measure of quality which does not presently exist. Since the basic merit of the scale is not significantly affected by the lack of linearity, no further consideration of this factor has been attempted.

Ordinal Versus Interval Scale:

An interval scale is to be desired, but the pilot rating scale proposed cannot be shown to be an interval scale. The authors accepted it as being ordinal. It is, however, an absolute scale and not a relative one. The pilot rating is given for a configuration in the context of its acceptability for the mission and not in terms of its goodness with respect to a configuration already evaluated. The concentration and effort required in performing each evaluation tends to erase from the pilot's memory the characteristics of preceding configurations. But pilots are concerned about rating something as excellent or optimum for fear that a subsequent configuration will be shown them which is significantly better than anything which they had previously considered possible. Just in case some fortunate evaluation pilot does encounter the optimum airplane, it allows him to use the rating "zero" to cover this possibility.

Words Versus Numbers:

The basic structure of the rating scale is completely dependent on words and their explicit definition. The fact that a numeral is associated with the evaluator's final decision is an expedient, a shorthand. One risk that is faced with a numerical scale is that the engineers will attempt to treat the pilot rating data with mathematical operations which are rigorously applicable only to a linear interval scale. No rigorous justification can be given to such actions, although some insight is sometimes gained. However, if analysis of specific pilot rating data depends on such mathematical operations, a strong argument should be made for the engineer to obtain a better or deeper understanding of his experimental data.

Differing Standards of Acceptance for the Same Mission:

One difficulty that has arisen in the use of pilot rating data can be illustrated by the following example. An evaluation program was conducted for the mission of the landing approach of a

commercial air transport. A careful definition was made of the mission, the program was run, and the results were published. One of the subjects, an airline pilot, subsequently remarked that his airline would not accept any airplane worse than a 4.5 rating for the landing approach. Yet, in the generally accepted interpretation of the rating scale, by giving ratings of 5 and 6, he had said that other airplanes which were unsatisfactory could still be considered acceptable for the intended use. An explanation is that this pilot is assigning a 4.5 rating floor to define the poorest airplane he will buy under the circumstances which he envisions at the moment. Such decisions as his must be made by any customer while considering that which is to be gained in terms of how much it will cost. It is easy to envision similar decisions being made to buy only that which is above the 3.5 boundary. And similarly, one can envision a reluctant decision to buy as low as the 6.5 boundary, but only if all other possibilities for purchase of a better airplane had been excluded. The basic pilot rating on which these decisions are based, however, must be strictly mission-oriented if the subsequent quality versus cost decisions are to be meaningful.

EXPERIMENTAL USE OF RATING OF HANDLING QUALITIES

The evaluation of handling qualities has a similarity to other scientific experiments in that the output data are only as good as the care taken in the design and execution of the experiment itself, and in the analysis and reporting of the results. There are two basic categories of output data in a handling qualities evaluation: the pilot objections (or pilot comment data) and the pilot ratings. Both items are important output data. An experiment which ignores one of the two outputs is discarding a substantial part of the output information.

As one might expect, the output data which are most often neglected are the pilot comments, primarily because they are quite difficult to deal with due to their qualitative form and, perhaps, their bulk. Ratings, however, without the attendant pilot objections, are only part of the story. Only if the deficient areas can be identified, can one expect to devise improvements to eliminate or attenuate the shortcomings. The pilot comments are the means by which the identification can be made.

There are several factors which have a strong influence on the quality of pilot evaluation data and a brief discussion of them follows.

MISSION DEFINITION

Explicit definition of the mission is probably the most important contributor to the objectivity of the pilot evaluation data. The mission is defined here as a use to which the pilot-airplane combination is to be put. The mission must be very carefully examined, and a clear definition and understanding must be reached between the engineer and the evaluation pilot as to their interpretation of this mission. This definition must include:

a. what the pilot is required to accomplish with the airplane, and

b. the conditions or circumstances under which he must perform the mission.

For example, the conditions or circumstances might include instrument or visual flight or both, type of displays in the cockpit, input information to assist the pilot in the accomplishment of the mission, etc. The environment in which the mission is to be accomplished must also be defined and considered in the evaluation, and could include, for example, the

presence or absence of turbulence, day versus night, the frequency with which the mission has to be repeated, the variability in the preparedness of the pilot for the mission, and in his level of proficiency.

SIMULATION SITUATION

The pilot evaluation is seldom conducted under the circumstances of the real mission. The evaluation almost inherently involves simulation to some degree because of the absence of the real situation. As an example, the evaluation of a day fighter is seldom carried out under the circumstances of a combat mission where the pilot is not only shooting at real targets but being shot back at by real guns. Therefore, after the mission has been defined, the relationship of the simulation situation to the real mission must be explicitly stated for both the engineer and the evaluation pilot so that each may clearly understand the limitations of the simulation situation.

The pilot and engineer must both know what is left out of the evaluation program, and also what is in that should not be in. The fact that the anxiety and tenseness of the real situation are missing, and that the airplane is flying in the clear blue of calm daylight air, instead of in the icing, cloudy, turbulent, dark situation of the real mission, will affect results. Regardless of what are selected for the evaluation tasks, the pilot must use his knowledge and experience to provide a rating which includes all considerations which are pertinent to the mission, whether provided in the tasks or not.

PILOT COMMENT DATA

One of the tendencies resulting from the use of a rating scale which is considered for universal handling qualities application is the assumption that the numerical pilot rating can represent the entire qualitative assessment. Extreme care must be taken against this oversimplification because it does not constitute the full data gathering process.

The pilot objections to the handling qualities are important, particularly to the airplane designer who is responsible for the improvement of the handling qualities. But, even more important, the pilot comment data are essential to the engineer who it attempting to understand and use the pilot rating data. If ratings are the only output data, one has no real way of assessing whether the objectives of the experiment were actually realized. Pilot comments supply a means of assessing whether the pilot objections (which lead to his summary rating) were related to the mission or resulted from some extraneous uncontrolled factor in the execution of the experiment, or from individual pilots focusing on and weighing differently various aspects of the mission. In order that the pilot comments be most useful, several details are important.

The comments must be given by the pilot in the simplest language. Engineering terms are generally to be avoided, unless they are carefully defined. The pilot should report what he sees and feels, and describe his difficulties in carrying out that which he is attempting. It is then important for the pilot to relate the difficulties which he is having in executing specific tasks to their effect on the accomplishment of the mission.

The pilot should be required to make specific comments in evaluating each configuration. These comments generally are in response to questions which have been developed in the discussions of the mission and simulation situation. The

pilot must also be free to make comments regarding his difficulties over and above the answers to the specific questions asked of him. In this regard, the test pilot should strive for a balance between a continuous running commentary and occasional comment in the form of an explicit adjective. The former often requires so much editing to find the substance that it is often ignored, while the latter may add nothing to the numerical rating itself.

The pilot comments must be taken during or immediately after each evaluation. For in-flight evaluations, this means that the comments should be recorded on a wire or tape recorder. Experience has shown that the best free comments are often given during the evaluation. If the comments are left until the conclusion of the evaluation, they are often forgotten. A useful procedure is to permit free comment during the evaluation itself and to require answers to specific questions in the summary comments at the end of the evaluation.

Questionnaires and supplementary pilot comments are most necessary to ensure that: (a) all important or suspected aspects are considered and not overlooked, (b) information is provided relative to why a given rating has been given, (c) an understanding is provided of the tradeoffs with which pilots must continually contend, and (d) supplementary comment that might not be offered otherwise is stimulated. It is recommended that the pilots participate in the preparation of the questionnaires. The questionnaires should be modified if necessary as a result of the pilots' initial evaluations. On occasion, it may be desirable to classify pilot comment by having the pilots select one of several ranked comments about a specific characteristic. Examples of two such classifications of specific pilot comment are shown in figures 7 and 8 for PIO tendency and effects of turbulence, respectively. Identification as shown in these examples (by number of letter) is for easy identification only, and not to be confused with the designation for pilot rating.

PILOT RATING DATA

The pilot rating is an overall summation of the net effect of all of the objections which the pilot has observed during the evaluation as they relate to the mission. It is emphasized that the basic question that is asked of the pilot conditions the answer that he provides. For this reason, it is most important to ensure that the objectives of the program are clearly stated and understood by all concerned, and that all criteria, whether established or assumed, be clearly defined. In other words, it is extremely important that the basis upon which the evaluation is established be firmly understood by pilots and engineers. Unless a common basis is used, one cannot hope to achieve comparable pilot ratings, and confusing disagreement will often result. Care must also be taken that criteria established at the beginning of the program carry through to the end. If the pilot finds it necessary to modify his tasks, technique or mission definition during the program, he must make it clear just when this change occurred.

A discussion of the specific use of a rating scale tends to indicate some disagreement among pilots as to how they actually arrive at a specific numerical rating. There is general agreement that the numerical rating is only a shorthand for the word definition. Some pilots, however, lean heavily on the specific adjective description and look for that description which best fits their overall assessment. Other pilots prefer to make the dichotomous decisions sequentially, thereby arriv-

ing at a choice between two or three ratings. The decision among the two or three ratings is then based upon the adjective description. In concept the latter technique is much to be preferred since it emphasizes the relationship of all quality decisions to the mission.

It is suggested that the actual technique used is somewhere between the two techniques above and not so different among pilots. In the past, the pilot's choice has probably been strongly influenced by the relative usefulness of the descriptions provided for the categories on one hand, and the numerical ratings on the other. The evaluation pilot is more or less continuously considering the rating decision process during his evaluation. He proceeds through the dichotomous decisions to the adjective descriptors enough times that his final decision is a blend of both techniques. It is therefore obvious that descriptors should not be contradictory to the mission-oriented framework.

Half ratings are permitted (e.g., rating 4.5) and are generally used by the evaluation pilot to indicate a reluctance to assign either of the adjacent ratings to describe the configuration. Any finer breakdown than half ratings is prohibited since any number greater than or less than the half rating implies that it belongs in the adjacent group. Any distinction between configurations assigned the same rating must be made in the pilot comments. Use of the 3.5, 6.5, and 9.5 ratings is discouraged as they must be interpreted as evidence that the pilot is unable to make the fundamental decision with respect to category.

As noted previously, the pilot rating and comments must be given on the spot in order to be most meaningful. If the pilot should later want to change his rating, the engineer should record the reasons and the new rating for consideration in the analysis, and should attempt to repeat the configuration later in the evaluation program. If the configuration cannot be repeated, the larger weight (in most circumstances) should be given to the on-the-spot rating since it was given when all the characteristics were freshest in the pilot's mind.

EXCUTION OF HANDLING QUALITIES EXPERIMENTS

Probably the most important item is the admonition to execute the experiment as it was planned. This requires careful attention to the conduct of the experiment so that the plans are actually executed in the manner intended. It is valuable for the engineer to monitor the pilot comment data as the experiment is conducted in order that he becomes aware of evaluation difficulties as soon as they occur. These difficulties may take a variety of forms. The pilot may use words which the engineer needs to have defined. The pilot's word descriptions may not convey a clear, understandable picture of the piloting difficulties. Direct communication between pilot and engineer is most important in clarifying such uncertainties. In fact, communication is probably the most important single element in the evaluation of handling qualities. Pilot and engineer must endeavor to understand one another, and cooperate to achieve and retain this understanding. The very nature of the experiment itself makes this somewhat difficult. The engineer is usually not present during the evaluation and, hence, he has only the pilot's word description of any piloting difficulty. Often, these described difficulties are contrary to the intuitive judgments of the engineer based on the characteristics of the airplane by itself. Mutual confidence is required. The engineer should be confident that the pilot will give him accurate, meaningful

data; the pilot should be confident that the engineer is vitally interested in what he has to say and trusts the accuracy of his comments.

It is important that the pilot have no foreknowledge of the specific characteristics of the configuration being investigated. This does not exclude information which can be provided to help shorten certain tests (e.g., the parameter variations are lateral-directional, only). But it does exclude foreknowledge of the specific parameters under evaluation. The pilot must be free to examine the configuration without prejudice, learn all he can about it from meeting it as an unknown for the first time, look clearly and accurately at his difficulties in performing the evaluation task, and freely associate these difficulties with their effects on the ultimate success of the mission. A considerable aid to the pilot in this assessment is to present the configurations in a random-appearing fashion.

The time which the pilot should use for the evaluation is difficult to specify a priori. He is normally asked to examine each configuration for as long as is necessary to feel confident that he can give a reliable and repeatable assessment. Sometimes, however it is necessary to limit the evaluation time to a specific period of time because of circumstances beyond the control of the researcher. If the evaluation time per pilot is limited, a larger sample of pilots or repeat evaluations will be required for similar accuracy, and the pilot comment data will be of poorer quality.

One final point is the state of mind of the evaluation pilot. He must be confident of the importance of the simulation program and join wholeheartedly into the production of data which will supply answers to the questions. Pilots as a group as strongly motivated toward the production of data to improve the handling qualities of the airplanes they fly. It isn't usually necessary to explicitly motivate the pilot, but it is very important to inspire in him confidence in the structure of the experiment and the usefulness of his rating and comment data. Pilot evaluations are probably one of the most difficult tasks that a pilot undertakes. To produce useful data involves a lot of hard work, tenacity, and careful thought. There is a strong tendency for the pilot to become discouraged in the course of his evaluations about their ultimate usefulness. He worries constantly about his assessments: their accuracy and repeatability. The pilot may feel that the engineer has the answers on a sheet of paper and he is merely testing the pilot as to his ability to search out the correct answers. Such feelings are added to by a lack of communication between the piloting and engineering organizations, and are to be avoided. Probably the best approach is to explicitly state to the pilot that only he knows the answers to the questions which are being asked, and he can arrive at these correct answers by carrying out the evaluation program. He must be reassured in the course of the program that his assessments are good, so that he gains confidence in the manner in which he is carrying out the program.

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CHAPTER 7

SPINS

7.1 DEFINITION

"Of the myriad of coupled motions which an airplane can perform, the spin stands out as being unique. When an airplane is stalled and left to itself, it will perform some sort of rolling, yawing and pitching motion which, if allowed to continue, may develop into a characteristic motion called a spin, in which the airplane descends rapidly toward the earth in a helical movement about the vertical axis at an angle of attack between the stall and 90°."[1] However, many different names are given to the unusual gyrations of high performance aircraft, e.g., post stall gyrations, inertia coupled maneuvers, horizontal (flat) spins and incipient spins.

The primary purpose of any spin program is to find out whether or not an aircraft can be satisfactorily and consistently recovered from incipient and fully developed spins using consistent pilot techniques. Best recovery techniques are also determined. Aircraft susceptibility to spin entry is another important characteristic investigated during a spin test. Information gained during the test program is published in the Flight Manual as specific spin characteristics, warnings, limitations, and recommended recovery procedures. Some of the unusual characteristics of a spin are:

1. Spins are conducted in a region beyond the airplane's normal operational envelope.
2. Once the airplane is flown into this region, its motion is not precisely predictable since the motion depends on past history as well as forces acting on it at any particular time.
3. The pilot may experience extreme accelerations and attitudes.

[1] Spin Testing USN High Performance Airplanes, John A. Niol.

These characteristics and safety considerations demand maximum preflight preparation.

7.2 SPIN TERMINOLOGY AND DESCRIPTION

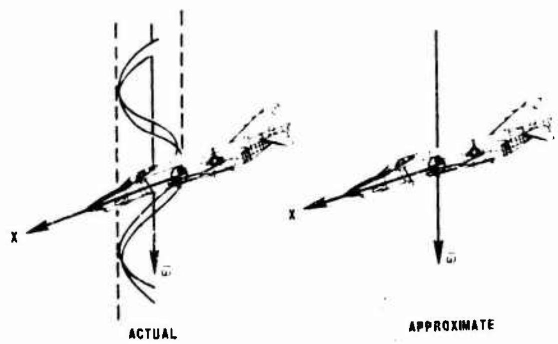
The spin is a complicated maneuver involving simultaneous roll, pitch and yaw rates and high sideslip angles. These motions can be analyzed by looking at the equations of motion and through computer studies. Predictions can be made as to the rate of rotation, angle of attack, and recovery techniques. However, these predictions are difficult to obtain and are not always accurate; therefore, most of the information needed to predict the spin motion and recovery techniques is obtained from wind tunnel and free flight model tests.

The correlation between wind tunnel tests and actual spins is generally good; however, spins obtained in wind tunnels are primarily steady state, and investigations of incipient spins (transition from stall to steady state) cannot be made. Actual steady state spins may be difficult to obtain due to mass distribution of the aircraft or engine gyroscopic effects. As

a result, correlation with wind tunnel data may not be obvious or there might be no correlation at all. It is important, therefore, to recognize the variance in conditions when comparing wind tunnel data with flight test data. Engine gyroscopic effects are not usually included in spin tunnel data although work has been done in this area, but predictions can be made as to the effect of engine rotation. Limitations also exist in the data obtained from free flight model tests; however, the data obtained from either or both of these methods are the most reliable available prior to actual flight tests of the aircraft. Their value should definitely not be minimized.

An aircraft spin is a coupled motion at extreme attitudes that requires all six equations of motion for accurate numerical analysis. Much can be gained, however, from a simplified approach using the equations singularly or in sets. The spin motion is actually a form of a tight helix as shown in figure 7.1.

FIGURE 7.1



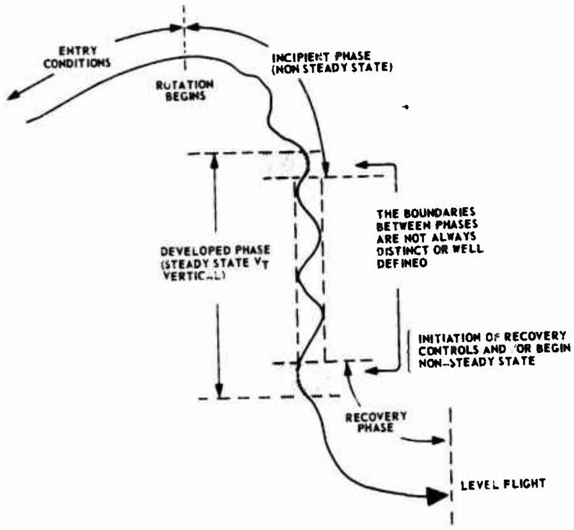
ACTUAL AND APPROXIMATE SPIN MOTION

Because a spin is predominately a rotary motion, we will be able to disregard translation and treat it as a pure rotation about the spin axis passing through the center of gravity. Moreover, in the analysis to follow, we will assume the linear velocity of the cg is vertical.

7.2.1 Spin Phase

A typical spin may be divided into the phases shown in figure 7.2.

FIGURE 7.2



SPIN PHASES

Each spin phase will be discussed in detail in later sections.

7.2.2 Spin Modes

In each phase (but primarily the steady state), an aircraft may exhibit more than one "mode." For example:

Flat, steep

Slow, fast

Erect, inverted

Oscillatory, non-oscillatory

and various combinations of the above modes.

7.2.3 Entry Conditions

Variations in airspeed, load factor and attitude at entry frequently have pronounced effects on the character of the ensuing spin. These differences are sometimes apparent only after several turns.

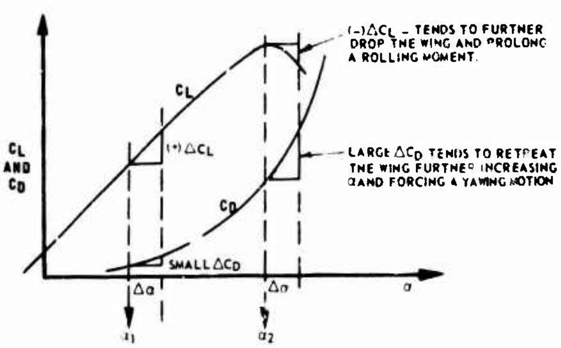
•7.3 INCIPIENT PHASE

Webster defines incipient as "just beginning to exist." In spin terminology, incipient is used to describe the non-steady state portion of the spin. Some aircraft never do exhibit a steady state (developed) mode and hence they could be considered to be in the incipient phase throughout the spin. This is particularly true of present day, high performance fighter aircraft. The term "post stall gyration" is sometimes used when the motion in this phase does not have a recognizable pattern. A number of high performance designs will recover consistently <u>only</u> in the incipient phase, where airspeed, angle of attack, and attitude are more favorable, and the angular momentum has not yet increased to a high value.

•7.4 AUTOROTATION

The aircraft spin is caused by, or initiated primarily by, the property of an aircraft to autorotate when operating at angles of attack above the stall. A developed spin involves a balance between aerodynamic and inertial moments. An aerodynamic "autorotation" is the forcing function that tends to keep the motion going.

FIGURE 7.3



SPIN INITIATION

If a wing is operating at α_1 (low angle of attack) and experiences a $\Delta\alpha$ due to wing drop, there is a restoring moment from the increased lift. If, on the other hand, a wing operating at α_2 (high α) experiences a sudden drop, there is a loss of lift and an increase in drag that tends to prolong the disturbance and sets up autorotation.

FIGURE 7.4

SPIN SUSTENANCE (AUTOROTATION)

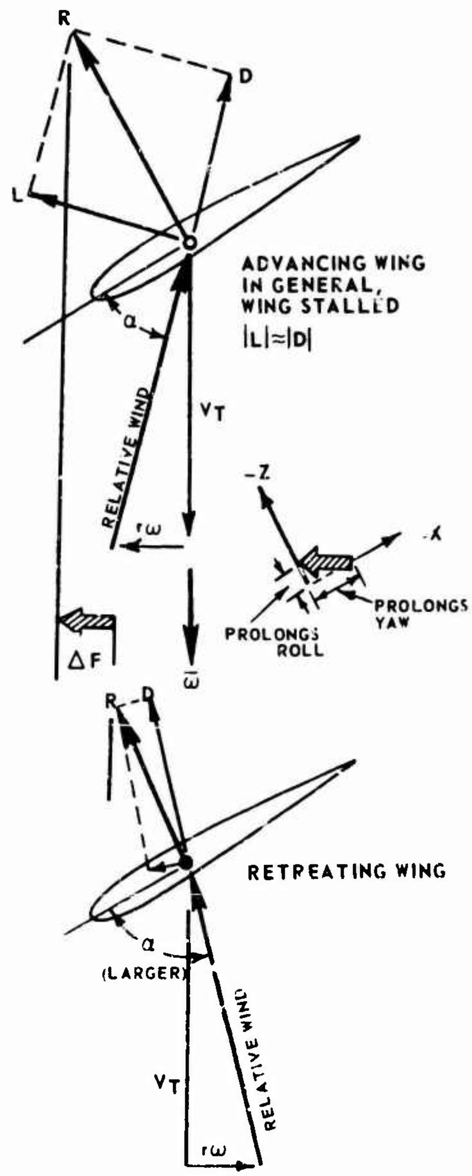


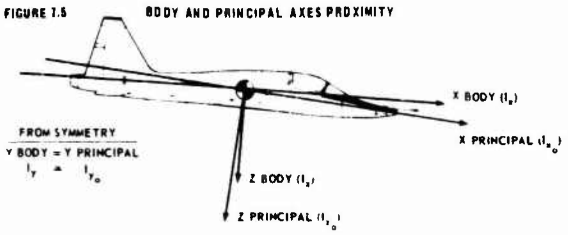
Figure 7.4 is an extremely simplified presentation of what causes a spin to persist (also see figure 7.12). Consider a right steady state (developed) spin with a vertical V_T. The effect of the rotation modifies V_T so as to produce a relative wind on the advancing wing which differs from the relative wind on the retreating wing. This difference causes the α of the retreating wing to be larger than the α of the advancing wing. Thus the advancing wing experiences a larger resultant aerodynamic force than does the retreating wing. The difference in the orientation and magnitude of the resultant forces can be represented by a force vector as shown in figure 7.4. When applied to the advancing wing, ΔF tends to keep the spin going as components of the force vector produce roll and yaw.

• 7.5 AIRCRAFT MASS DISTRIBUTION

7.5.1 Principal Axes

For every rigid body there exists a set of principal axes for which the products of inertia are zero and one of the moments of inertia is the maximum possible for the body. For a symmetrical aircraft, this principal axis system is frequently quite close to the body axis system. For the purpose of this course, the small difference in displacement is neglected, and the principal axes are assumed to lie along the body axes. Figure 7.5 illustrates what the actual difference might be.

FIGURE 7.5 BODY AND PRINCIPAL AXES PROXIMITY



7.5.2 Relative Magnitude of the Moments of Inertia

Because aircraft configurations are "flattened" into the XY plane, I_z is invariably the maximum moment of inertia. I_x is greater or less than I_y depending on the aircraft's mass distribution. The relative magnitudes of the moments of inertia are shown in figure 7.6.

7.5.3 Radius of Gyration

$$I_x = K_x^2 M = \sum (y^2 + z^2)\Delta M = \int r_x^2 dm \quad (7.1)$$

$$I_y = K_y^2 m = \sum (x^2 + z^2)\Delta M = \int r_y^2 dm \quad (7.2)$$

$$I_z = K_z^2 M = \sum (x^2 + y^2)\Delta M = \int r_z^2 dm \quad (7.3)$$

Where K_x, K_y and K_z are each a radius of gyration, and M is the total aircraft mass. The center of gyration of a body with respect to an axis is a point at such a distance (K) from the axis that, if the entire mass of the body were concentrated there, its moment of inertia would be the same as that of the body. The radius of gyration of a body with respect to an axis is the distance (K) from the center of gyration to the axis. Another equation is:

$$K = \sqrt{I/M} \quad (7.4)$$

7.5.4 Relative Aircraft Density

μ is defined to be a non-dimensional parameter comparing the aircraft density to the density of air.

$$\mu = \frac{M/Sb \text{ slugs/ft}^3}{\rho \text{ slugs/ft}^3} = \frac{M}{\rho Sb} \quad (7.5)$$

Where M is the total aircraft mass, S is the wing area and b is the wing span.

7.6 EQUATIONS OF MOTION

Because the spin is a complex motion, the analytical treatment requires certain simplifying assumptions. In general, they are not too restrictive for a "first look," and good qualitative information can be obtained. The following observations apply to a developed spin and lead to the formulation of a simplified mathematical analysis.

1. In a well developed steady state spin, the apparent average rotation rate and the rate of descent are essentially constant.

2. The linear velocity is approximately vertical.

3. If the spin is oscillatory, the oscillations tend to be approximately periodic.

7.6.1 The Mathematical Analysis

1. The total rotational energy of the aircraft and the translational kinetic energy are constant.

2. Statement (1) implies that the time average of the summation of the external forces is zero.

3. Statement (1) also implies that the loss in potential energy is being absorbed in aerodynamic damping so that the net change in rotational energy is zero.

The mathematical analysis provides the simplifying assumption that the time average of the external forces is zero. This requirement and the assumption that the linear velocity vector is vertical means that we need only consider the three moment equations that were developed in chapter I. These equations are repeated below and will be utilized for most of the analytical developments.

$$G_x = \dot{p}I_x + qr(I_z - I_y) - (\dot{r} + pq)I_{xz} \quad (7.6)$$

$$G_y = \dot{q}I_y - pr(I_z - I_x) + (p^2 - r^2)I_{xz} \quad (7.7)$$

$$G_z = \dot{r}I_z + pq(I_y - I_x) + (qr - \dot{p})I_{xz} \quad (7.8)$$

Section 7.5.1 discussed the assumption that the body axes and principal axes coincide. Actually, this is a very good assumption and leads to a considerable simplification, in that we can set $I_{xz} = 0$. Expanding the general applied moments to include an aerodynamic moment, a gyroscopic moment and a miscellaneous term, we have:

$$\begin{aligned} G_x &= L + L_{gyro} + L_{other} \\ &= \dot{p}I_x + qr(I_z - I_y) \end{aligned} \quad (7.9)$$

$$\begin{aligned} G_y &= M + M_{gyro} + M_{other} \\ &= \dot{q}I_y - pr(I_z - I_x) \end{aligned} \quad (7.10)$$

$$\begin{aligned} G_z &= N + N_{gyro} + N_{other} \\ &= \dot{r}I_z + pq(I_y - I_x) \end{aligned} \quad (7.11)$$

Rearrangement of the above equations leads to expressions relating the body angular accelerations in terms of the contributing terms.

	AERO	INERTIAL COUPLING (sometimes called gyrodynamic term)	GYROSCOPIC TERM (an engine effect)	MISC (rockets, spin chutes, etc.)	
$\dot{p} =$	$\dfrac{L}{I_x}$	$+ \left(\dfrac{I_y - I_z}{I_x}\right) q\,r$	$+ \dfrac{L_{gyro}}{I_x}$	$+ \dfrac{L_{oth}}{I_x}$	(7.12)
$\dot{q} =$	$\dfrac{M}{I_y}$	$+ \left(\dfrac{I_z - I_x}{I_y}\right) p\,r$	$+ \dfrac{M_{gyro}}{I_y}$	$+ \dfrac{M_{oth}}{I_y}$	(7.13)
$\dot{r} =$	$\dfrac{N}{I_z}$	$+ \left(\dfrac{I_x - I_y}{I_z}\right) p\,q$	$+ \dfrac{N_{gyro}}{I_z}$	$+ \dfrac{N_{oth}}{I_z}$	(7.14)

Determination of the aerodynamic moments is extremely complex. The aircraft is operating out of its normal environment, and the aerodynamic moments vary radically with time. In general, these moments cannot be accurately defined.

We can also express the body angular accelerations in terms of the aerodynamic coefficients and the relative aircraft density by defining the following terms:

$$\frac{L}{I_x} = \frac{\frac{1}{2}\rho V^2 Sb}{K_x^{\,2} M} C_\ell = \frac{V^2}{\frac{2M}{\rho Sb} K_x^{\,2}} C_\ell = \frac{V^2}{2\mu K_x^{\,2}} C_\ell \tag{7.15}$$

$$\frac{M}{I_y} = \frac{V^2}{2\mu K_y^{\,2}} C_m \tag{7.16}$$

$$\frac{N}{I_z} = \frac{V^2}{2\mu K_z^{\,2}} C_n \tag{7.17}$$

(For the longitudinal pitching case, it is necessary to use b instead of c in the coefficient, or alternately, to use c instead of b in the relative density, μ.) Thus:

$$\dot{p} = \frac{V^2 C_\ell}{2\mu K_x^{\,2}} + \left(\frac{I_y - I_z}{I_x}\right) qr + \frac{L_{gyro}}{I_x} + \frac{L_{other}}{I_x} \tag{7.18}$$

$$\dot{q} = \frac{V^2 C_m}{2\mu K_y^{\,2}} + \left(\frac{I_z - I_x}{I_y}\right) pq + \frac{M_{gyro}}{I_y} + \frac{M_{other}}{I_y} \tag{7.19}$$

$$\dot{r} = \frac{V^2 C_n}{2\mu K_z^{\,2}} + \left(\frac{I_x - I_y}{I_z}\right) pq + \frac{N_{gyro}}{I_z} + \frac{N_{other}}{I_z} \tag{7.20}$$

We now have developed all the equations necessary for our analysis. These equations are not new, but simply rearranged forms of the three basic moment equations. Further manipulation will be done, but with the goal of understanding the fundamental relationships required in spin motion analysis.

7.7 DEVELOPED PHASE

A developed spin involves a balance of aerodynamic and intertial moments and forces. The developed phase may never be achieved in some aircraft; however, we will discuss this phase in our analysis and assume that most of the conditions of steady state are attained. Due to the trend of current designs, the steady developed spin has practically been eliminated and has been replaced by a cyclic, large motion oscillation. A steady state spin mode can exist only when α is greater than the stall, C_m is negative and the slope of C_m versus α is stable, i.e., negative and constant.

In general, the developed phase displays the following characteristics:

1. Vertical descent at approximately constant airspeed - - - -
   ΔH/turn $\approx$ constant.
2. $\bar{V}_T$ and $\bar{\omega}$ nearly coincident (vertical).
3. $\dot{p}$, $\dot{q}$ and $\dot{r}$ average approximately zero. Hence $\bar{\omega}$ is constant - - - -
   Δ time/turn $\approx$ constant.
4. On the average, wings level - - - -
   $\phi \approx 0$ and $q \approx 0$.
5. Varying amounts of oscillation about these average values may be observed. Figure 7.7 summarizes these results.

It is very important to realize that the $\bar{\omega}$ vector lies in the XZ plane and has no component along the Y axis. This is a result of the assumption that the aircraft spins in a wings level condition. A very steep, nose down spin will be characterized by a large rolling velocity (p) and a small yawing velocity (r). A flat spin would be characterized by a small rolling velocity (p) and a large yawing velocity (r).

Other Generalities that frequently apply are:

6. Spins tend to exhibit more yaw than roll; thus they tend to have higher yaw rates than roll rates.
7. High performance "modern" designs spin flatter than do older conventional designs. This, primarily, is a result of the moments of inertia distribution in that modern design aircraft are fuselage loaded and the more conventional designs are neutrally or wing loaded.
8. High performance designs exhibit more oscillations. This stems from the dynamic considerations concerned with the moments of inertia and the interchanges between the external aerodynamics and the inertia distribution of the aircraft.

Several of the above characteristics will be discussed. The observations are important and provide the assumptions necessary in the development of the simplified analysis.

FIGURE 7.8

AIRCRAFT MASS DISTRIBUTION

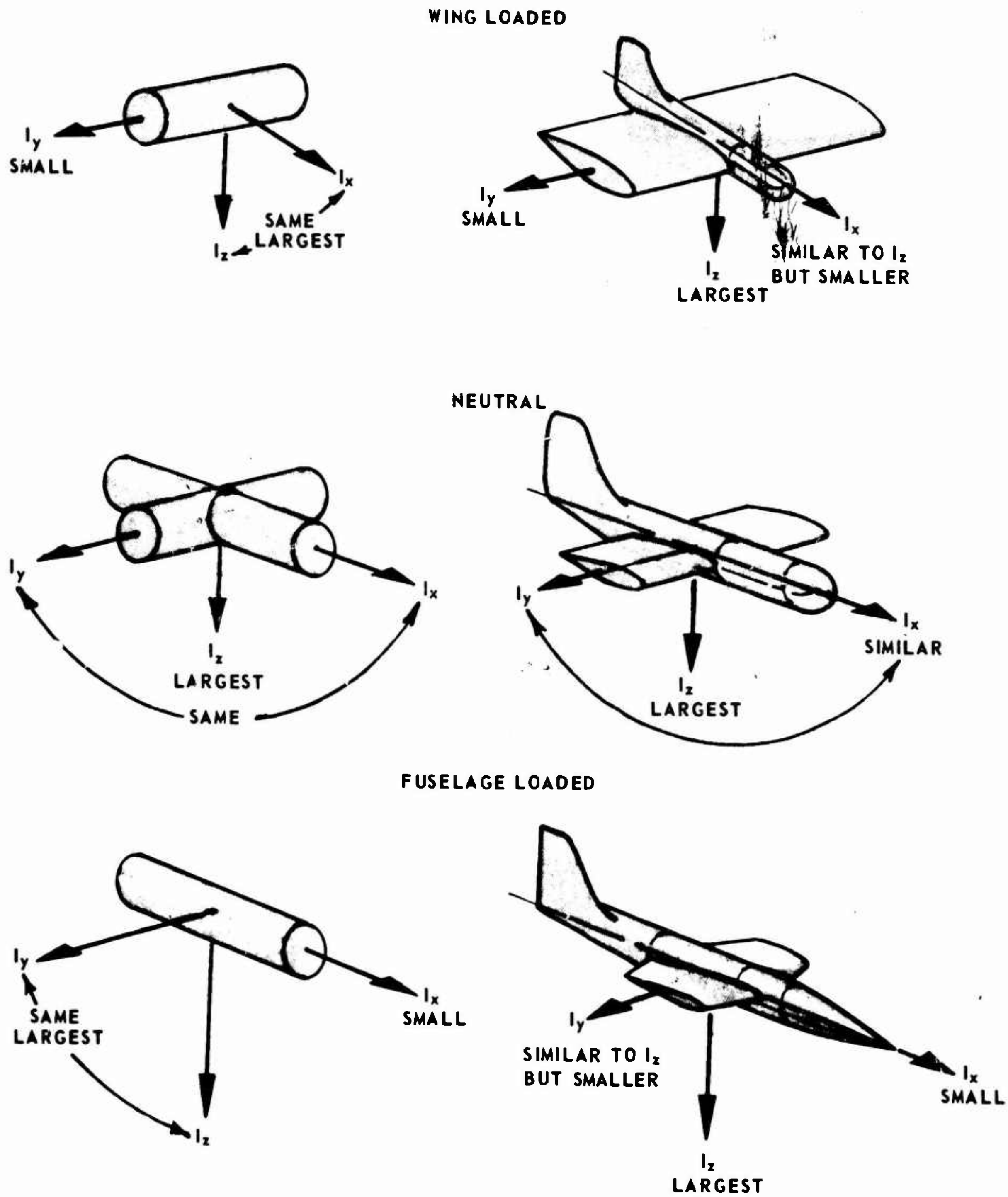
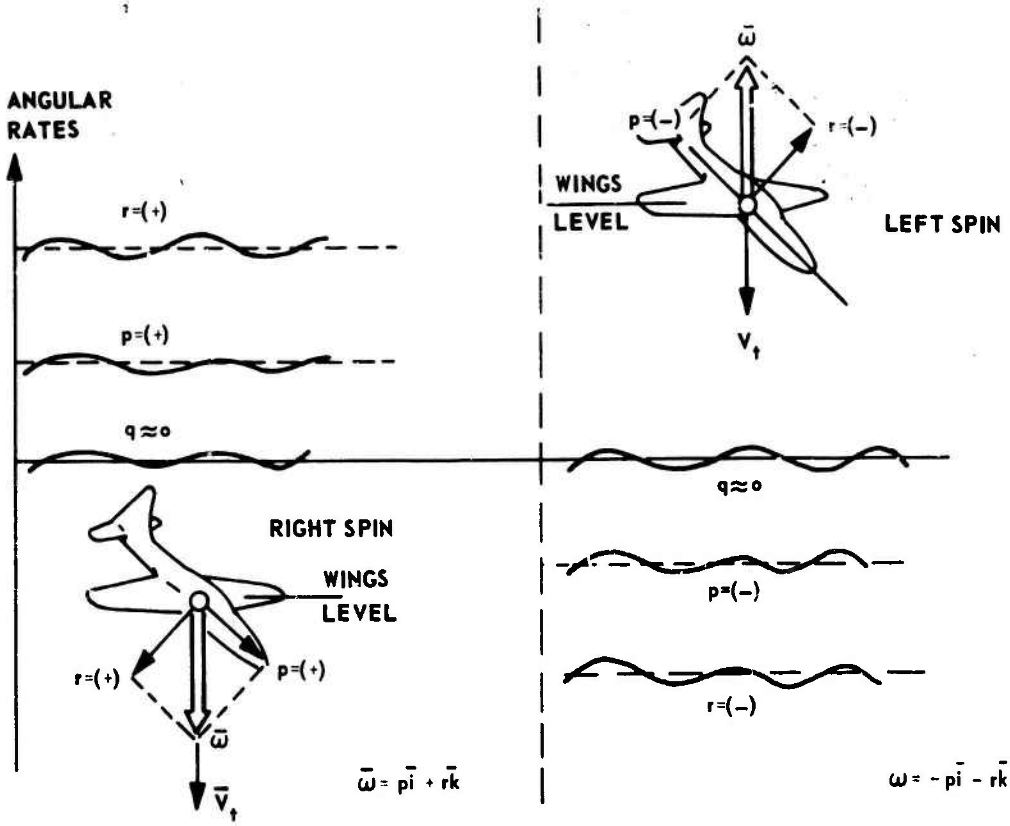


FIGURE 7.7

SPIN VECTOR COMPONENTS FOR RIGHT AND LEFT SPIN



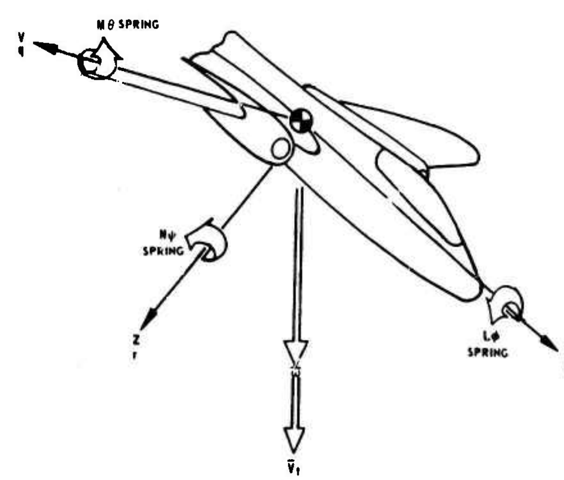
7.8 THE MATHEMATICAL MODEL

The previous assumptions lead to the development of a mathematical model which is shown in figure 7.8. Some of the spin characteristics can be proven analytically and others have been gained from experience in wind tunnel and flight testing. The remaining observations cannot be rigorously proven and no attempt is made to prove these latter characteristics.

In an analytical treatment of the preceding assumptions, the spinning aircraft can be analyzed as a series of coupled, or uncoupled, ordinary second order differential

FIGURE 7.8

KINETIC ENERGY POTENTIAL ENERGY



equations. The model consists of a reference frame, X Y Z, that is moving vertically downward at a constant velocity. The body is also rotating about a vertical axis through the cg of the aircraft at a constant angular velocity, $\bar{\omega}$. The aircraft, in general, oscillates about this reference condition and the rotational potential energy is alternately stored and released by the torsional springs. The torsional springs are, in reality, the aerodynamic moments that provide a balance between the inertia moments and the aerodynamic moments.

The conservation of energy for conservative systems is:

KINETIC ENERGY + POTENTIAL ENERGY = CONSTANT

and for dissipative systems is:

KINETIC ENERGY +	POTENTIAL ENERGY +	LOSSES = CONSTANT
$KE_{rotation} = \frac{1}{2} (\bar{\omega} . \bar{H})$	$PE_{rotation}$ (torsion springs)	STRUCTURAL DAMPING & FLEXING
$KE_{linear} = \frac{1}{2} M V_T^2$	PE_{linear} (height above datum)	AERODYNAMIC DAMPING
		DRAG

It is observed that an aircraft in the developed spin is behaving as a conservative system in that the cg is moving through the air at constant velocity (KE_{linear} = constant) and with constant rotational energy ($KE_{rot} + PE_{rot}$), with the decrease in PE_{linear} exactly in balance with the dissipated losses. From this and similar discussion and proofs, the previously mentioned conclusions about spin characteristics may be drawn; however further analysis will not be attempted with this method.

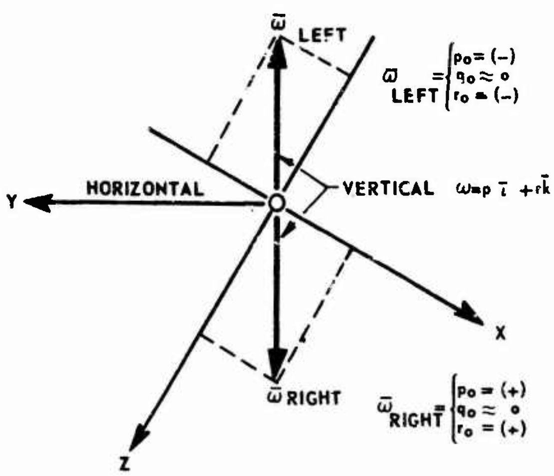
7.8.2 The Reference Motion is Wings Level

For conventional aircraft, $\bar{\omega}$ lies in the XZ plane and the reference motion is a combination of yaw and roll only. The bank angle is, on the average, zero, and no component of $\bar{\omega}$ is reflected on the Y axis, i.e., the reference pitch rate, $q_o \approx 0$.

It should be noted that the moments and body rates with the "o" subscripts are reference rates

from which small pertubations will be taken.

FIGURE 7.9



SPIN VECTORS FOR RIGHT AND LEFT SPINS

7.8.3 Aircraft Tend to Spin About the Z Axis

It can be proven that a system which has no external moments or forces tends to rotate about its largest principal axis, which in the case of an aircraft, is the Z axis. In an actual spinning aircraft, the external moments are not zero and thus the aircraft spins about some intermediate axis which provides a minimum energy condition. Whether an airplane spins steep or flat, and what its rate of rotation will be, are primarily dependent upon the yawing moment and pitching moment characteristics of the airplane. These characteristics will be discussed more later. The spin rotation and angle of attack also can be influenced by the gyroscopic moment produced by the rotating parts of the engine. Because these parts continue to rotate at a fairly high rate even though the engine is throttled back, the gyroscopic effect of the engine on the developed spin and subsequent recovery therefrom must be given consideration. This topic will also be covered in more detail. In general terms, we can say that spins tend to exhibit more yaw rate than roll rate.

The mathematical model suggests that the aircraft possess constant rotational kinetic energy and this demands that the body acceleration rates be zero, or $\dot{p} = \dot{q} = \dot{r} = 0$. Moreover, we earlier assumed that the body axes and the principal axes coincided, and thus the products of inertia are zero, thus:

$$\bar{G} = G_x + G_y + G_z$$

where

$$G_x = qr(I_z - I_y)$$

$$G_y = -pr(I_z - I_x)$$

$$G_z = pq(I_y - I_x)$$

The observation of spins also leads to another simplifying assumption, namely that a spin is usually wings level, or $\phi \approx 0$ and $q \approx 0$.

Thus, as a first start, the equations simplify to:

$$G_x = 0$$

$$G_y = -pr(I_z - I_x)$$

$$G_z = 0$$

or

$$\bar{G} = -pr\ (I_z - I_x)$$

$$G_y = -M = pr(I_z - I_x) \qquad (7.21)$$

For an aircraft, $(I_z - I_x)$ can never be zero, hence if $G_y = 0$ then p must be zero, in which case $\bar{\omega} = r\bar{k}$ and the spin is flat ($\bar{\omega} = p\bar{i}$ is excluded by the definition of a

spin). If the spin is not flat, then both p and r exist and from figure 7.9, always have the same algebraic sign. Because $(I_Z - I_X)$ is always positive, examination of equation (7.21) shows that G_Y must always be negative (or zero). Neglecting M_{gyro} and M_{other}, we see that a (-M) is required for spins other than those which are flat (horizontal).

CONCLUSIONS (Steady state spin with wings level)

$\bar{G}$ = constant (may be zero)

If zero - flat spin about Z axis results (p = 0).

If not zero - $\bar{G}$ consists only of pitching moment and conventional spin results.

7.8.4 Fuselage - Loaded Aircraft Tend to Spin Flatter Than Wing-Loaded Aircraft

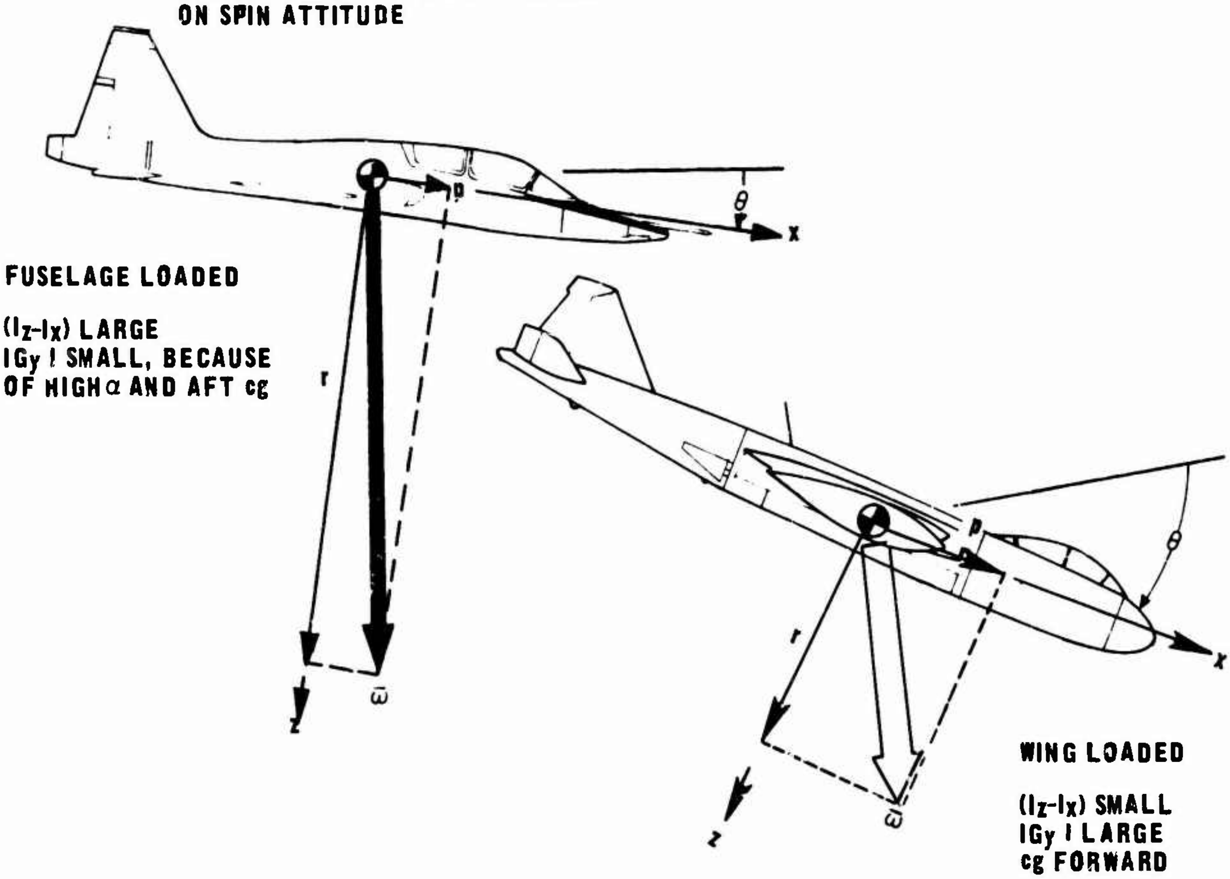
The degree of flatness varies with the relative magnitude of I_Z and I_X. This can be seen solving equation (7.18) for p:

$$p = \frac{|G_y|}{r(I_z - I_x)}$$

as p decreases, $\bar{\omega}$ is closer to Z making the spin flatter.

FIGURE 7.10

EFFECT OF I_Z AND I_X MAGNITUDES ON SPIN ATTITUDE



7.8.5 Fuselage - Loaded Aircraft Tend to Exhibit More Oscillation

On aircraft where I_y is approximately equal to I_z in magnitude, the wing down case represents only a small perturbation in rotational energy. In the limit, if $I_y = I_z$, the reference spin could be wing down, since any axis in the YZ plane would be a maximum principal axis. Although bank angle may thus be easily disturbed, the requirement for $\phi = 0$ provides a constant restoring tendency which leads to periodic oscillations in ϕ, and through the inertial coupling terms, to the other two axes as well (i.e., all 3 moment equations become active) Again the equations of motion can be examined for confirmation.

$$G_x = \dot{p} I_x + q r (I_z - I_y)$$

$$G_{x_o} = \dot{p}_o I_x + q_o r_o (I_z - I_y)$$

$$(G_x - G_{x_o}) = (\dot{p} - \dot{p}_o) I_x + (q r - q_o r_o)(I_z - I_y)$$

Assume perturbations in roll won't significantly change r_o, hence $r \approx r_o$ and

$$\Delta G_x = \Delta\dot{p} I_x + \Delta q (I_z - I_y) r_o \qquad (7.22)$$

$$\Delta\dot{p} = \frac{\Delta G_x}{I_x} - \Delta q \left(\frac{I_z - I_y}{I_x}\right) r_o \qquad (7.23)$$

The second term on the RHS serves to damp oscillations in that it reduces the ability or perturbations in rolling moment (ΔG_x) to produce perturbations in roll acceleration ($\Delta\dot{p}$). For fuselage loaded aircraft, where ($I_z - I_y$) is small, the damping is much reduced.

7.9 GYROSCOPIC THEORY

A gyro, in its simplest form, may be thought of as a rapidly spinning rotor of substantial moment of inertia, supported on some kind of mount which allows freedom of tilt of the spin axis relative to the base on which it is mounted.

By virtue of its rotation, a gyroscope tends to maintain its axis of spin aligned with respect to inertial space. That is, unless an external torque is applied, the gyro spin axis will remain stationary with respect to the fixed stars. If a torque is applied about an axis which is transverse to the spin axis, the rotor turns about a third axis which is at right angles to the other two (precession). On removing this torque the rotation (precession) ceases - unlike an ordinary wheel on an axle which keeps on rotating after the torque impulse is removed.

These phenomena, all somewhat surprising when first encountered, are consequences of Newton's laws of motion. The precession behavior represents obedience of the gyro to Newton's second law expressed in rotational form, which states that torque is equal to the time rate of change of angular momentum.

$$\bar{L} = \bar{\omega} \times \bar{H} \qquad (7.24)$$

where

L = applied torque

H = rotor angular momentum

ω = angular velocity of precession

and

$H = I\omega_s$

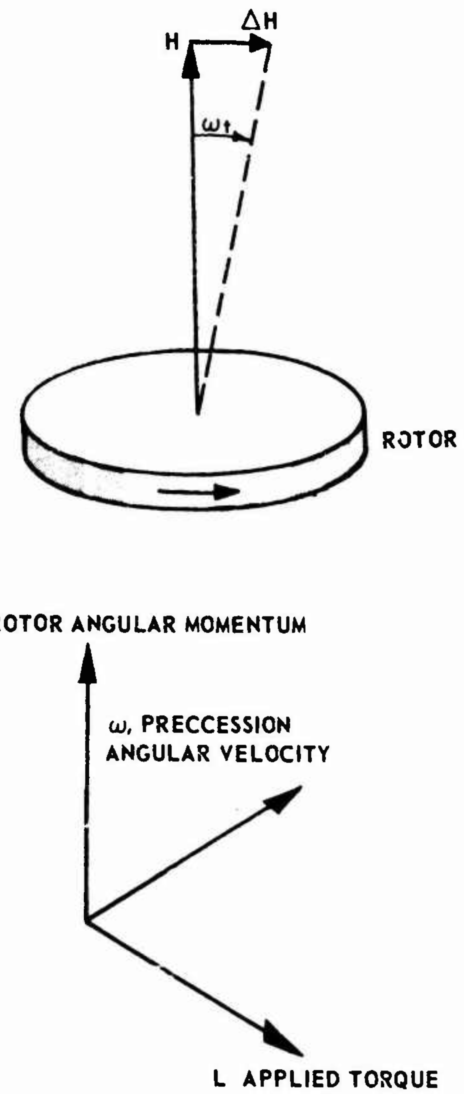
where

I = rotor moment of inertia

ω_s = rotor angular spin rate

The direction of procession for a gyro when a torque is applied is given by the vector equation (7.24). This direction is such that the gyro spin axis tends to align itself with the total angular momentum vector, which in this case is the vector sum of the angular moment due to the spinning rotor and the angular momentum due the applied torque, ΔH, where $\Delta H = L\Delta t$. This is shown in figure 7.11, below.

FIGURE 7.11



The law of precession is a reversible one. Just as a torque input results in an angular velocity output (precession), an angular velocity input results in a torque output along the corresponding axis.

7.9.1 Gyro Axes

Three gyro axes are significant in describing gryo operation; the torque axis, the spin axis and the precession axis. These are commonly referred to as input (torque), spin, and output (precession). The direction of these axes is shown in

FIGURE 7.12

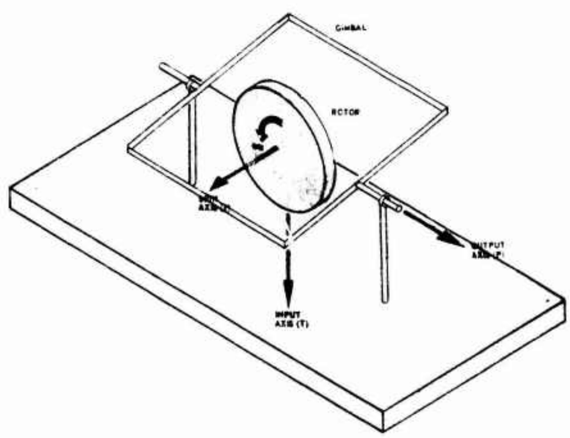
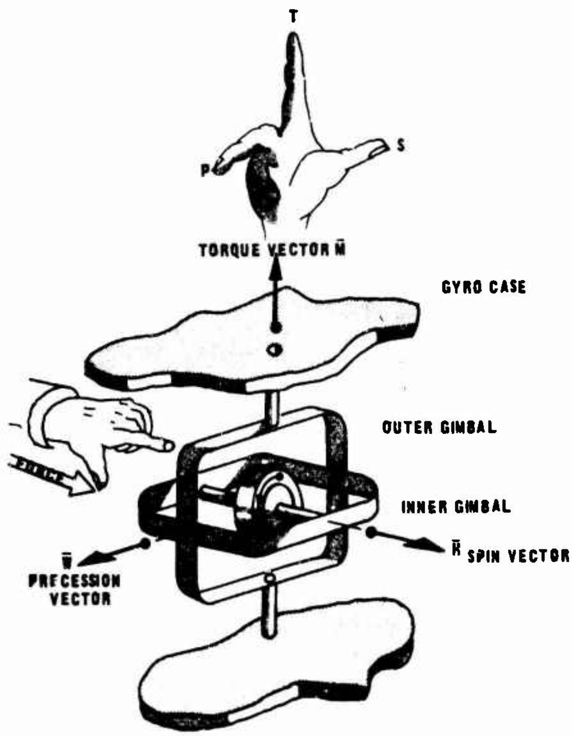


figure 7.12 and is such that the spin axis rotated into the input axis gives the output axis direction by the right hand rule.

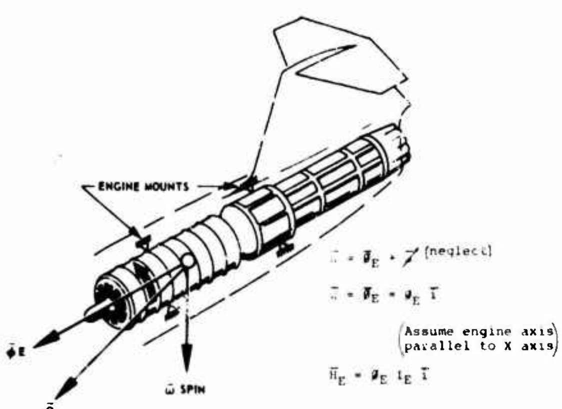
The direction of rotational vectors such as spin, torque, and precession can be shown by means of the right hand rule. If the curve of the fingers of the closed right hand point in the direction of rotation; the thumb extended will point along the axis of rotation. For gyro work, it is convenient to let the thumb, forefinger, and middle finger represent the spin, torque, and precession axes respectively. This is illustrated in figure 7.13.

FIGURE 7.13



•7.10 GYROSCOPIC MOMENTS (Engine)

FIGURE 7.14



$$\bar{G}_{gyro} = \bar{H}_E \times \bar{\omega}$$

$$I_E \, \phi \, \bar{i} = \bar{H}_E$$

$\bar{H}_E$ along X axis

Any moment about X axis has no gyro effect, i.e., $\bar{H}_E \times \bar{p} = 0$

$$M_{gyro} = - I_E \phi_E r \tag{7.25}$$

$$N_{gyro} = I_E \phi_E q \tag{7.26}$$

$$L_{gyro} = 0 \tag{7.27}$$

•7.11 BALANCE OF MOMENTS

$$C_\ell = \frac{2\mu K_x^2}{V^2} \left[\dot{p} - \left(\frac{I_y - I_z}{I_x} \right) q \, r - \frac{L_{oth}}{I_x} \right] \tag{7.28}$$

$$C_n = \frac{2\mu K_z^2}{V^2} \left[\dot{r} - \left(\frac{I_x - I_y}{I_z} \right) p \, q - \frac{I_{eng} \phi_{eng}}{I_z} q - \frac{N_{oth}}{I_z} \right] \tag{7.29}$$

$$C_m = \frac{2\mu K_y^2}{V^2} \left[\dot{q} - \left(\frac{I_z - I_x}{I_y} \right) p \, r + \frac{I_{eng} \phi_{eng}}{I_y} r - \frac{M_{oth}}{I_y} \right] \tag{7.30}$$

For the reference motion we have set ($\dot{p}_o = \dot{q}_o = \dot{r}_o = \dot{q}_o = 0$) and neglecting miscellaneous moments, we find:

$$C_\ell = 0 \tag{7.31}$$

$$C_n = 0 \tag{7.32}$$

$$C_m = -\frac{2\mu K_y^2}{V^2}\left[\left(\frac{I_z - I_x}{I_y}\right) p_o\, r_o - \frac{I_E\, \emptyset_E}{I_y}\, r_o\right] \tag{7.33}$$

Autorotative Forcing Moments

Aerodynamic damping

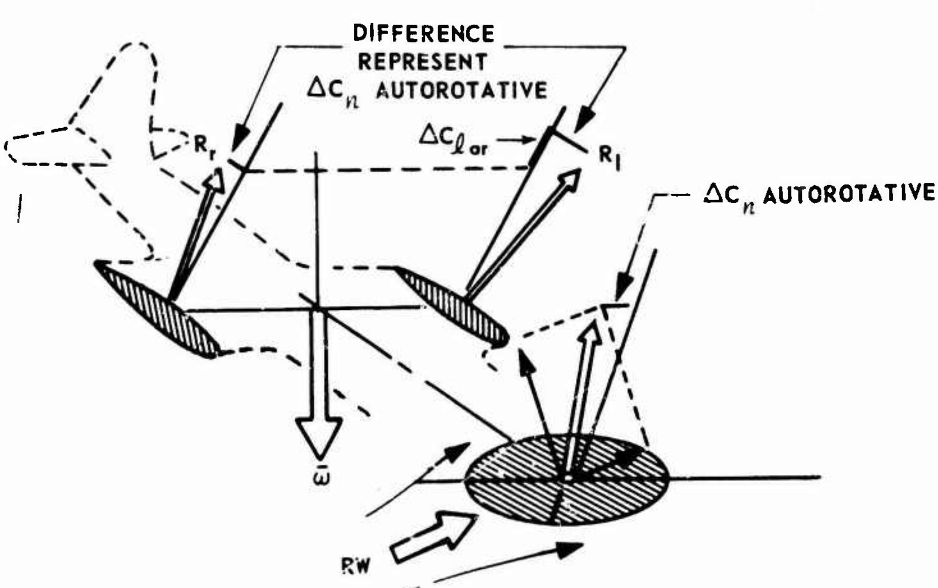
$$C_{\ell_{autorotative}} + C_{\ell_p} P_o = 0 \tag{7.34}$$

$$C_{n_{autorotative}} + C_{n_r} r_o = 0 \tag{7.35}$$

$$C_m = -\frac{2\mu K_y^2}{V^2}\left[\left(\frac{I_z - I_x}{I_y}\right) p_o\, r_o - \frac{I_E\, \emptyset_E}{I_y}\, r_o\right] \tag{7.36}$$

Aerodynamic Pitching Moment

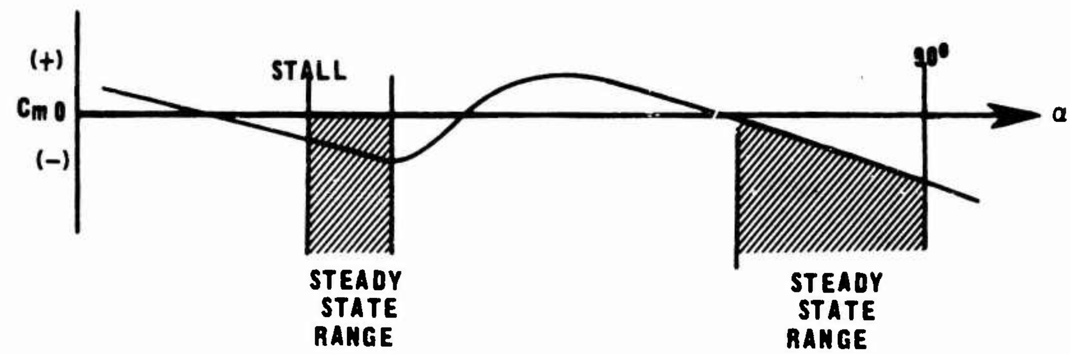
FIGURE 7.15



C_ℓ & C_n AUTOROTATIVE

FIGURE 7.16

STEADY STATE SPIN RANGES



The aerodynamic coefficients and p, r are all functions of α. C_m versus α plots may be examined for ranges of α where steady state spins will most likely stabilize. The following conditions must be present for a steady state spin to exist:

1. α must be above stall.

2. C_m must be negative.

3. Slope of C_m versus α must be stable, i.e., negative. (Slope must also be relatively constant.)

Rotation rate has been seen to be related to C_m and α (as well as to $C_{\ell_{autorotative}}$, C_{ℓ_p}, $C_{n_{autorotative}}$ and C_{n_r}). The actual relationship involves a balance between C_m and the product p r, but this may be replaced with the following expression:

$$G_y = -pr(I_z - I_x) \qquad (7.37)$$

$$M - I_E \phi_E r = -(I_z - I_x)\, p\, r$$

(Eq. 7.13 during steady state)

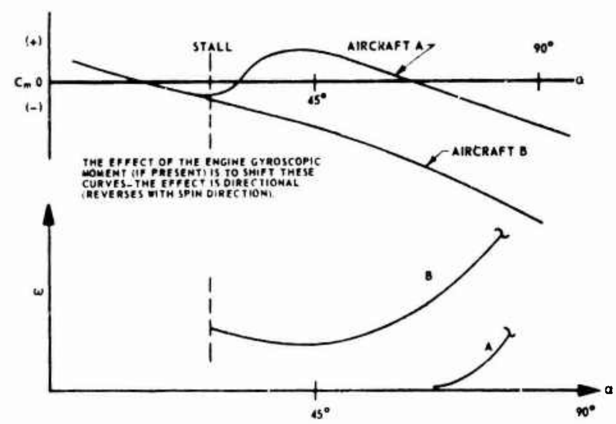
$$p = \omega \cos\alpha \qquad r = \omega \sin\alpha$$

$$p\, r = \omega^2 \sin\alpha \cos\alpha = \omega^2 \frac{1}{2} \sin 2\alpha$$

(trig identity)

$$\omega^2 = \frac{-M + I_E \phi_E r}{\frac{1}{2}(I_z - I_x)\sin 2\alpha} \qquad (7.38)$$

FIGURE 7.17



Both fast and slow flat spins are observed. For example, for the aircraft represented below (figure 7.18),

FIGURE 7.18

ω^2 approaches the indeterminate form 0/0 (equation 7.38) at high α's and is useless in predicting rotation rate. The rate will be dependent (as usual) on what α the

small C_m = (-) can sustain, and on the values of yaw and roll damping.

Great care must be exercised when using equation 7.38 as the equation is only a trend equation. It is not valid for large angles of attack. The main emphasis should be placed on the evaluation of the change in $\bar{\omega}$ that can result with variations in the pitching moment, -M, and in the engine gyroscopic effects.

7.12 RECOVERY

Obtaining developed spins today is generally difficult but, when obtained, the same factors that make it difficult to obtain the spin may also make it difficult to recover from the spin. Current and future aircraft designs may be compromised too much for their intended uses to provide adequate aerodynamic control for termination of the developed spin; also, there is a rising problem of pilot disorientation associated with developed spins. As a result, the incipient phase must be given more attention than it has been in the past, and preventing the developed spin by proper control utilization while the airplane is still in the incipient phase of the spinning motion may become a primary factor.

The current aircraft have weights which are appreciably larger and have moments of inertia about the Y and Z axes which may be 10 times as large as those of World War II aircraft. With the accompanying high angular momentum, it is difficult for a spin to be terminated as effectively as a spin of the earlier airplanes by aerodynamic controls which generally are of similar size. Furthermore, controls which are effective in normal flight may be inadequate for recovery from the spin unless sufficient consideration has been given to this problem in the design phase.

In brief, an airplane is considered to have recovered from a spin when the angle of attack at the center of gravity is below the stall. Usually, as this is achieved, the airplane enters a steep pullout dive without rotation; in some cases, however, it may be turning or rolling in a spiral dive or an aileron roll. Also, the airplane may roll or pitch to an inverted attitude from the erect spin and may still have some rotation, but it is out of the original erect spin. Thus, the precise definition for the termination of a spin can be rather ambiguous.

The effect of any control in bringing about spin recovery depends upon the moments that the controls provide and upon the effectiveness of those moments in producing a change in angular velocity and thus an upsetting of the spin equilibrium. The effectiveness of the applied moment in upsetting the spin equilibrium, in turn, is influenced by the magnitudes of the moments in balance in the developed spin. The effectiveness of the moments also depends greatly upon the mass distribution of the airplane.

Experience has shown that application of a yawing moment about the Z body axis to oppose the spin rotation is the most effective manner of terminating the spin and bringing about recovery. Thus, the effectiveness of a rudder deflection which generally creates a direct yawing moment on the spin, is dependent upon the magnitude of the yawing moment produced and upon the ability of this moment to affect the existing motion. Similarly, it appears that elevator effectiveness and aileron effectiveness, in the final analysis, depend upon their ability to alter the yawing moments acting on the spin. The most effective way to influence

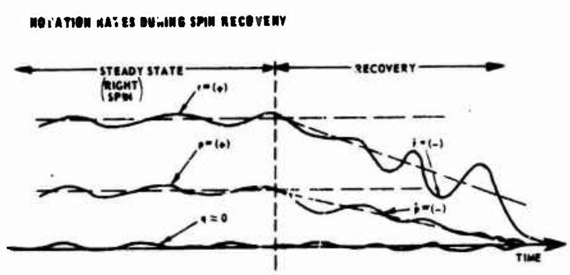
the spin and to bring about recovery is to obtain a yawing moment by applying a moment about an axis about which there is the least resistance to a change in angular velocity (least moment of inertia). By definition, there are no accelerations associated with the steady state spin; however, in order to recover from a spin, angular accelerations are needed to oppose the motion of the spin.

The balanced condition of the developed spin must be disturbed in order to effect a recovery;

To do this, prolonged angular accelerations in the proper direction are needed. Several methods for obtaining these accelerations are available but not all are predictable. Also the accompanying effects of some methods are adverse or potentially hazardous.

Below: The general methods available for generating anti-spin moments are presented with the applicable terms of the general equations

FIGURE 7.19



1. Modify aerodynamic moments
   a. With flight controls
   b. Configuration changes (gear, flaps, strakes)

2. Reposition the aircraft attitude on the spin axis

$$\dot{p} = \frac{V^2}{2\mu K_x^2} C_\ell + \frac{I_y - I_z}{I_x} q r + \frac{L_{oth}}{I_x}$$

$$\dot{q} = \frac{V^2}{2\mu K_y^2} C_m + \frac{I_z - I_x}{I_y} p r - \frac{I_E \phi_E}{I_y} r + \frac{M_{oth}}{I_y}$$

$$\dot{r} = \frac{V^2}{2\mu K_z^2} C_n + \frac{I_x - I_y}{I_z} p q + \frac{I_E \phi_E}{I_z} q + \frac{N_{oth}}{I_z}$$

3. Variations in Engine Power

4. Spin chutes Spin Rockets

POSSIBLE ENGINE EFFECTS

1. ΔV_T (during spin and recovery)

2. $\Delta\alpha$ or $\Delta\theta$ from $\dot{q}$

3. $\Delta\omega$ from equation (7.38)

   (Nos. 2 and 3 can't handle together - just predict separate parts)

4. $\Delta\dot{r}$

   a. No effect if $q = 0$

   b. If $q_{ss} \neq 0$, the engine can affect ability to produce recovery $\dot{r}$

5. Other -

   a. Chance of oscillatory tendencies

   b. NASA experience shows that when considering engine (or any) effects, you should EXPECT THE HURT NOT THE HELP.

MOVEMENT OF THE CONTROLS

<u>MOVEMENT OF THE CONTROLS</u>

$$\underbrace{\delta_r \qquad \delta_a \qquad \delta_e}$$

<u>MAY</u> OR <u>MAY NOT</u> PRODUCE CHANGES IN AERO MOMENTS

?

BECAUSE OF LARGE α AND LARGE β - ALSO SURFACE BLANKING EFFECTS

$$\underbrace{\Delta C_n \qquad \Delta C_\ell \qquad \Delta C_m}$$

THESE CHANGES <u>MAY</u> OR <u>MAY NOT</u> PRODUCE SIGNIFICANT CHANGES IN THE ANGULAR RATES

?

MAY BE TOO SMALL OR OF TOO SHORT A DURATION

$$\underbrace{\dot{\psi} \qquad \dot{\phi} \qquad \dot{\theta}}$$

$$\underbrace{\text{RELATIVELY SMALL, SHORT DURATION CHANGES IN p, q, r, MAY OCCUR THROUGH EULER TRANSFORM}}$$

$$\left[\begin{array}{l} p = \dot{\phi} - \dot{\psi}\sin\theta \\ q = \dot{\theta}\cos\phi + \dot{\psi}\sin\phi\cos\theta \\ r = \dot{\psi}\cos\phi\cos\theta - \dot{\theta}\sin\phi \end{array}\right]$$

Small effects but may be sufficient

LARGER, LONGER DURATION CHANGES IN p, q, r MAY BE OBTAINED THROUGH REPOSITIONING OF THE AIRCRAFT ON THE SPIN AXIS

MOST EFFECTIVE

Experience has shown that usually the best way to achieve spin recovery is through the reduction of $\dot{r}$. Therefore the following equation provides a powerful tool for getting a prediction on what aileron and rudder to use during recovery.

$$\dot{r} = \frac{V^2}{2\mu K_z^2} C_n + \left(\frac{I_x - I_y}{I_z}\right) p\,q$$

(neglecting the N_{gyro} and N_{other} terms) (7.20)

Equation (7.20) will be used to determine the control movements needed to recover from a spin.

The first term, $V^2/2\mu K^2 C_n$, was predominant in World War II aircraft where the mass was distributed equally along the wings and fuselage. Changes in this aerodynamic term were large while the inertia term

$$\left(\frac{I_x - I_y}{I_z}\right) p\,r$$

was small since I_x I_y. Therefore, the predominant recovery control on these aircraft was the rudder which would create large yawing moments in a direction to oppose the spin. Also for these aircraft, $(I_z - I_x)$ was a small positive term which meant that a less negative aerodynamic pitching moment would create a slower rotation rate (equation 7.38). For this reason, recovery from a spin in the T-6 or T-28 was stick back, rudder against the spin until rotation stopped and then stick forward.

In recent years, mass distribution along the fuselage by heavy jet engines and increased wing loading (short, stubby wings) have tended to make the changes in the inertial term, $(I_x - I_y/I_z)p\,q$ much more important. For example, I_x = 6,439 slug - ft^2 and I_y = 8,825 slug - ft^2, I_z = 17,149 for the T-28, while I_x = 3,554 slug - ft^2 and I_y = 41,615 slug - ft^2, I_z = 42,633 slug - ft^2 for the F-104. This shows that the term $(I_x - I_y/I_z)$ will have approximately six times the effect on the F-104 as it will on the T-28. In addition to this, the control surfaces have become smaller with much less rudder available. Thus it becomes extremely important that the gyrodynamic inertia term become anti-spin (negative for a right spin) for recovery.

The sign of the inertia term,

$$\left(\frac{I_x - I_y}{I_z}\right) p\,q$$

in a right spin for a fuselage loaded aircraft is negative due to I_y being larger than I_x. Therefore, the magnitude of the quantity needs to be increased and the same time continued negative. This can be done by controlling the algebraic sign of the pitching and rolling velocities. A positive pitch is needed and can be attained in two ways; stick back, and ailerons right (right spin). The ailerons right or with the spin repositions the aircraft axes on the spin axis so that some component of $\bar{\omega}$ is felt along the positive Y axis. The same technique can be used to prove that "ailerons with" in a left spin also produces the correct moment about the Z axis to recover from a spin.

7.12.1 Summary of Design Trend in Modern High Performance Aircraft

7.12.1 Summary of Design Trend in Modern High Performance Aircraft	
Conventional A/C	High Performance Fighters
Low wing loading (W/S small)	High wing loading
Ordinary span (b)	Small span (b)
$\mu = \frac{M}{\rho S b}$ = small	μ = large
C_n relatively large due to large δ_r, larger vertical tails	C_n - reduced, restricted δ_r smaller verticals blanking at high α's
$\frac{I_x - I_y}{I_z} \approx 0$	$\frac{I_x - I_y}{I_z}$ large negative

7.12.2 Summary of Recovery Controls for Past and Present Aircraft

7.12.2 Summary of Recovery Controls for Past and Present Aircraft

SPIN CHARACTERISTICS & RECOVERY PROCEDURES
VARY PRIMARILY WITH MASS DISTRIBUTION

(I_x - I_y) is only one important parameter

Control \ A/C	F-106	F-102	F-104	F-8U T-38 F-105	T-33	T-6 T-28	B-57	B-17
δ_r	N[1]	A	A	A	A	A	A	A
δ_a	W	W	W	W	N[3]	N	A	A
δ_e	N	N	FWD[2]	AFT	AFT	AFT	FWD	FWD

- ⟵ 0 ⟶ +

(I_x - I_y)

[1]Blanking

[2]Pitch-up

[3]δ_a with not too effective & also tumble danger

Superscripts indicate designs with special problems.

7.12.3 Example Recovery Analysis

The following analysis will aid in understanding the steps involved in predicting optimum recovery techniques.

Given: F-104 in a right, steady state spin with $\phi = 0$.

Predict: Proper control positions for recovery.

Analysis: Equation (7.20) provides the powerful tool for spin recovery control prediction:

$$\dot{r} = \frac{V^2}{2\mu K_z^2} C_n + \left(\frac{I_x - I_y}{I_z}\right) pq \quad (7.20)$$

1. A negative $\dot{r}$ is needed for spin recovery.

PREDICT: LEFT RUDDER (aerodynamic term)

2. $I_x - I_y/I_z$ is negative as the F-104 is fuselage loaded.

3. $q \approx 0$ (steady state spin).

4. p is positive in a right spin.

Therefore, a positive pitch rate (q) is needed to generate a negative $\dot{r}$.

PREDICT:

a. Stick aft (not desirable in the F-104 due to possible pitch up on recovery). Aft stick would have a very small effect.

b. AILERON WITH (RIGHT). This is the big effect as it repositions the aircraft on the $\bar{\omega}$ vector such that there is a component of $\bar{\omega}$ on the Y axis. This results in a positive pitch rate (q). Ailerons are generally much more effective than the rudder in the spin recovery of fuselage loaded aircraft.

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CHAPTER

8 ROLL COUPLING

8.1 INTRODUCTION

Divergencies experienced during rolling maneuvers have frequently been referred to as "Inertia Coupling." This leads to a misconception of the problems involved. The divergence experienced during rolling maneuvers is complex because it involves not only inertia properties, but aerodynamic ones as well. It is the intention of this chapter to offer a physical explanation of the more important causes of roll coupling and to also introduce a brief mathematical tool to aid in predicting roll coupling divergencies.

Coupling results when a disturbance about one aircraft axis causes a disturbance about another axis. An example of uncoupled motion is the disturbance created by an elevator deflection. The resulting motion is restricted to pitching motion and no disturbance occurs in yaw or roll. An example of coupled motion is the disturbance created by a rudder deflection. The ensuing motion will be some combination of both yawing and rolling motion. Although all lateral-directional motions are coupled, the only motion that ever results in coupling problems large enough to threaten the structural integrity of the aircraft is coupling as a result of rolling motion. Thus our study of "roll coupling."

Although there are numerous contributions to the roll coupling characteristics of an aircraft, aeroelastic effects, etc., this chapter will only consider three: (1) inertia coupling, (2) the I_{xz} parameter, (3) aerodynamic coupling. These effects occur simultaneously in a very complicated fashion. Therefore, the resulting aircraft motion cannot be predicted by analyzing these effects separately. The complicated interrelationship of these parameters can best be seen by analyzing the aircraft equations of motion.

Roll
$$\frac{\sum L}{I_x} = \dot{p} + qr\left(\frac{I_z - I_y}{I_x}\right) - (\dot{r} + qp)\frac{I_{xz}}{I_x} \tag{8.1}$$

Pitch
$$\frac{\sum M}{I_y} = \dot{q} + pr\left(\frac{I_x - I_z}{I_y}\right) + (p^2 - r^2)\frac{I_{xz}}{I_y} \tag{8.2}$$

Yaw
$$\frac{\sum N}{I_z} = \dot{r} + pq\left(\frac{I_y - I_x}{I_z}\right) + (qr - \dot{p})\frac{I_{xz}}{I_z} \tag{8.3}$$

Drag
$$\frac{\sum F_x}{m} = \dot{u} + qw - rv \tag{8.4}$$

Lift
$$\frac{\sum F_z}{m} = \dot{w} + pv - qu \tag{8.5}$$

Side
$$\frac{\sum F_y}{m} = \dot{v} + ru - pw \tag{8.6}$$

This analysis will be based on equations 8.1 - 8.3. Equations 8.4 - 8.6 are not important in an analysis of roll coupling. Consider equations 8.1 - 8.3. In each case,

the first term in the equations represents the aerodynamic contribution, the second term the interial contribution, and the third term the I_{xz} parameter. It can be seen that the relationships involved could never occur singularily and that they actually occur in conjunction with one another to either become additive and aggravating or opposing and thus alleviate the tendency to diverge.

"Divergence" in roll coupling is manifested by a departure from the intended flight path that will result in either loss of control or structural failure. As defined, this "divergence" is what we are concerned with in roll coupling. Smaller roll coupling effects that do not result in divergence will not be considered. It should be noted that divergence about any one axis will be closely followed by divergence about the others.

In attempting to explain the subject of roll coupling, this chapter will first explain the physical aspects of inertia coupling, aerodynamic coupling, and the I_{xz} parameter. No attempt will be made to physically analyze these combined motions. Next, a mathematical model for roll coupling will be developed that will permit determination of the approximate roll rate that will drive an aircraft to divergence.

8.2 INERTIAL COUPLING

The problem of inertial coupling did not manifest itself until the introduction of the century series aircraft. As the modern fighter plane evolved from the conventional fighter, such as the F-51 and F-47, through the first jet fighter, the F-80, and then to the F-100 and other century series aircraft, there was a slow but steady change in the weight distribution. During this evolution, more and more weight became concentrated in the fuselage as the aircraft's wings became thinner and shorter. This shift of weight caused relations between the moments of inertia to change. As more weight is concentrated along the longitudinal axis, the moment of inertia about the x-axis decreases while the moments of inertia about the y and z axes increase. This phenomena increases the coupling between the lateral and longitudinal equations. This can be seen by examining equation 8.2.

$$\frac{\sum M}{I_y} = \dot{q} + pr\left(\frac{I_x - I_z}{I_y}\right) + (p^2 - r^2)\frac{I_{xz}}{I_y} \tag{8.2}$$

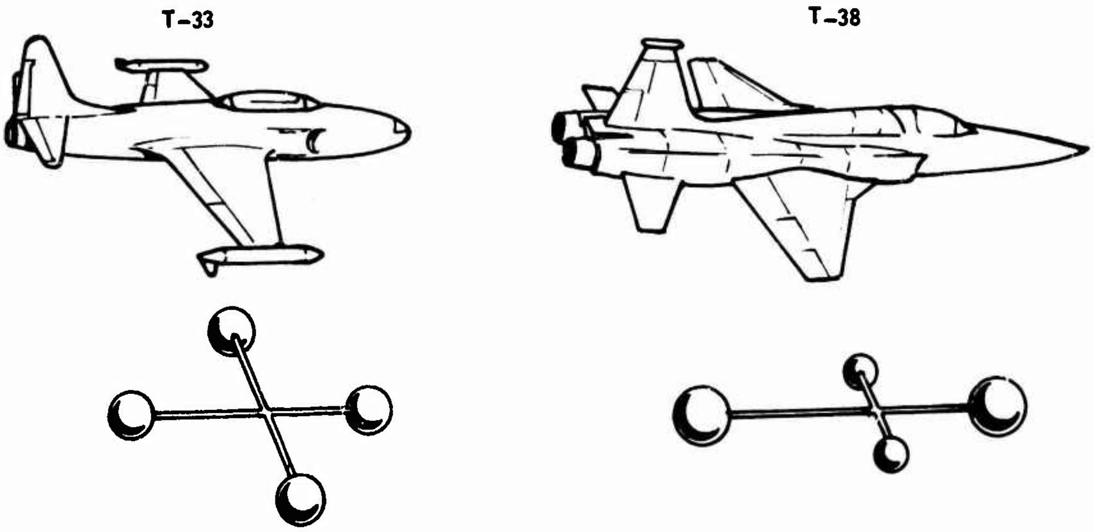
As I_x becomes much smaller than I_z, the moment of inertia difference term $(I_x - I_z)/I_y$ becomes large. If a rolling moment is introduced, the term $pr\ (I_x - I_z)/I_y$ may become large enough to cause an uncontrollable pitching moment.

Modern fighter design is characterized by a long slender, high-density fuselage with short, thin wings. This results in a roll inertia which is quite small in comparison to the pitch and yaw inertia. The more conventional low speed airplane may have a wingspan greater than the fuselage length, and a great deal of weight concentrated in the wings. A comparison of these configurations is presented in figure 8.1.

It can be seen that the conventional design presents considerable resistance to rotation about the x-axis. Thus, with this design, one would not expect high roll rates. On the other hand, it can be seen that the modern design presents a relatively small resistance to rotation about the x-axis. Thus, with this design, one could expect to attain high rates of roll. It has been shown that high roll rates

Figure 8.1

CONVENTIONAL AND MODERN AIRCRAFT DESIGNS

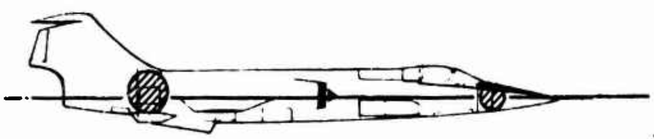


enhance the tendency toward inertial coupling.

This analysis of inertia coupling will consider rolls about two different axes: The inertia axis, and the aerodynamic axis. The inertia axis is formed by a line connecting the aircraft's two "centers of inertia." Refer to figure 8.2.

Figure 8.2

AIRCRAFT INERTIA AXIS

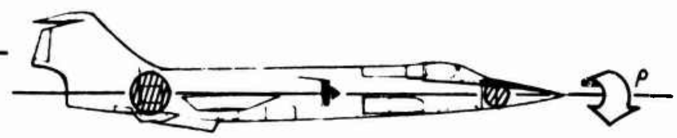


The aerodynamic axis is simply the stability x-axis first introduced in the investigation of the left hand side of the equations of motion. It is merely the line of the relative wind. Aircraft rotation in a roll is generally assumed to be about this axis. To visualize this, recall that to produce a rolling moment, a differential in lift must be created on the wings. By definition, the differential lift created must be perpendicular to the relative wind. Therefore, the aircraft will roll about the relative wind, or aerodynamic axis.

First, consider a roll when the aerodynamic and inertia axis are coincident. Figure 8.3.

Figure 8.3

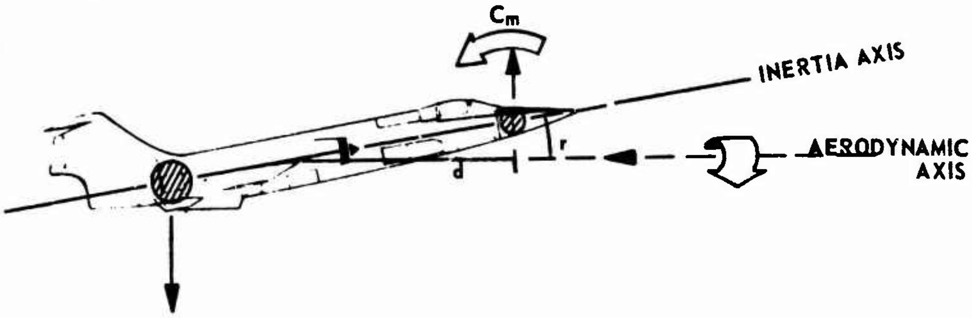
AERODYNAMIC AND INERTIA AXIS COINCIDENT



In this case, there is no force created by the centers of inertia that will cause the aircraft to be diverted from its intended flight path, and no inertia coupling results. Now, observe what happens when the inertia axis is displaced from the aerodynamic axis.

Figure 8.4

AERODYNAMIC AND INERTIA AXIS NON-COINCIDENT



As the aircraft is rotated about the aerodynamic axis, centrifugal force will act on the centers of inertia. Remembering that centrifugal force acts perpendicular to the axis of rotation, it can be seen that a moment will be created by this centrifugal force. For the case depicted in figure 8.4, where the aerodynamic axis is depressed below the inertia axis, a pitch up will result. Conversely, if the aerodynamic axis is above the inertia axis, a pitch down will result.

To appreciate the magnitude of the moment thus developed, refer to figure 8.4 and consider the following:

$$\text{Centrifugal Force} = \frac{mV_T^2}{r} \qquad (8.7)$$

$$V_T = r\omega = rp \qquad (8.8)$$

Therefore,

$$C.F. = mrp^2$$

the moment created by this centrifugal force is

$$M = (C.F.)(d) = mrp^2 d \qquad (8.9)$$

For modern designs, m is large. Also, r will be larger than for a high aspect ratio wing. (The aircraft will operate at a higher angle of attack.) As previously discussed, p will be large. Also, for long fuselages, d will be large. Thus, the moment created by inertia coupling will be large.

8.3 THE I_{xz} PARAMETER

Three products of inertia I_{xy}, I_{yz} and I_{xz} appear in the equations of motion for a rigid aircraft. By virtue of symmetry, I_{xy} and I_{yz} are both equal to zero. However, the product of inertia I_{xz} can be of an appreciable magnitude and can have a significant effect on the roll characteristics of an aircraft.

The parameter, I_{xz}, can be thought of as a measure of the uniformity of the mass distribution about the x-axis. The axis about which I_{xz} is equal to zero is defined as the principle inertia axis, and the mass of the aircraft can be considered to be concentrated on this axis.

If the I_{xz} parameter is not equal to zero, then the principle inertia axis is not aligned with the aircraft x-axis. A typical modern aircraft design can be represented by two centers of mass in the xz plane designated m_1 and m_2 in figure 8.5.

It can be seen that if the aircraft is rolled about the aerodynamic axis, a pitch down will result. This phenomena is produced by exactly the same centrifugal force effects that produced inertial coupling. However, it should be noted that in this case the x-axis is aligned with the aerodynamic axis and that the pitching moment is a result of the inclination of the principle inertia axis. Thus, when an aircraft is rolled about any axis which differs from its principle inertia axis, pitching moments develop which tend to cause the aircraft to depart from its intended flight path. Depending on its orientation, the I_{xz} parameter can either aggravate or oppose inertial coupling.

To appreciate the magnitude of this parameter, consider figure 8.6.

From equation 8.9, the moment produced by the forward center of mass is,

$$M_1 = (C.F.)(x_1) = m_1 x_1 p^2 z_1 \qquad (8.10)$$

Similarly, the moment produced by the aft center of mass is,

$$M_2 = m_2 x_2 p^2 z_2 \qquad (8.11)$$

The total pitch moment is therefore,

$$M_T = m_1 + m_2 = m_1 x_1 p^2 z_1 + m_2 x_2 p^2 z_2 \qquad (8.12)$$

$$M_T = p^2 (m_1 x_1 z_1 + m_2 x_2 z_2) \qquad (8.13)$$

Figure 8.5

PRINCIPLE INERTIA AXIS

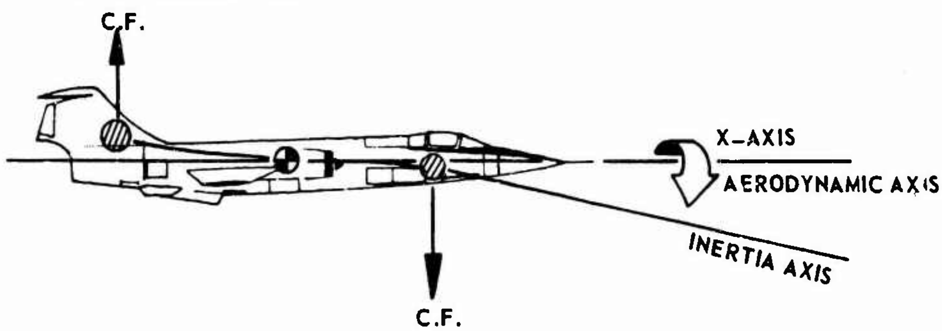
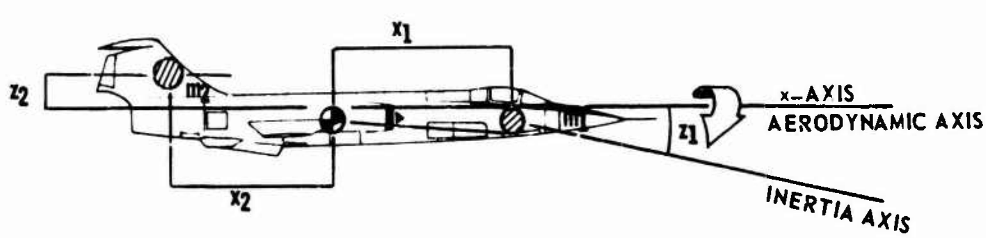


Figure 8.6

PRINCIPLE INERTIA AXIS BELOW AERODYNAMIC AXIS



But for a simplified system,

$$I_{xz} = m_1x_1z_1 + m_2x_2z_2 \qquad (8.14)$$

Therefore,

$$M_T = p^2 I_{xz} \qquad (8.15)$$

Thus, it can be seen that the magnitude of the pitching moment thus developed depends on the roll rate and the magnitude of the I_{xz} parameter relative to the roll axis.

The product of inertia, I_{xz}, is not only of concern because of its introduction of a pitching moment, but also because it plays an important role in determining the aircraft's cross over speed from adverse to complimentary yaw. Primarily, an aircraft's cross over speed is determined by the relationship of the induced and parasite drag generated by a wing during a rolling maneuver. However, this primary effect is mitigated somewhat by the effect of the I_{xz} parameter.

As previously shown, the I_{xz} parameter can cause a pitching moment during a rolling maneuver. However, since the aircraft is rolling, gyroscopic effects come into play, and this moment results in a yawing motion. From figure 8.6, it can be seen that when the principle inertia axis is below the roll axis, a pitch down will result. For the right roll depicted, this pitching moment would result in a yaw left. Thus, in this case, the I_{xz} parameter would contribute to the adverse yaw tendency of the aircraft. In actual practice however, it will be found that the aerodynamic axis generally lies below the principle inertia axis throughout most of the flight envelope. Thus the I_{xz} parameter will generally cause the aircraft to transition from adverse to complimentary yaw at an earlier speed. For a relative comparison of values, the F-102 has an I_{xz} of 3,500 slugs-feet^2 and a crossover speed of 268 knots. The F-100 has an I_{xz} of 942 slugs-feet^2 and experiences complimentary yaw in roll above 360 knots.

8.4 AERODYNAMIC COUPLING

This analysis of roll coupling is not concerned with all aerodynamic coupling terms (C_{n_p}, $C_{n_{\delta_a}}$, C_{ℓ_r}, $C_{\ell_{\delta_r}}$, etc.). Only the "kinematic coupling" aspects of aerodynamic coupling will be considered.

Kinematic coupling may be considered as an actual interchange of α and β during a rolling maneuver. This interchange is an important means by which the longitudinal and lateral motions are capable of influencing each other during a rapid roll.

To understand how this interchange of α for β occurs, consider figure 8.7.

In this figure the aircraft is assumed to have either infinitely large inertia or negligible stability. Thus, it will roll about its principle inertia axis. In (I) the aircraft initiates a roll from a positive angle of attack. In (II) the initial angle of attack is converted to a positive sideslip angle of equal magnitude after 90° of roll. In (III) the aircraft has again exchanged β and α and after 180° of roll has an angle of attack equal in magnitude but opposite in sign to the original α. The interchange continues and in (IV) this $-\alpha$ is converted to $-\beta$.

Next, consider an aircraft with infinitely large stability in pitch and yaw or negligible inertia. Refer to figure 8.8.

Figure 8.7

KINEMATIC COUPLING. STABLE ROLLING OF AN AIRCRAFT WITH INFINITELY LARGE STABILITY IN PITCH AND YAW OR NEGLIGIBLE INERTIA

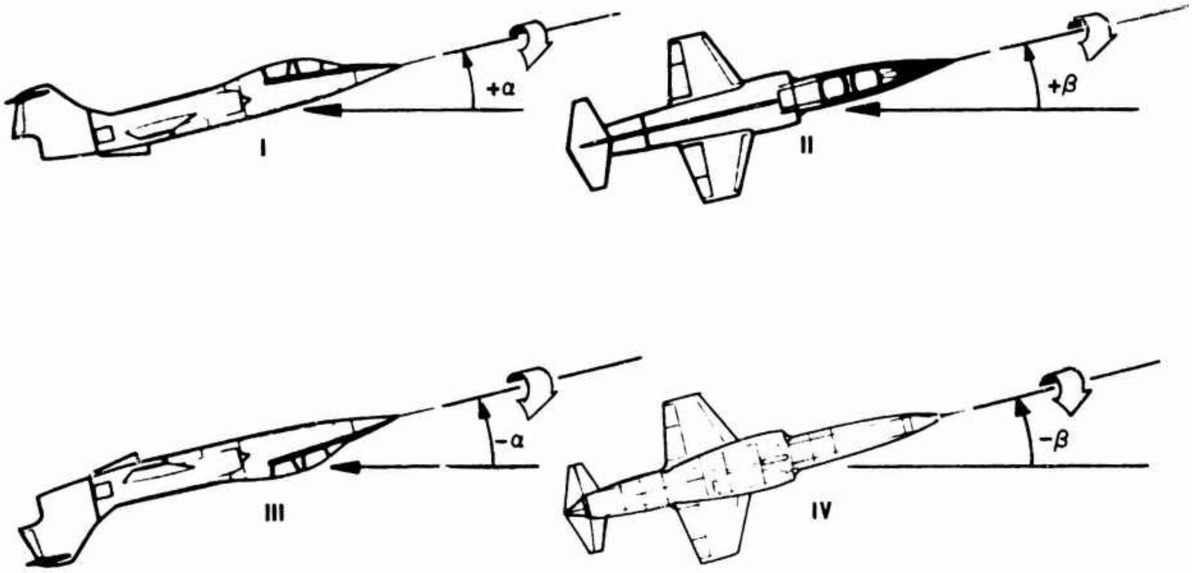
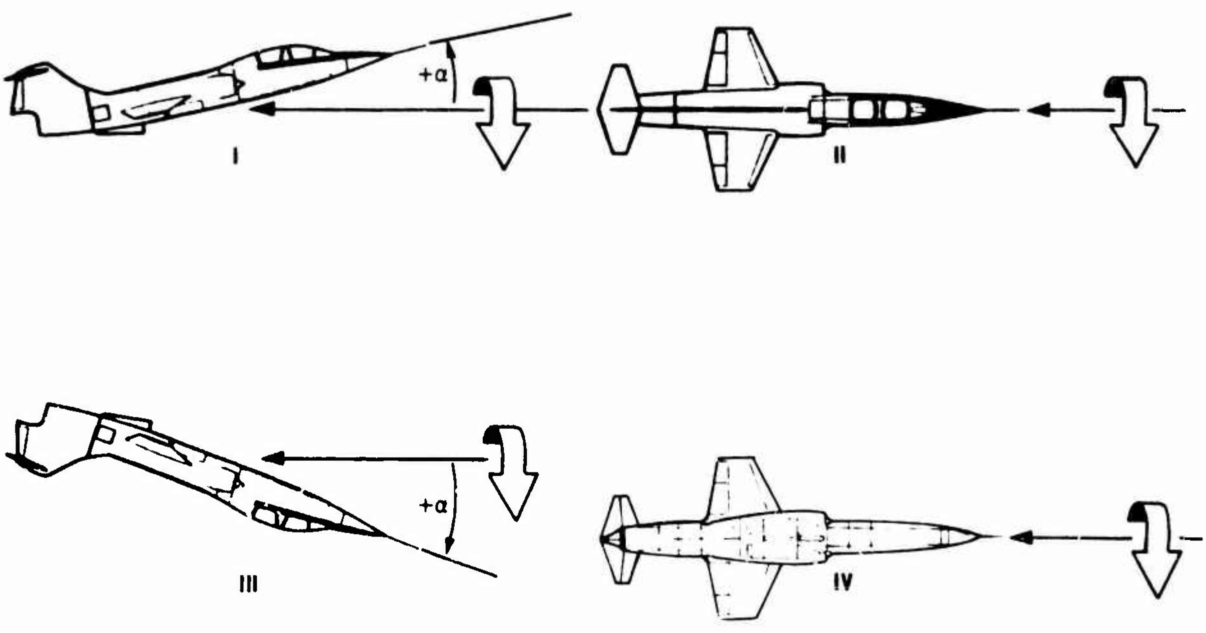


Figure 8.8

KINEMATIC COUPLING. STABLE ROLLING OF AN AIRCRAFT WITH INFINITELY LARGE STABILITY IN PITCH AND YAW OR NEGLIGIBLE INERTIA



In this case, the aircraft will roll about its aerodynamic axis, and no interchange of α or β will occur. However, in this situation it is possible for inertia coupling to occur.

The situation depicted in figure 8.8 never actually occurs but can be fairly well approximated by an aircraft possessing large magnitudes of C_{n_β} and C_{m_α} which is rolled at a relatively slow roll rate. However, in most cases, the actual aircraft motion will lie somewhere between the two extremes depicted.

Two empirical relationships can be stated:

$$\dot{\alpha} = -Kp\beta \qquad (8.16)$$

$$\dot{\beta} = Kp\alpha \qquad (8.17)$$

These relationships show that any roll rate will cause an interchange of α and β, the exact amount depending on the relative values of the moments of inertia and C_{m_α} and C_{n_β}. It can also be seen that for a given aircraft, the rate of interchange of α and β depends on the roll rate.

It has been shown that whenever α and β exist, a rolling maneuver will generate inertia coupling. The relative value of C_{m_α} and C_{n_β} will determine just what axis an aircraft will actually roll about, and thus how much interchange of α and β will occur.

Subsequent to a disturbance in pitch or yaw from an aircraft's equilibrium condition, a finite period of time will be required for the natural aircraft stability to reduce the disturbance to zero. The frequency of this response is a function of the value of C_{n_β} and C_{m_α}.

It will be shown in dynamics that,

$$f_{n_\beta} = (f)\, C_{n_\beta} \qquad (8.18)$$

$$f_{m_\alpha} = (f)\, C_{m_\alpha} \qquad (8.19)$$

Assume that an aircraft is rolled at a rate that creates a disturbance in β at a rate equal to the maximum rate that the natural aircraft stability can damp out the disturbance. Thus,

$$\dot{\beta} = Kp\alpha = f_{n_\beta} \qquad (8.20)$$

In this case there would be no buildup of β, and a condition of neutral stability in yaw would result. However, if the roll rate were increased slightly above this value then successively larger increases in β would occur and divergence would result. This analysis can also be followed through for an initial disturbance in α. It is not important which diverges first, α or β, since any divergence about one axis will quickly drive the other divergent. As a matter of interest however, supersonically C_{n_β} decreases more rapidly than C_{m_α} and therefore, most modern aircraft will diverge in yaw first.

It can be shown on an analog computer that when $C_{m_\alpha} = C_{n_\beta}$ a stable condition will exist at all roll rates. This is often referred to as a "tuned condition," and is a possible dodge for an aircraft designer to utilize in a critical flight area. However, it is difficult to capitalize on this occurrence because of the wide variation of the stability derivatives with Mach number.

It may be that an aircraft will possess stability parameters such that a roll coupling problem exists at a given roll rate. How-

ever, if a relatively long time is required before large values of α and β are generated, then the aircraft may be rolled at the maximum value by restricting the aircraft to one 360 degree roll. In this situation, the aircraft is diverging during the roll, but at such a slow rate that by the time the aircraft has rolled 360 degrees, the maximum allowable α or β of the aircraft has not been exceeded.

8.5 AUTOROTATIONAL ROLLING

It has been shown that during rolling maneuvers large angles of sideslip may occur as a result of roll coupling and kinematic coupling. At negative angles of attack, kinematic coupling may cause the vertical tail to produce large rolling moments in the original direction of roll. In such a case, it may not be possible to stop the aircraft from rolling, although the lateral control is held against the roll direction. This is known as autorotational rolling. In this situation, positive "G" would facilitate recovery. As the angle of attack is increased to a positive value, kinematic coupling will result in a moment that opposes the original direction of roll, thus alleviating the tendency for autorotational rolling.

For an appreciation of roll coupling difficulties, computer studies have demonstrated that in quite realistic designs, the critical roll rate for the occurrence of such phenomenon as autorotational rolling can be as low as 20 degrees per second.

8.6 A MATHEMATICAL ANALYSIS OF ROLL DIVERGENCE

The roll coupling characteristics of high performance modern fighters are thoroughly investigated by analog simulation prior to flight testing. However, smaller test programs may not have this capability. It is the intent of this section to provide the test pilot with a practical mathematical tool to aid in determining the roll rate at which an aircraft will start to diverge due to roll coupling. If the critical roll rate thus determined is attainable in the aircraft, then the test pilot should avoid higher rates of roll until a complete analog analysis can be conducted. Thus, this mathematic tool will enable the test pilot to identify potentially hazardous areas.

This mathematical analysis is based on certain simplifying assumptions. They are:

1. Velocity remains constant during the roll maneuver, $\dot{u} = 0$.

2. Roll rate is constant, $\dot{p} = 0$.

3. v, w, q, r are small therefore their products are negligible.

4. Engine gyroscopic effects are negligible.

5. The rudder and elevator are fixed in their initial trim position.

6. Aerodynamic parameters are negligible with the exception of M_α, M_q, N_β, N_p.

When these assumptions are applied to the equations of aircraft motion, the following results are obtained:

$$\frac{\sum F_x}{m} = \cancel{\dot{u}} + \cancel{qw} - \cancel{rv} = 0 \qquad (8.4)$$

$$\frac{\sum L}{I_x} = \cancel{\dot{p}} + \cancel{qr}\left(\frac{I_z - I_y}{I_x}\right) - (\dot{r} + qp)\,\frac{I_{xz}}{I_x} \qquad (8.1)$$

It can be shown that:

$$(\dot{r} + qp) = -\ddot{\theta}\ \text{SIN}\ \phi \approx 0 \qquad (8.21)$$

Therefore, in light of the assumptions made, equation 8.1 will be approximately equal to zero.

$$\frac{\sum M}{I_y} = \dot{q} + pr\left(\frac{I_x - I_z}{I_y}\right) + (p^2 - \cancel{r}^2)\ \frac{I_{xz}}{I_y} \qquad (8.2)$$

Thus,

$$M_\alpha \cdot \alpha + M_q \cdot q = \dot{q} + pr\left(\frac{I_x - I_z}{I_y}\right) + p^2\ \frac{I_{xz}}{I_y} \qquad (8.22)$$

$$\frac{\sum N}{I_z} = \dot{r} + pq\left(\frac{I_y - I_x}{I_z}\right) + (q\cancel{r} - \cancel{\dot{p}})\ \frac{I_{xz}}{I_z} \qquad (8.3)$$

Thus,

$$N_\beta \cdot \beta + N_r \cdot r = \dot{r} + pq\left(\frac{I_y - I_x}{I_z}\right) \qquad (8.23)$$

The lift and side force equations will average zero throughout a roll.

$$\frac{\sum F_z}{m} = \dot{w} + pv - qu = 0 \qquad (8.5)$$

$$\frac{\sum F_y}{m} = \dot{v} + ru - pw = 0 \qquad (8.6)$$

To get equations 8.5 and 8.6 into a more suitable form, recall that for small α and β,

$$\alpha = \frac{w}{u} \qquad (8.24)$$

$$\beta = \frac{v}{u} \qquad (8.25)$$

Assuming $\dot{u} = 0$

$$\dot{\alpha} = \frac{\dot{w}}{u} \qquad (8.26)$$

$$\dot{\beta} = \frac{\dot{v}}{u} \qquad (8.27)$$

Thus, equation 8.5 becomes,

$$\frac{\dot{w}}{u} + p\frac{v}{u} - q = \dot{\alpha} + p\beta - q = 0 \qquad (8.28)$$

Equation 8.6 becomes,

$$\frac{\dot{v}}{u} + r - p\frac{w}{u} = \dot{\beta} + r - p\alpha = 0 \qquad (8.29)$$

To further streamline equations 8.22 and 8.23, the following substitutions are made,

$$A = \frac{I_y - I_x}{I_z}$$

$$B = \frac{I_z - I_x}{I_y}$$

To recap, simplifying assumptions have reduced the aircraft equations of motion to the following for rolling maneuvers:

$$\dot{\alpha} + p\beta - q = 0 \qquad (8.28)$$

$$\dot{\beta} + r - p\alpha = 0 \qquad (8.29)$$

$$M_\alpha \cdot \alpha + M_q \cdot q - \dot{q} + prB = p^2\ \frac{I_{xz}}{I_y} \qquad (8.30)$$

$$N_\beta \cdot \beta + N_r \cdot r - \dot{r} - pqA = 0 \quad (8.31)$$

These equations can be recognized as four linear differential equations expressed in terms of the variables α, β, q, r. To examine the stability of these equations, it is only necessary to find out if the transient solution of the equations delays with time. To do this, first express the equations in Laplace notation:

$$s\alpha + p\beta - q = 0 \quad (8.32)$$

$$s\beta + r - p\alpha = 0 \quad (8.33)$$

$$M_\alpha \cdot \alpha + (M_q - s)\, q + prB = 0 \quad (8.34)$$

$$N_\beta \cdot \beta + (N_r - s)\, r - pqA = 0 \quad (8.35)$$

The characteristic equation of this set of simultaneous, non-homogeneous, differential equations is identical to the determinate of the coefficients. The expanded determinate yields:

$$\begin{aligned} & s^4 \qquad (8.36) \\ & + (-M_q - N_r)\, s^3 \\ & + (-M_\alpha + M_q N_r + N_\beta + p^2 + p^2 AB)\, s^2 \\ & + (M_\alpha N_r - M_q N_\beta - M_q p^2 - N_r p^2)\, s \\ & + M_\alpha p^2 A + M_q N_r p^2 - N_\beta B p^2 + ABp^4 - M_\alpha N_\beta \\ & = 0 \end{aligned}$$

For convenience, the following substitutions will be made:

$$a_3 = -M_q - N_r \quad (8.37)$$

$$a_2 = -M_\alpha + M_q N_r + N_\beta + p^2 (1 + AB) \quad (8.38)$$

$$a_1 = M_\alpha N_r - M_q N_\beta - p^2 (M_q + N_r) \quad (8.39)$$

$$a_o = p^2 (M_\alpha A + M_q N_r - N_\beta B + ABp^2) - M_\alpha N_\beta \quad (8.40)$$

Thus, equation 8.36 becomes

$$s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_o = 0 \quad (8.41)$$

If any roots of equation 8.41 have positive real parts, then the motion will be unstable and the aircraft will diverge. In order that there be no roots with positive real parts, it is necessary but not sufficient that:

1. All coefficients have the same sign.

2. None of the coefficients vanish.

If both of these conditions are met, then the equation must be examined further.

The coefficients of the characteristic equation will be examined in light of the following assumptions:

1. The aircraft possesses positive static stability in pitch and yaw, thus $M_\alpha = (-)$, $N_\beta = (+)$.

2. The aircraft possesses positive damping in pitch and yaw, thus, $M_q = (-)$, $N_r = (-)$.

3. The aircraft mass distribution is such that $I_Z > I_X$ and $I_Y > I_X$, thus $A = (+)$, $B = (+)$. (This mass distribution is typical of a modern fuselage loaded aircraft.)

In view of these assumptions, it can be seen that the coefficients a_2, a_2, a_1 will be positive. However, a_o may be either positive or negative. If a_o is examined and found to be negative, the resulting rolling maneuver will be unstable. If, however, a_o if found to be positive, the coefficients must be investigated further by means of the Routh-Hurwitz Stability Criterion. A brief description of the mechanics involved in the Routh-Horwitz method follows:

The first step is to arrange the polynomial coefficients into two rows: The first row will consist of the first, third, fifth coefficients, etc., and the second row will consist of the second, fourth, sixth coefficients, etc. The following example is presented:

$$\begin{array}{ccccc} a_o & a_2 & a_4 & a_6 & a_8 \\ a_1 & a_3 & a_5 & a_7 & a_9 \end{array}$$

The next step is to form the following array of numbers obtained by the indicated operations. The example shown is for a sixth-order system.

$$F(s) = a_o s^6 + a_1 s^5 + a_2 s^4 + a_3 s^3 + a_4 s^2 + a_5 s^1 + a_6$$

$$\begin{array}{cccc}
a_o & a_2 & a_4 & a_6 \\
a_1 & a_3 & a_5 & 0 \\
\dfrac{a_1 a_2 - a_3 a_o}{a_1} = A & \dfrac{a_1 a_4 - a_o a_5}{a_1} = B & \dfrac{a_1 a_6 - a_o \times 0}{a_1} = a_6 & 0 \\
\dfrac{Aa_3 - a_1 B}{A} = C & \dfrac{Aa_5 - a_1 a_6}{A} = D & \dfrac{A \times 0 - a_1 \times 0}{A} = 0 & 0 \\
\dfrac{CB - AD}{C} = E & \dfrac{Ca_6 - A \times 0}{C} = a_6 & \dfrac{C \times 0 - A \times 0}{C} = 0 & 0 \\
\dfrac{ED - Ca_6}{E} = F & 0 & 0 & 0 \\
\dfrac{Fa_6 - E \times 0}{F} = a_6 & 0 & 0 & 0
\end{array}$$

The last step is to investigate the signs of the numbers in the first column in this array. If all of the elements in the first column are positive, the system is stable. If one or more of the elements is negative, the system is unstable.

When the Routh-Hurwitz Criterion is applied to equation 8.41, the following equations result. They must both be positive if the system is to be stable.

$$a_3 a_2 - a_1 = 0 \qquad (8.42)$$

$$a_1 a_2 a_3 - a_o a_3^2 - a_1^2 = 0 \qquad (8.43)$$

In summary, to determine if a given roll rate will result in a stable aircraft motion:

1. Determine the value of a_o (equation 8.40). If it is negative the system is unstable.

2. Determine the values of a_1, a_2, a_3 (equations 37 - 39).

3. Solve equation 8.42. If negative the system is unstable.

4. Solve equation 8.43. If negative the system is unstable.

5. If steps 1-4 yield no negative results, the system is stable for the roll rate investigated.

Although somewhat unwieldly, in the absence of adequate analog simulation, the foregoing system will yield fairly accurate results. It can serve to warn the test pilot of a perilous situation.

8.7 CONCLUSIONS

As an aircraft's inertias are disproportionately increased in relation to its aerodynamic stabilities in pitch and yaw, the aircraft will be liable to pitching and yawing motions during rolling maneuvers. The more typical case is a divergence in yaw by virtue of an inadequate value of C_{n_β}.

The peak loads resulting from roll coupling generally increase in proportion to the initial incidence of the principle inertial axis and progressively with the duration of the roll and the rapidity of aileron application at the beginning and the end of the maneuver. The most severe cases naturally should be expected in a flight regime of low C_{n_β} and high dynamic pressures.

The rolling pull-out maneuver in a high performance aircraft is especially dangerous. It combines many unfavorable features: High speed hence high roll rate capability; high acceleration which favors poor coordination and inadvertent excitation of transients by the pilot; and high dynamic pressures which at large values of α and β may break the aircraft.

Most high performance aircraft incorporate roll rate limiters in addition to angular damping augmenters. In these aircraft a lateral control with enough power for low speed is almost certain to be too powerful for high speeds. Fortunately, limiters of various kinds are not too difficult to incorporate in a fully powered control system.

It is obvious that flight testing in suspected regions of roll coupling warrant a cautious methodical approach and must be accompanied by thorough computer studies that stay current with the flight test data. The only way that the pilot can discover the

exact critical roll limit in flight is when he exceeds it, which is obviously not the approach to take. Because of this, flight tests are generally discontinued when computer studies indicate that the next data point may be "over the line."

The following example is cited. The Bell X-2 rocket ship in 1956 was launched from its mother ship at Edwards. The pilot flew a perfect profile but the rocket engine burned a few critical seconds longer than the engineers predicted, resulting in a greater speed (Mach 3.2) and greater altitude (119,800 feet) than planned. Unknown to the pilot, he was progressively running out of directional stability. When he was over the point at which he had preplanned to start his turn toward Roger's Dry Lake he actuated his controls. The X-2 went divergent with a resultant loss of control. The accident investigation revealed the cause to be a greater loss in directional stability than planned, resulting in divergent roll coupling.

A combination of reasonable piloting restrictions coupled with increased directional stability has provided the solution to roll coupling problems in the present generation of aircraft. The problem is one of understanding since a thinking pilot would no more exceed the roll limitations imposed on an aircraft than he would the structural "G" limitations.

Besides pilot education, some other schemes to eliminate roll coupling divergencies are:

1. Roll rate limiters.

2. Angular damping augmenters.

3. Placarded roll limits such as

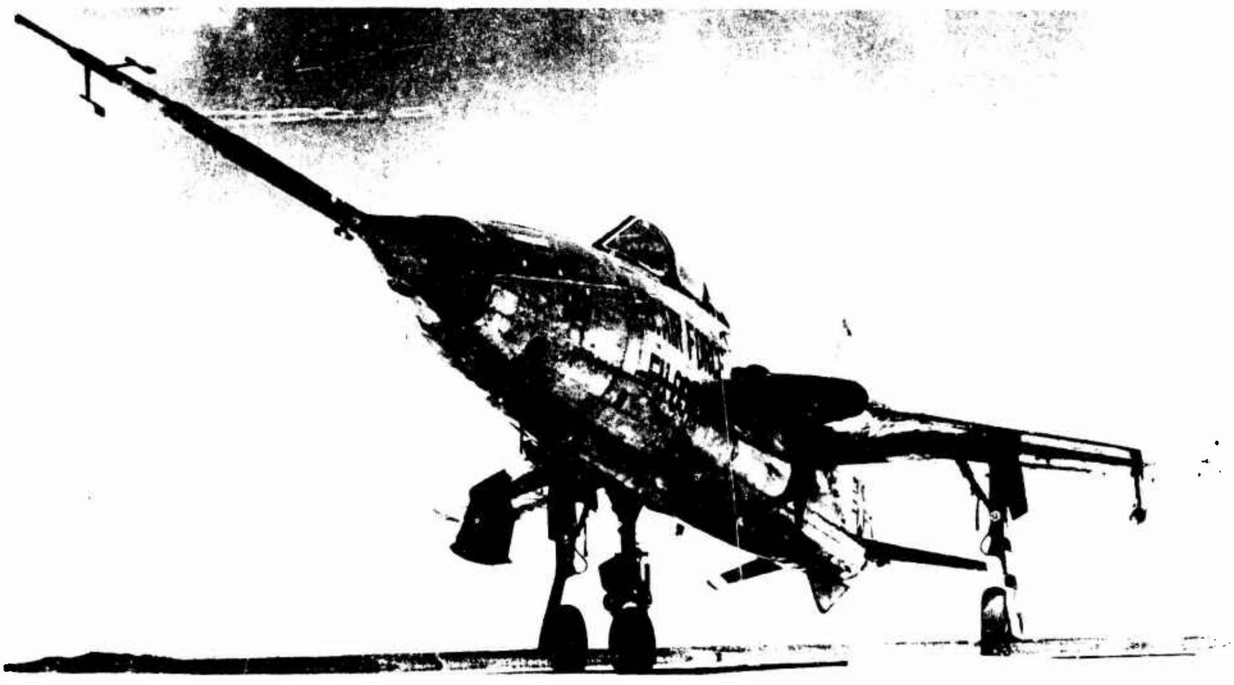
    a. "G" limits.

    b. Total allowable roll at maximum rate.

    c. Altitude limits.

    d. Mach limits.

    e. Flap position limits.



CHAPTER 9

CONTROL SYSTEMS

ABBREVIATIONS AND SYMBOLS FOR THIS CHAPTER

C_{h_δ}	hinge moment coefficient (restoring)	
C_{h_α}	hinge moment coefficient (floating)	
$C_{h_{\delta_t}}$	hinge moment coefficient (tab)	
q	"dynamic pressure"	lbs/ft^2
q_c	compressible dynamic pressure	lbs/ft^2
δ	elevator deflection	degrees or radians
V	airspeed	knots
$d\delta_e/dn$	elevator angle per g	
n	normal acceleration	ft/sec^2
dF_s/dn	stick force per g	
$d\delta_e/dv$	elevator angle to airspeed ratio	
$d\delta_r/d\beta$	rudder angle per degree of sideslip	
δ_e	elevator deflection	degrees or radians
C_{n_r}	Yaw damping derivative	
C_{n_β}	Yawing moment coefficient with sideslip angle	
C_{n_p}	Yawing moment coefficient with rolling velocity	

9.1 INTRODUCTION

In the broad sense the aircraft flight control system consists of all the mechanical, electrical and hydraulic elements which convert cockpit control forces and motions into aerodynamic control surface deflections or action of other control devices which in turn change the orientation of the vehicle. The flight control system together with the power plant control system, enables the pilot to "fly" his aircraft - that is, to place it at any desired flight condition within its capability.

The power plant control system acts as a thrust metering device, while the flight control system varies the moments about the aircraft center of gravity. Through these control systems the pilot is able to vary the velocity, normal acceleration, sideslip, roll rate, and other parameters within the aircraft's envelope. How easily and effectively he can accomplish his task is a measure of the suitability of his control systems. An aircraft with exceptional performance characteristics is virtually worthless if it is not equipped with at least an acceptable flight control system.

Two important control system characteristics are the magnitude of cockpit control forces and deflections. Figure 9.1 shows approximate limitations of the pilot's physical effort. The limitations shown in this figure are much greater than those considered desirable for normal flying. However, if the control forces required for normal aircraft maneuvers are pleasantly light it will usually be possible to overstress the aircraft by misuse of the controls. Conversely, an aircraft that displays control characteristics that prevent exceeding the design limitations would ordinarily be considered unacceptably heavy.

FIGURE 9.1
APPROXIMATE PILOT PHYSICAL LIMITATIONS

CASE	AILERON STICK	AILERON WHEEL	ELEVATOR STICK	ELEVATOR WHEEL	RUDDER
MAX ALL OUT EFFORT FOR VERY SHORT TIME (2 HANDS)	90	120	180	120	440
MAX PERMISSIBLE (2 HANDS) FOR SHORT TIME (1 HAND)	– 50	80 50	100 70	110 70	200
MAX COMFORTABLE (2 HANDS) FORCE SHORT TIME (1 HAND)	– 20	30 20	– 30	40 30	60
LARGEST HAND OR FOOT MOVEMENT FOR FULL TRAVEL	–10 IN	–20 IN	–9 IN	–9 IN	–5 IN

Having noted the approximate capabilities of a pilot with respect to effort, there is another important factor to which he reacts, that of control "harmony." Control harmony is a rather nebulous quantity and cannot be discussed merely in terms of relative forces required for the three controls, it is also necessary to take into account the aircraft's response characteristics. Therefore, no idealized ratios can be established for relative heaviness of the controls. However, as one would suspect after reference to figure 9.1, aileron force should ordinarily be less than elevator force which in turn is usually expected to be less than the rudder force required. In any case control harmony is a matter of opinion, and opinions change with time and pilots. Since the pilot-control system acts on an aircraft with specific static and dynamic stability properties, it follows that the characteristics of the closed loop system must be found satisfactory.

In flying an aircraft the pilot frequently makes use of such references as the movement about the horizon and the magnitude of accelerations felt, but these indications are available to him after he is already in the maneuver. He can only conclude that this maneuver is too violent or too mild after he has felt or seen its magnitude. Needless to say this could be rather untimel for effective control of the ai aft, consequently there is a need for something to forewarn the pilot of aircraft motion that

will take place as a result of his use of the cockpit controls. Control movement and force are two of the pilot's inputs to the stick, of which he is usually aware. In some flight regimes the stick movement can be considerable but at high speeds and aft cg conditions, the stick movements may be barely perceptible and for this reason it is generally conceded that control force is the most important indication of the magnitude of the maneuver. It follows that there should be no reversal of these forces in order that the pilot is never required to reverse his thinking. It would seem that stick deflections should never reverse either, but it may be possible to relax such a requirement in the portion of the aircraft envelope where stick movement is so small as to be unnoticeable. If the stick movement is infinitesimal the pilot simply senses that he must pull to slow down and is probably unaware of the fact that the stick may have moved forward a fraction of an inch.

To summarize the general requirements of the aerodynamic control system; two conditions must be met if the pilot is to be given suitable command over his airplane.

1. It must be capable of actuating the control surface.

2. It must provide the pilot with a "feel" that bears a satisfactory relationship to the aircraft's reaction.

There are numerous variations in the designs of aircraft control systems. However, these systems may be rather simply classified. The first classification in which the majority of the modern aircraft control systems fall is that in which the moments about the aircraft center of gravity are created by varying the camber or angle of attack of the aerodynamic lifting or stabilizing surfaces. The most familiar example, is the use of trailing edge flaps on the wings, horizontal and vertical stabilizer, called ailerons, elevator and rudder, respectively. Another subclass of aerodynamic control is the spoiler which has come into rather wide use in recent years for lateral control. This device creates rolling moments by changing the energy of the airflow over one wing thereby resulting in a differential lift between the two wings. These aerodynamic systems can be further broken down into "reversible" and irreversible systems. These systems can be simple mechanical controls in which the pilot supplies all of the force required to move the control surface. This type system is called "reversible" since all of the forces required to overcome the hinge moments at the control surface are transmitted to the cockpit controls. The system may have built into it a mechanical, hydraulic or some other type of boosting device, which supplies some specific proportion of the control force. Systems of this nature are generally called "boosted control systems." However, they are still considered "reversible." Even though the force required of the pilot is less than the control surface hinge moments the force required is proportional to these moments. In other words the pilot furnishes a fraction of the force required to overcome the hinge moments throughout the aircraft's envelope. The control system is said to be irreversible if the pilot through his cockpit controls, actuates a hydraulic, electronic, or some other type of device which in turn moves the control surface. In this system the aerodynamic hinge moments at the control surface are no longer transmitted to the stick. Here, without artificial feel devices the pilot would feel only the force required to actuate the valves or sensing devices of his powered control system. Because of this, artificial feel, which approximates the feel that the pilot

senses with a "reversible" system must be added.

A second category of control system is that of reaction controls. In this system small jets are usually located near the extremities of the aircraft where the length of the moment arm is greatest. These jets may be fired in order to create pitching, rolling, or yawing moments. Reaction controls are used extensively for control of vehicles in the very low speed or very high altitude flight regimes where aerodynamic controls are ineffective. Since this control system is used primarily in areas where aerodynamic damping is very light or in some cases almost non-existent, it becomes apparent that a different technique is required in the use of the cockpit controls. When an angular velocity about the cg is established by use of one of the reaction jets, this velocity tends to continue undiminished until an equal and opposite pulse is introduced. Therefore, it is obvious that a considerable change in ground rules is required for the pilot to intelligently evaluate such a control system.

Other control system categories might include the use of inertial or magnetic control devices not only to guide, but to control the orientation of a vehicle in space.

Since in the final analysis it is the pilot-control system - aircraft combination which must be satisfactory in order for the aircraft itself to be considered acceptable, this chapter will concern itself with not only the various types of control systems mentioned above, but also with the pilot himself.

9.2 SERVOMECHANISMS

From experience it has been shown that in simple tasks the difference in performance between one pilot and another is generally small, becoming negligible when such tasks are repetitive. As the complexity of the task is gradually increased his performance becomes more inconsistent. Then at each progressively more difficult stage the performance depends on the variance of adaptability of pilots, the degrees of predictability of the system they operate, their individual nervous and muscular characteristics and also their psychological and physiological condition at the time. In spite of all this the human pilot is popular with servo engineers for use as an adaptive servo. The human, however, has some limiting characteristics which are virtually constant. They are his reaction time (normally between 0.2 and 0.3 seconds) and his neuro-muscular lag (0.10 to 0.16 seconds). These virtually fixed characteristics represent some of his most serious liabilities as a servo element and restrict system performance. Therefore, the control system designer must design for the physical limitations of the pilot as well as for the airplane characteristics.

A servomechanism can be defined as an automatic control system which senses the output of a system, compares this output with the desired output, and if a difference exists - causes the output to be changed until it equals the desired value. Familiar examples of servomechanisms are automatic tracking radar, and thermostatically controlled air conditioning units. Another example is the human body when performing even the simplest of tasks. A boy catching a ball senses the direction of motion of the ball with his eyes and moves his body, arms and hands until the ball enters his glove. He is subconsciously but continually comparing the relative position and motion of the ball and glove and correcting the motion of the glove until the ball enters it. A pilot flying an aircraft, may be thought of as part

of a multi-channel servomechanism. He is constantly monitoring and - through the flight control system - adjusting angles of pitch and bank, airspeed, altitude, heading, and often a number of other parameters as well.

Before further considering the pilot as a servomechanism, a brief discussion of the servomechanism is in order. The servomechanism is based on the detection of a difference between the existing output of a system and the desired output, and a resulting correction of the output toward the desired value. A signal (such as a voltage), representing the output is "fed back" to a sensing device which detects this difference which is called the error signal. The route by which the existing output is fed back to the sensing device is called the feedback loop. A control system containing one or more feedback loops is called a closed loop system. A system which does not contain a feedback element is called an open loop system. Examples of a closed loop and open loop system are shown in figures 9.2 and 9.3.

FIGURE 9.2
CLOSED LOOP SYSTEM

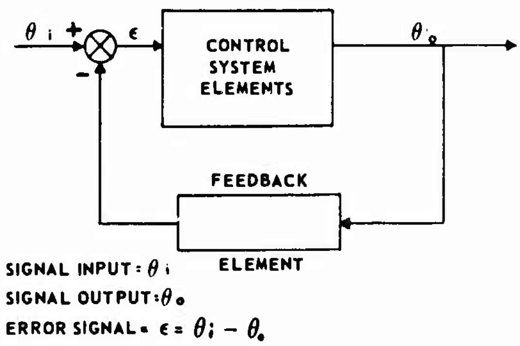
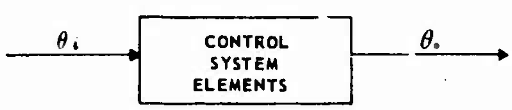


FIGURE 9.3
OPEN LOOP SYSTEM



An open loop system does not qualify as a servomechanism. An example of an open loop system is an aircraft with the controls free to float. An example of a closed loop system is an aircraft under the control of a pilot. Typical block diagrams for these two systems are shown in figures 9.4 and 9.5.

FIGURE 9.4
CLOSED LOOP SYSTEM

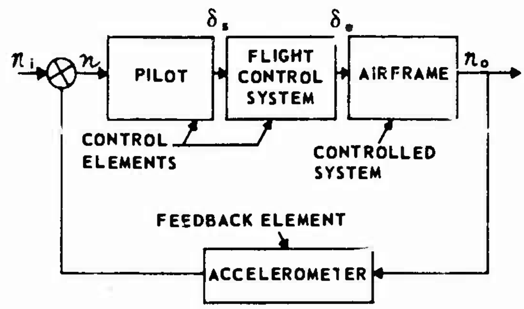
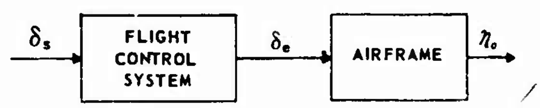


FIGURE 9.5
OPEN LOOP SYSTEM

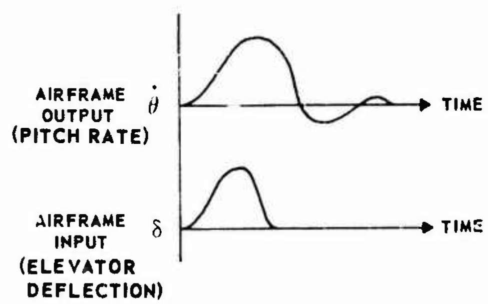


In the closed loop case, the input is a desired normal acceleration of say 2.0 g's. If the aircraft is at only 1.8 g's (the output) the pilot reads and interprets the accelerometer and pulls back on the stick. Thus, he is both a sensing and a control element. He senses an error signal of 0.2 g and imparts an aft deflection to the stick. This aft motion of the stick produces a trailing edge up elevator deflection which in turn, produces an increase in normal acceleration. When the actual normal acceleration (the output) equals the desired normal acceleration (the input) the error signal will then be zero and the pilot will stop the aft movement of the stick.

In the open loop system, the stick is deflected to a position which is estimated to produce a normal acceleration of 2.0 g's. The elevator is moved to a corresponding angle by the flight control system, and the airplane is given a normal acceleration which may or may not be exactly 2.0 g's. Since there is no feedback, no further refinement is possible. The normal acceleration obtained with this stick deflection depends on many variables including altitude, airspeed, cg position, etc.

It is quite possible for an aircraft to exhibit satisfactory open loop characteristics and yet be unsatisfactory when the pilot is introduced into the system and the loop is closed. For example, suppose that the stick-fixed longitudinal dynamic characteristics of an aircraft were such that an elevator pulse produced a pitch rate trace as shown in figure 9.6.

FIGURE 9.6
OPEN LOOP DYNAMIC TIME HISTORY



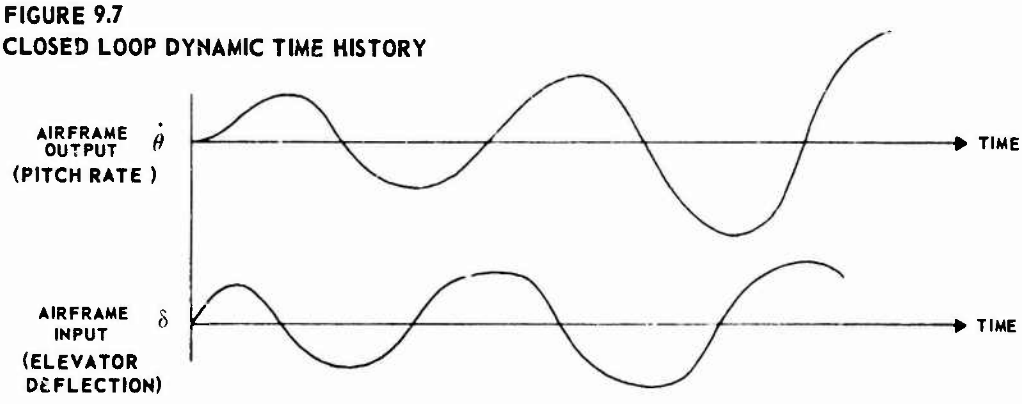
This open loop dynamic time history indicates stable, moderately-damped longitudinal dynamics. If the pilot were required to control the aircraft quite closely, however, such as in low-level, high-speed formation flying, the closed loop dynamic time history might indicate an entirely different situation. The resulting pilot-induced oscillations might lead to normal accelerations which would produce structural damage to the aircraft. The results seen in figure 9.7 could well be caused by the designer failing to take into account that the normal pilot response time was similar to the short period in a portion of the aircraft's envelope.

Thus the job of matching airframe and pilot characteristics with suitable flight control system characteristics is not an easy one. But if the airplane designer does not bear in mind the basic characteristics and limitations of the pilot when he designs the aircraft and control system, the results might well prove disappointing.

9.3 BOOSTED CONTROL SYSTEMS

As the size and speed of aircraft increase, the hinge moments and thus the control forces increase accordingly. Neglecting Mach effects, if the dimensions and speed of a given aircraft are both doubled,

FIGURE 9.7
CLOSED LOOP DYNAMIC TIME HISTORY

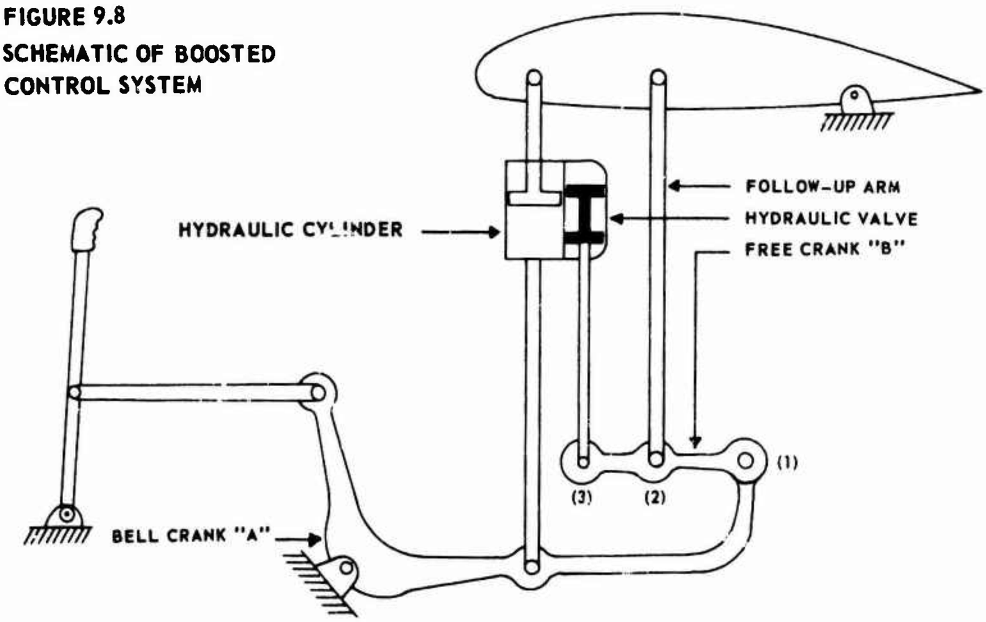


the control forces will be multiplied by a factor of thirty-two. Although aerodynamic balancing could alleviate the problem up to a point, the difficulty in the past of achieving the production tolerances necessary to produce a Ch_δ of say, 0.0002 per degree, and the erratic variations in hinge moment in the transonic regime led to the development of boosted control systems. In this type of system the pilot is required to produce only a fraction of the force required to overcome the hinge moments. However, he does produce a specified portion of this force and the force required of the pilot is still approximately proportional to the hinge moments on the control surface. The control system then, is still a reversible one in that the hinge moments are fed back to the stick.

One method of control boosting is shown in figures 9.8 and 9.9. This system uses the hydraulic method of amplifying which requires the addition of a hydraulic cylinder on a follower arm. As in the unboosted system the pilot moves the stick aft and the trailing edge of the elevator deflects up.

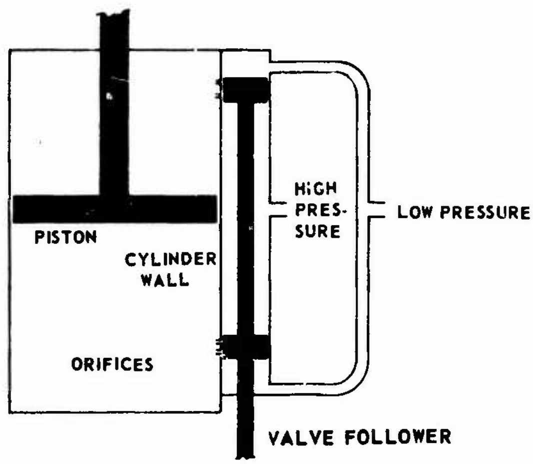
The process however causes a hydraulic pressure drop across the piston causing the cylinder to collapse and affecting additional elevator deflection. As the cylinder collapses a follow-up linkage reduces the pressure drop across the piston to zero and the follower arm contraction ceases. Starting from an initially trimmed condition with zero stick force the pilot pulls back on the stick and holds. There is an accompanying up elevator produced, identical to that which would be produced in an unboosted system. This up elevator moves the follow-up arm down but point (2) does not move as far as point (1) and thus the free crank "B" rotates clockwise relative to bell crank "A". This produces an upward motion at point (3) relative to the horizontal arm of the bell crank "A" and the valve slides toward the elevator, thereby unporting two orifices in the cylinder wall astraddle the piston head. High-pressure oil is fed to the orifice on the rod side and the oil pressure drop across the piston forces it to collapse and therefore shorten the follower arm achieving an additional up elevator deflection. As the elevator moves up, the follow-

FIGURE 9.8
SCHEMATIC OF BOOSTED CONTROL SYSTEM



up arm goes down and point (2) moves down relative to point (1). This action rotates the free crank "B" counterclockwise and the valve slides down on the cylinder until the orifices are closed and the action is complete.

FIGURE 9.9
HYDRAULIC VALVE



The hydraulic valve consists of a double piston and cylinder arranged so that a slight motion in either direction of the valve follower opens two orifices straddling the hydraulic cylinder piston head. If the valve follower is forced upward the high-pressure orifice on the rod side and the low-pressure orifice on the face side are opened. If the valve follower is forced downward the reverse action takes place. (See figure 9.9.)

Some advantages of the boosted control system are:

1. Aerodynamic feel from the control surface is retained.

2. In event of power failure the controls can be operated manually by the pilot, at least over a limited flight regime.

Some disadvantages are:

1. Unless the boost ratio is variable, control forces may still be excessive at high speeds (low boost ratio) or too low at low indicated airspeeds (high boost ratio).

2. Normal antiflutter mass balance is still required.

3. A certain amount of aerodynamic balance is still required for manual operation.

4. Power failure in out-of-trim flight will produce very high control forces.

On aircraft which have a wide speed range, the difficulty of avoiding the problems mentioned in (1) makes full power operation of controls desirable.

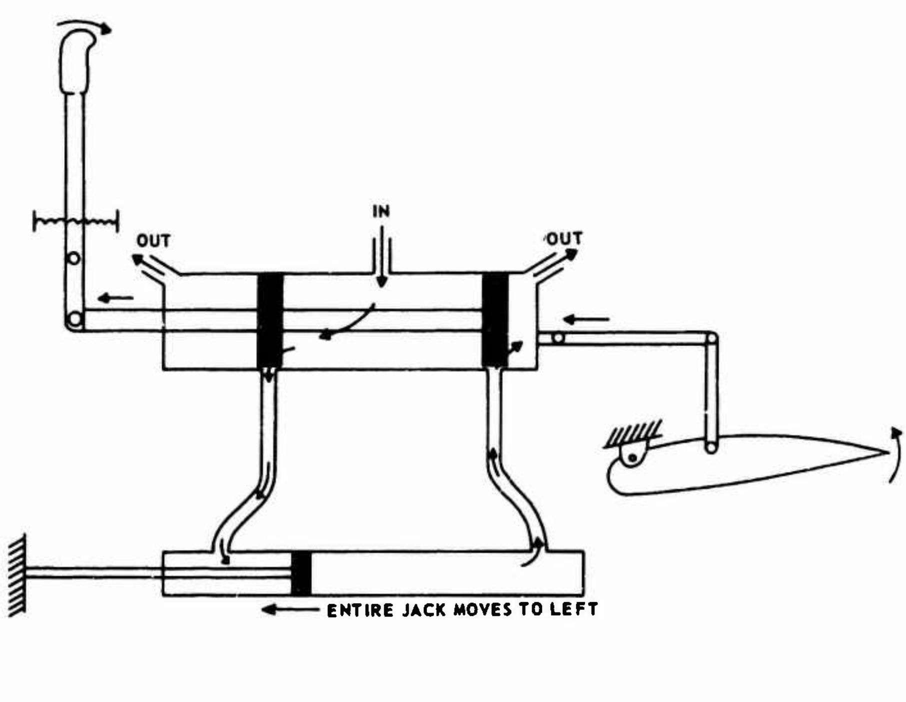
9.4 POWERED CONTROL SYSTEMS

The aircraft designer can, by proper use of aerodynamic balance and/or boost, enable the pilot to cope with control hinge moments of extremely large magnitude. These methods are practical as long as the control derivatives (Ch_α, Ch_δ, Ch_{δ_t}) remain essentially constant. But as aircraft approach sonic speed they run into a region where the hinge moments vary widely with very small speed changes, and as they accelerate to supersonic speeds the hinge moment characteristics settle down but at substantially different values than those experienced in subsonic flight. This is due primarily to the aft shift in center of pressure. Therefore, the designer divorces the pilot from the control surface by giving him a control system in which he activates the flight control surface indirectly through a mechanical or electrically actuated system - usually hydraulically operated. A system such as this would require

the pilot to overcome forces which originate from friction of the control linkages, valves, etc., and not directly from control surface movement. It is fairly obvious that these forces would not be a satisfactory measure of aircraft response. Thus the designer must supply the pilot with an artificial feel system. One of the purposes of this section is to discuss some of the relatively simple devices used in irreversible control systems. The following section will deal with the more complex problem of feel systems.

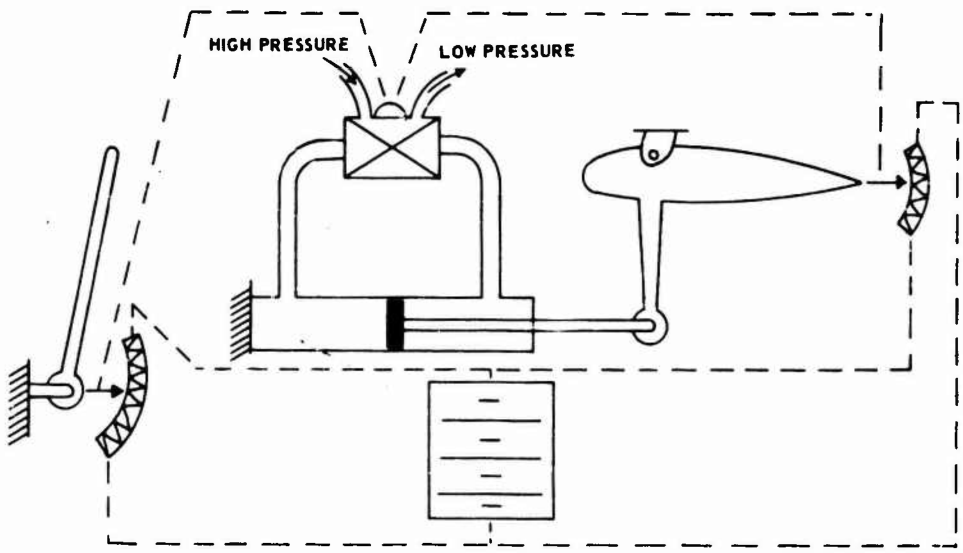
A schematic of a typical powered system is shown in figure 9.10. The pilot, by pulling the stick aft, positions the upper piston or valve in the jack. The incoming hydraulic fluid forces the lower piston to the right with respect to the jack. Since the piston is fixed to the structure, the jack must move to the left. This movement, in turn leaves the control surface trailing edge up. As the jack moves to the left the ports of the upper cylinder are again covered and the control deflection ceases. If the pistons in the upper valve have only a small overlap the control surface will follow the stick deflection quite closely. However, this type of system, while capable of handling very large hinge moments, is not capable of handling unlimited loads. If the aerodynamic hinge moment on the control surface exceeds the mechanical hinge moments which can be generated by the powered control system, the system encounters a "jack stall."

FIGURE 9.10
POWERED IRREVERSIBLE CONTROL SYSTEM



A variation of the powered control can be made because it is not necessary to make use of mechanical valve linkage. An electrically operated servo valve can be used to meter the hydraulic fluid to the cylinder, by supplying it with a voltage proportional to stick deflection (see figure 9.11).

FIGURE 9.11
POWER OPERATED CONTROL WITH ELECTRIC SERVO



To fulfill the positional requirement of stick-elevator gearing, a feedback voltage in the ratio of elevator deflection is also fed back to the servo. Stick motion is not the only way to obtain control surface deflection. Since only a voltage is required, a stick-mounted strain gage generating voltage proportional to stick force will command surface deflection. It is generally accepted that the power systems using mechanical linkage rather than electrical signals are somewhat more reliable but are less adaptive to sophisticated damping systems and automatic weapons delivery systems. In most cases it is wise to choose the least complex system which is capable of doing the task required.

The most important advantages of using powered irreversible control systems are:

1. The pilot is divorced from the large and often erratic control forces required at high speeds.

2. Aerodynamic balancing is not usually required. The rigidity associated with powered control systems reduces the requirement for mass balancing, although some attention must still be given to the problem in order to avoid high speed flutter.

The primary disadvantages are:

1. An artificial feel system must be incorporated into the aircraft control system.

2. The increased complexity of the control-feel system combination reduces system reliability.

3. It is often difficult or impractical to build into the system a standby capable of reverting to manual reversible control.

9.5 AIRCRAFT FEEL SYSTEMS

Aircraft feel was discussed at some length in section 9.1, however, several additional comments will now be made before some of the artificial devices that affect feel are described.

The control system characteristics, together with the airframe stability characteristics, are determined by airframe design, the control-feel system is the place where the handling qualities may be patched up - or ruined.

The aircraft-feel system ordinarily includes stability augmentation devices, since they definitely affect aircraft handling qualities, but because of their dual function and unique character they will be considered separately.

The feel characteristics of a reversible control system may certainly be altered in order to improve handling qualities. In fact two devices for doing just that were discussed previously in this course, i.e., the downspring and bobweight. In the following paragraphs, though, the control system will be assumed to be irreversible and feel systems artificial.

9.6 MECHANICAL CHARACTERISTICS OF CONTROL SYSTEMS

Mechanical characteristics of control systems which affect feel will now be discussed before going into artificial devices which are incorporated in order to improve feel.

Breakout Force:

This term defines the force necessary to be applied to the stick before it moves. The source lies in the cumulative affect of the following:

1. The mechanical friction of the control circuit, the feel unit, and the valve.

2. The force due to viscous flow past the valve in its neutral position and/or valve centering spring, and

3. The preload of the feel unit.

If the breakout force is high, it will tend to produce overshooting in the desired small and rapid control pressures (e.g., during tracking or instrument flight) because of the pilot's neuromuscular and reaction lag. As the pilot exerts pressure on the cockpit control to overcome this force he is likely to continue this pressure for a short period after the friction is overcome and thus take the control past the desired value. These effects will be aggravated if the force level immediately following breakout force becomes noticeably lower because of lower running friction.

Backlash:

Mechanical play in the control system resulting from cable stretch, valve overlap, etc., is called by various names - backlash, lost motion, free play, mechanical hysteresis. If it reaches annoy-

ing proportions it is frequently called "slop." The cockpit-control/control-surface relationship is not constant. This phenomenon makes the pilot work even harder to maintain steady flight or make small corrections. The combined effects of breakout force and backlash on a system where the pilot is attempting to make a small correction is shown in figure 9.12.

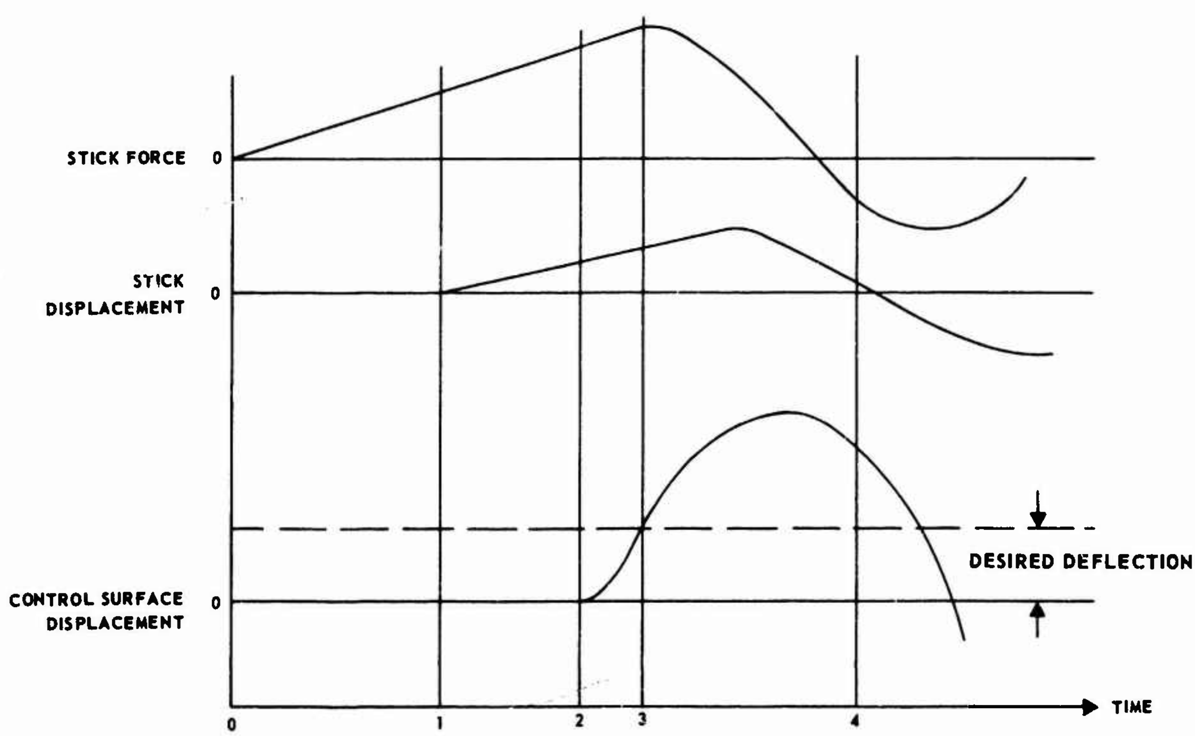
The pilot starts applying a force at time "0." At time "1" he has applied enough force to overcome the breakout force and the stick starts to move. At time "2" the stick has moved far enough to take up the mechanical play in the system and the control surface starts to move. At time "3" the control surface reaches the desired deflection. Just after this time, the pilot starts to release force, but the control surface has moved past the desired point until time "4" when he has applied enough force in the opposite direction to again overcome friction, backlash, etc. Such a system would thus cause the pilot to overcontrol.

If the initial aircraft response is slow and damping is high, the overshoot is likely to be negligible; however, rapid aircraft response to control pressures will result in marked overshoot, often culminating in pilot induced oscillations. In more severe cases of breaking forces and backlash, exciting landing flareouts may result.

Centering:

Good centering of the cockpit control and the control surface is a requirement for any satisfactory aerodynamic control system. Centering may be defined as the degree to which the pilot's control and

FIGURE 9.12
EFFECTS OF BREAKOUT FORCE AND BACKLASH



control surface will return to the trimmed position after the cockpit control has been displaced and released. For a reversible system the primary centering tendency comes from the aerodynamic hinge moments. For an irreversible system a feel spring would have the same effect. Good centering aids the pilot in making small momentary corrections from a trimmed flight condition. Sources of trouble occur when the pilot releases the force on the cockpit control, but the control valve does not center, causing the control surface to continue to move. The end result is again to make the pilot work harder to maintain steady flight or to make small corrections since he must now consciously take out each correction.

Phase Lag:

In virtually all control systems there is a finite time delay between a cockpit control deflection and the corresponding flight control deflection. In most cases, this time is extremely small. If the flight control system is relatively large and complicated, however, this time lag can become significant. It may be that this control system lag plus the airplane response time could be relatively large. In such cases, if the pilot wished to make a series of rapid control inputs, he might find the aircraft motions out of phase with his desires. Control system lag plus aircraft response time is the phase lag.

Another associated problem which can develop is that the natural frequency of the cockpit control or control system may be near the natural frequency of the associated airplane modes of motion in a portion of the performance envelope. If, for instance, the centering spring is weak, in order to produce a light stick force gradient, the natural frequency of the control system may be low enough to couple with the short period or Dutch roll mode.

Trim:

Trimming is accomplished on virtually all artificial feel systems by shifting the no-load position of the feel device. The trimming device should have enough authority to reduce the control forces to zero throughout the flight envelope. The rate of movement of trim should be determined by the runaway trim condition at V_{max}.

9.7 ARTIFICAL FEEL SYSTEMS

In this section, a brief description of some typical artificial feel systems will be given.

Without artificial feel in the irreversible control systems, the stick is free to flop around at will.

Simple Spring:

The simplest type of feel system is that in which the cockpit control is restrained from movement in either direction by a linear spring. The control force is then simply proportional to control deflection.

$$\Delta F = K\Delta\delta \tag{9.1}$$

It is worth noting that this type is often called a "bungee." It is unfortunate, but a downspring is also sometimes called a bungee. (In this chapter the term bungee will be used to mean a centering or feel spring.) Although the simple spring does give the pilot some feel of the aircraft response, the stick force versus control surface deflection is a constant regardless of airspeed (ignoring aeroelastic and Mach effects). Therefore, for aircraft with wide speed ranges their use for longitudinal control is unsatisfactory, since

the stick force per "g" is lowest for high speeds and highest for low speeds. However, this type of feel is often used in aileron systems, and thus explains the relatively high roll rates obtained in some of our high performance aircraft at high subsonic airspeeds.

q-Feel:

If the feel system can be made a function of dynamic pressure, by use of bellows, a servo device, or any other means, then the control force will be a function of control deflection and dynamic pressure.

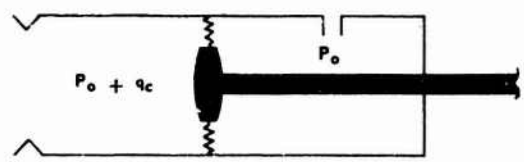
$$\Delta F = Kq\Delta\delta \qquad (9.2)$$

Such a system produces a stick force per g which is invariant with airspeed. Another characteristic of this system is that in regards to speed stability, the stick force varies as a function of "q." Thus "q" feel is an attempt to approximate the feel that is obtained in the reversible system. This device is commonly used for feel in the longitudinal control of many modern aircraft.

There are some problems that reduce the utility of this system somewhat. Since the feel is based on dynamic pressure and not Mach number, the matching of the system to perform correctly at subsonic speeds will result in much larger forces supersonically because nearly twice as large control movements are needed for the same response. Other problems inherent in the "q" system are compressibility and position error effects in the transonic region, however, these can be alleviated to some extent by suitable scheduling, or by a Mach cutoff through a specific range of the transonic region.

One method of mechanizing the $Kq\Delta\delta$ of feel forces is to produce a force proportional to q_c by ram air bellows and piston combination shown in figure 9.13.

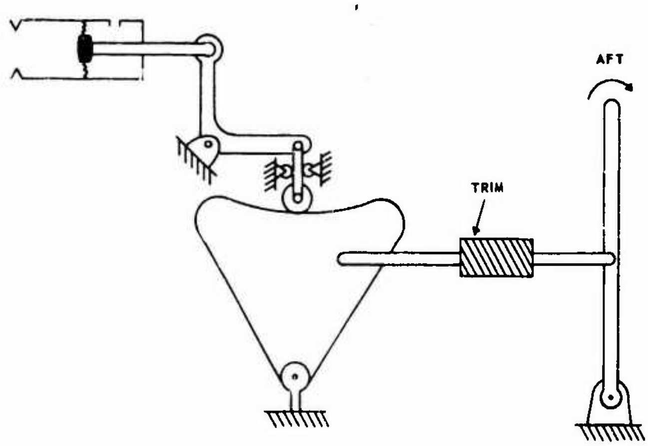
FIGURE 9.13
RAM AIR BELLOWS AND PISTON



The pressure drop across the piston is the difference between total and static pressure. Hence the force on the piston is approximately the product of the face area and the compressible dynamic pressure.

The q_c force is fed through a rocking cam to the cockpit control as shown in figure 9.14.

FIGURE 9.14
RAM AIR FEEL SYSTEM



Ram air provides at all times a force on the cam. As the stick moves aft the cam rotates clockwise and the q_c force creates a counterclockwise moment on the cam which must be balanced by a positive stick force.

V - Feel:

When the feel system is made a function of airspeed, then the control force will be a function of control deflection and airspeed.

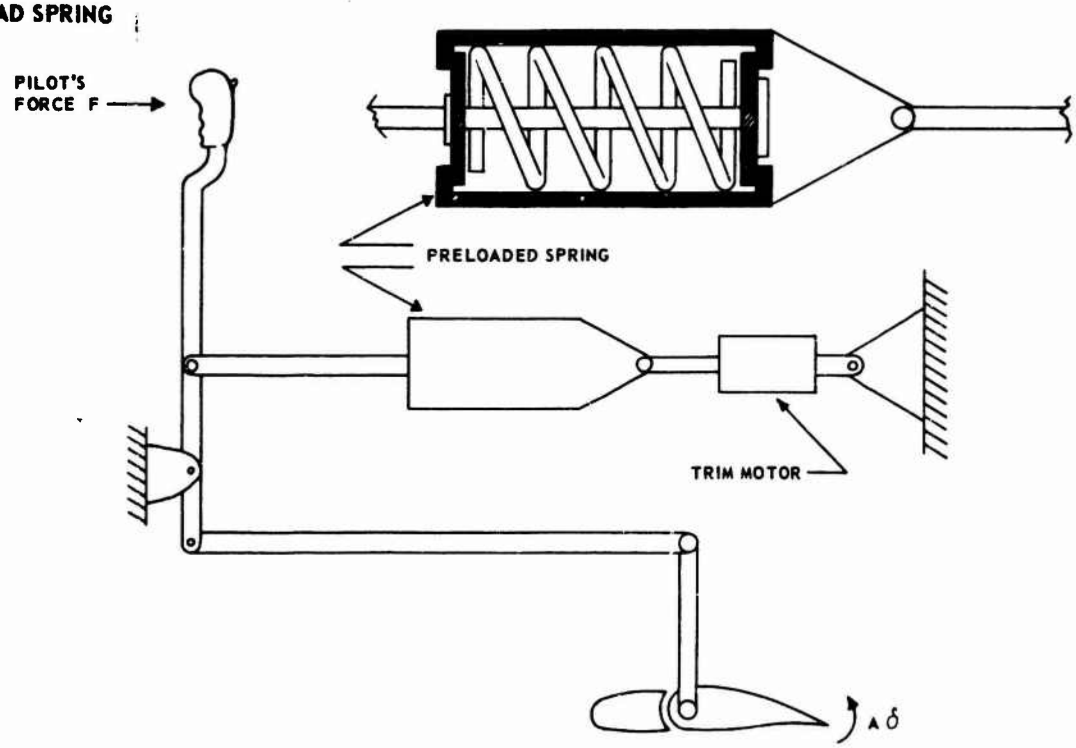
$$\Delta F = KV\Delta\delta \qquad (9.3)$$

This system, primarily used by the British, is desirable for much the same reason as the q - feel system, and similarly runs into some of the same difficulties in the transonic regions.

Preloaded Spring:

If a significant amount of friction exists in a flight control system, the control centering is likely to be poor. One method of eliminating this problem is to use a preloaded spring on the artificial feel system. A preloaded spring is one which has already been compressed, usually in a cylinder, in such a way that some specified force must be applied to compress it further, as indicated in figure 9.15. Thus, in order for the pilot to move the stick at all, he must supply this force. The magnitude of the preload can be adjusted to just exceed the force required to overcome system friction, thus assuring good centering. The stiffness of the spring can be chosen to give the desired stick force gradient.

FIGURE 9.15
PRE-LOAD SPRING



If a very light stick force gradient is desired, then the feel characteristics of the infinite stick force gradient through trim produced by the preload may be objectionable. In this case, a

vigorous friction reduction program would probably be a better solution to the centering problem than the preloaded spring.

Nonlinear Gearing:

In this arrangement, the ratio of control surface movement to movement of the stick is small near the neutral position changing with displacement to a high ratio near the limits of travel. In this system spring feel is often used with either linear or nonlinear force gradients versus displacement. A typical example of nonlinear gearing is used on the F-100 slab tail in conjunction with a nonlinear spring. The original F-100A with a linear gear, nonlinear spring provided the pilot with oversensitive control at high speed. To remedy this situation, a nonlinear gear was designed. As a result, the amount of stabilizer angle available within one inch forward and aft from the stick neutral position reduced from a total of 7 1/2 to 3 degrees.

There are certain disadvantages inherent in the use of nonlinear gearing. In the longitudinal axis it is desirable that the center of the reduced ratio of control movement occurs at high speed around the trim position. Now if large trim changes are present, say due to cg movement, the trim at high speed would not correspond to the optimum position in the nonlinear slope and the handling characteristics would change. Another problem arises in lightening of force with increasing "g." This is ordinarily solvable with nonlinearity in the feel springs.

Variable Gearing:

In a variable gear ratio control the ratio of pilot's control movement to surface movement is altered in flight to fit the desired response characteristics in a given flight regime. This principle of control is applied in the B-58. The Hustler uses a variable gear system which is varied automatically with dynamic pressure and g loading.

Stability Augmentation:

It is frequently necessary to augment the natural stability of an aircraft by synthetic means. This may be because the damping of some mode is too low, or because of actual instabilities, static or dynamic, that are present in some flight regime.

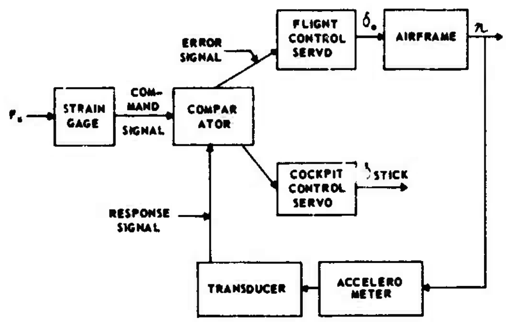
One type of a control damper produces a control force requirement which is proportional to the rate of deflection of the cockpit control or control surface. Usually this damper is a piston which must be moved in a cylinder which contains oil which must be forced through orifices. One possible use of a damper might be to prevent pilots "beating the bobweight." One difficulty in installing such a device is that this damper might also prevent the pilot from making necessary rapid corrections when needed; say, upon landing the aircraft.

From the aerodynamic standpoint, stability augmentation consists of altering one or more of the stability derivatives by automatically displacing one or more of the control surfaces in response to motion of the airframe. For example, if the aircraft is statically unstable longitudinally ($C_{m_\alpha} > 0$), then it could in principle be stabilized by sensing α, and producing an elevator deflection proportional and opposite to it; i.e., let $\delta_e = k\alpha$, then

$$\Delta C_{m\alpha} = - \frac{\partial C_m}{\partial \delta_e} \frac{\partial \delta_e}{\partial \alpha} = - K C_{m_{\delta_e}} \qquad (9.4)$$

A common example of the stability augmentor is the yaw damper. In its simplest form, it increases the damping in the derivative C_{n_r}, but can also be designed to alter C_{n_β} and C_{n_p}.

FIGURE 9.16
SCHEMATIC OF A RESPONSE FEEL SYSTEM



Other names occasionally used for this system are "Synthetic Feel" or "Stick Steering."

If the desired response (say, g's) to a cockpit input (usually force) is sensed, and if through an appropriate servo device the flight control surface deflection is varied in order to correct the output to the desired response, the pilot has "response feel."

$$F - KX \text{ (desired response)} \tag{9.5}$$

For example, suppose the longitudinal control system of an aircraft worked as follows (figure 9.16):

1. The pilot exerts a force on the stick.

2. The stick force is converted to a voltage - the signal.

3. This command signal voltage is fed to a comparator, then to a flight control position servo, and also to a cockpit control position-servo.

4. These position-servos convert the signals to valve movements, which direct fluid to the appropriate side of pistons, causing flight control and stick movement. (Thus the name "stick steering." The pilot thinks he is moving the stick, but really it is being driven by the cockpit control position-servo.)

5. The flight control movement produces a desired aircraft response, say normal acceleration.

6. This response is sensed by an accelerometer and converted into a voltage - the "response signal."

7. This response signal voltage is sent to the comparator where command and response signals are continually compared. The difference is called error signal. The comparator must be given the desired stick force per g in order to compare the command response signals. Thus the comparator is the origin and heart of the control system.

8. The error signal in the comparator causes further servo and control motion until the response and command are equal. Then the error signal is zero and no further control motion takes place until the pilot demands it by changing stick force.

As a byproduct of this type of control system, the aircraft which might otherwise have undesirable characteristics can be stabilized by introduction of a damping loop in the circuitry. In this case a rate gyro (sensing pitch rate in longitudinal oscillations in poorly damped flight regions) could be made to operate a fast response servo to deflect the elevator in a stabilizing sense, without the pilot being aware of the corrections being made.

9.8 ARTIFICAL FEEL SYSTEM RESPONSE

Typical response for a number of the feel systems just described are shown in the following figures, where

S = Spring Feel

V = V - Feel

q = q - Feel

R = Response Feel

FIGURE 9.17
STICK FORCE PER "g"

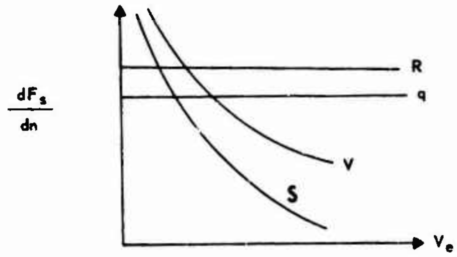


FIGURE 9.18 ICK FORCE
OUT-OF-TRIM STICK FORCE

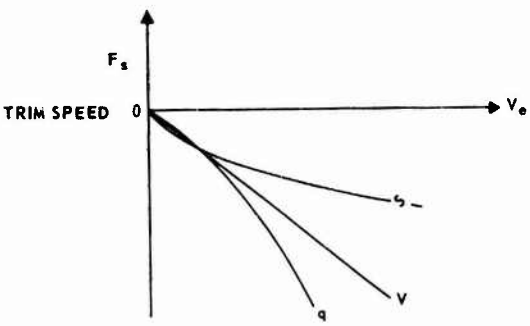


FIGURE 9.19
AILERON FORCE PER UNIT ROLL RATE

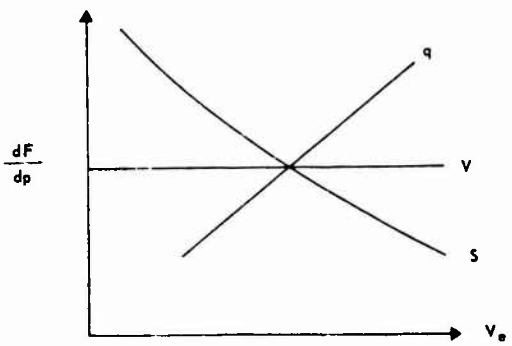
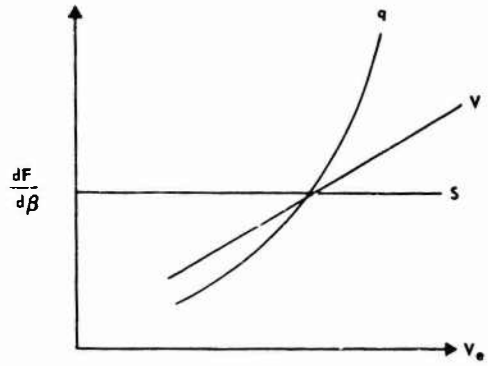


FIGURE 9.20
RUDDER FORCE PER UNIT SIDESLIP ANGLE

(Assuming $d\delta_r/d\beta = K$)



The stick force per g plot (figure 9.17) can be analyzed in the following way:

$$\frac{d\delta_e}{dn} = \frac{K_1}{V^2} \quad \text{For all feel systems except the response feel system} \tag{9.6}$$

$$\frac{dF_s}{d\delta_e} = K_2 \quad \text{For spring feel} \tag{9.7}$$

$$\frac{dF_s}{d\delta_e} = K_3 V \quad \text{For V - Feel} \tag{9.8}$$

$$\frac{dF_s}{d\delta_e} = K_4 V^2 \quad \text{For q - Feel} \tag{9.9}$$

Since $d\delta_e/dn$ is not a function of n, the derivatives can be multiplied

$$\frac{dF_s}{dn} = \frac{dF_s}{d\delta_e}\frac{d\delta_e}{dn} \tag{9.10}$$

$$\frac{dF_s}{dn} = \frac{K_1 K_2}{V^2} = f\left(\frac{1}{V^2}\right) \text{ for spring feel} \tag{9.11}$$

and

$$\frac{dF_s}{dn} = \frac{K_1 K_3}{V} = f\left(\frac{1}{V}\right) \text{ for V - feel} \tag{9.12}$$

and

$$\frac{dF_s}{dn} = K_1 K_4 = \text{constant for q - feel} \tag{9.13}$$

For the response feel $dF_s/dn = K_5$. Where K_5 is the desired stick-force per g which is built into the comparator.

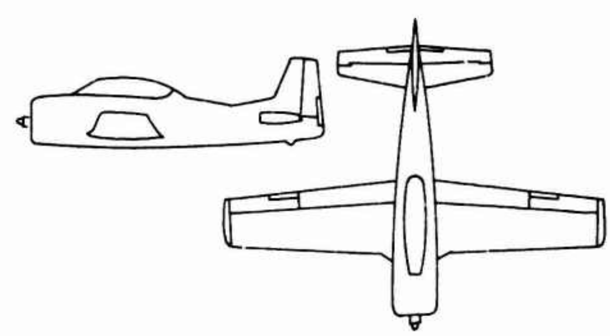
The reader may find it worthwhile to analyze the other response curves in a similar manner. Warning: Since $d\delta_e/dV$ is a function of velocity the method of multiplying will not work. A little reflection should convince the reader that a response feel system that senses a parameter such as normal acceleration or rolling acceleration will require no force by the pilot to maintain a steady state change of airspeed or roll rate. The shape of the response curves for a response feel system would depend entirely on what was built into the comparator or what additional feel device was added.

A glance at the stick force per g plot shows that the spring and q-feel curves are quite different. The q - spring gives the pilot a constant stick-force per g throughout the aircraft speed range. This constant measure of response is generally considered to be superior to the variable response encountered when the feel is provided by a simple linear spring. In the latter case, the stick force gradient might well be too high in the pattern and/or dangerously low at high speeds where the aircraft is usually capable of generating its limit load factor. The advantage of the simple spring is its simplicity and reliability. The designer must decide what degree of sophistication, and therefore, complexity must go into the system. The test pilot must then pass judgment on his decision.

9.9 CONTROL SYSTEM EXAMPLES

The previous paragraphs have discussed the different classifications of control systems and how they are used to give the pilot control of his aircraft. It is of interest now to look at some of the actual control systems of a number of aircraft and observe how the different designers have made use of the various types of controls to give the pilot a desirable control system. Descriptive summaries of the control systems of fourteen different aircraft are given in the following paragraphs. There are other aircraft that might have been included such as the F-100, F-106, B-66, etc. However, their control systems were so similar to one or more of the systems given that they were excluded. The reader might consider some of the aircraft described in this section rather outmoded, however, this selection includes the various types of control systems. Further, it is wise to consider the possibility of future development of subsonic aircraft to be employed in small brush-fire type wars, where the use of the more simple control systems might be advantageous from a reliability standpoint.

NORTH AMERICAN T-28A



The control system of the T-28A is a simple reversible type that consists of conventional elevators, ailerons, and a rudder.

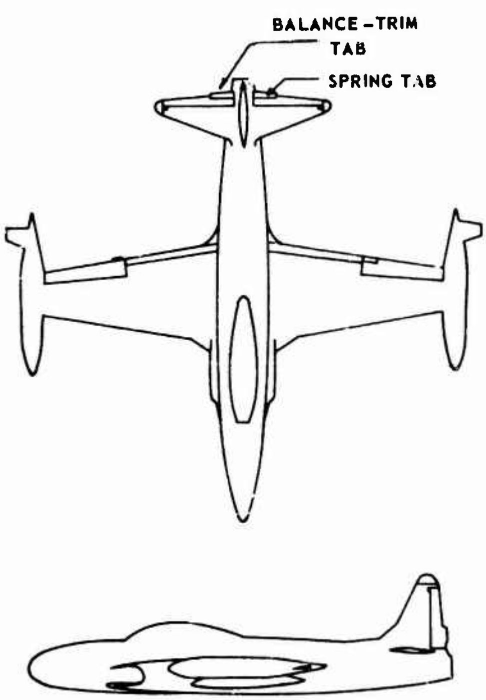
The elevators use aerodynamic balancing in the form of an overhung balance. Mass balancing of the elvator is accomplished by means of a negative bobweight located in the bell crank mechanism. This not only counterbalances the elevator, but reduces elevator

stick forces during accelerated flight. Elevator trim is provided by means of a manually operated trim tab on each elevator.

Aerodynamic balancing is incorporated in the aileron by means of a simple internal seal. Trim is provided by means of a manually-operated trim tab, one on each aileron.

The rudder incorporates aerodynamic balancing by means of a setback hinge. A trim tab located on the trailing edge of the rudder provides directional trim.

LOCKHEED T-33A

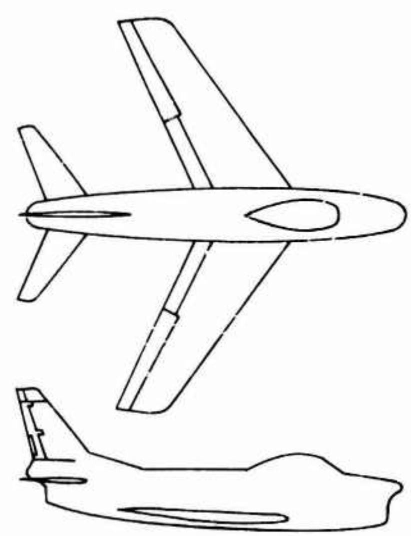


The T-33A has a reversible control system that includes several modifications for improving stick forces or pilot feel.

Longitudinal control is provided by conventional elevators. Aerodynamic balancing is incorporated in the elevators in the form of a spring tab and a balance tab on each elevator. A small shielded horn balance mounted outboard on each elevator is used for mass balancing of the elevators. Elevator unbalance plus a downspring are used in the control system to increase stick free stability. Trim is provided by an electric actuator that moves the balance tab.

Lateral control is provided by conventional ailerons that are hydraulically boosted. This boost is necessary since the ailerons are not aerodynamically balanced and stick forces are considerable without the hydraulic boost. Lateral trim is provided by an electrically-actuated tab located on the trailing edge of the left aileron.

NORTH AMERICAN F-86F



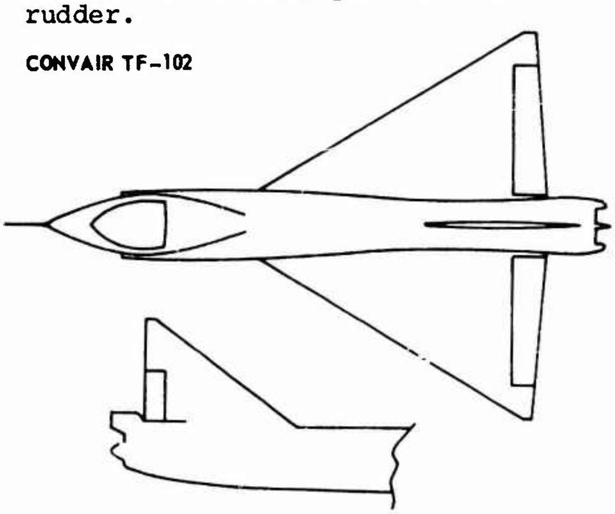
The lateral and longitudinal control systems of the F-86F are irreversible. The directional control system is reversible. The longitudinal control consists of a hydraulically-operated elevator and horizontal stabilizer that are interconnected and operate as one unit. Artificial feel is provided by means of a downspring and a bobweight. The main purpose of

the bobweight is to increase the gradient of stick force versus load factor. Longitudinal trim is accomplished by repositioning the downspring by an electric actuator. This, in effect, changes the neutral (no load) position of the control stick.

Lateral control is provided by hydraulically operated ailerons. Artificial feel is provided by a bungee. Lateral trim is accomplished in the same manner as longitudinal trim.

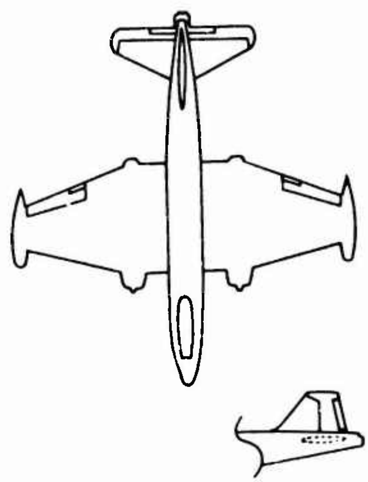
Directional control is provided by a mechanically-operated rudder. Rudder trim is accomplished by an electrically-actuated tab on the rudder. Mass balancing is incorporated in the rudder by the addition of weight in a small horn balance in the top part of the rudder.

CONVAIR TF-102



The TF-102 has a hydraulically-operated irreversible control system. Being a delta wing aircraft, the TF-102 utilizes elevons instead of ailerons and elevator control surfaces. The elevons when moved coincidentally act as elevators and when moved differentially act as ailerons. This is accomplished by a mixer assembly which consists of a bell crank assembly which rotates to provide aileron action and moves fore and aft to provide elevator action. A rudder mounted on the vertical stabilizer is used for directional control. Aileron artificial feel is provided the pilot by means of a bungee. Elevator feel is provided by a bungee, a bobweight, and a variable feel force cylinder that incorporates ram air pressure. This is better known as a "Q-feel" system. The bobweight is not only used to increase the gradient of stick force versus load factor, but to counter the negative bobweight effect of the control column. Rudder feel is provided by the same method as the elevator. Trim is accomplished by repositioning the neutral (no load) positions of the control stick and rudder pedals. In addition, an automatic trim servo is used in the control system to compensate for unstable stick force gradients in the transonic speed region.

MARTIN B-57E



The longitudinal control system is a reversible type and consists of mechanically-operated conventional elevators. Aerodynamic balancing is used on the elevators in the form of a spring tab, unshielded horn balance, and trailing edge strips on each elevator. Mass

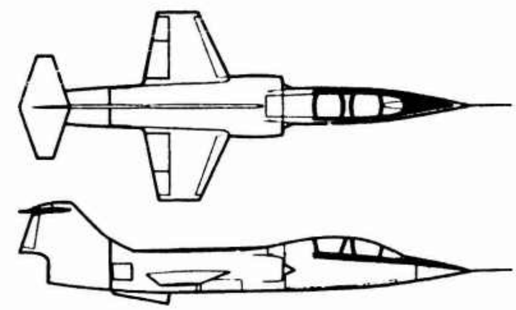
balancing of the elevator is included in the horn balances. The trailing edge strips are used on the elevator to produce a favorable response effect. Longitudinal trim is provided by an electric actuator that changes the angle of incidence of the horizontal stabilizer.

The lateral control system consists of mechanically operated conventional ailerons. A spring tab mounted on each aileron is used for aerodynamic balancing. Lateral trim is provided by an electric actuator that, in effect, shifts the neutral position of the control wheel by varying the elongation of a spring at the base of the control column.

Directional control is provided by a conventional rudder that is normally operated by hydraulic pressure. In the event of hydraulic pressure failure, the system becomes reversible and the rudder can be operated manually. The rudder is balanced aerodynamically by means of a balance tab, unshielded horn balance, and a beveled trailing edge. The rudder is mass balanced by addition of weight in the horn balance.

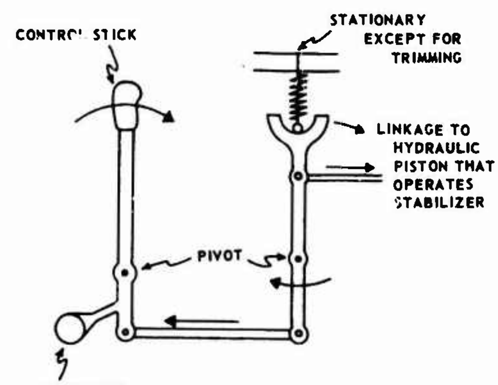
When the rudder is operated by hydraulic pressure, the system becomes irreversible and requires artificial feel. The artificial feel used is a V-feel system which varies the mechanical advantage of the rudder in proportion to the speed of the aircraft, producing high rudder pedal forces at high speeds and low rudder pedal forces at low speeds. When the rudder is operated manually, trim is provided by using the balance tab as a combined trim and balance tab. When the rudder is hydraulically operated, the balance tab is used only as a balance tab and trim is provided by an actuator in the feel system which varies the neutral position of the rudder pedals.

LOCKHEED F-104B

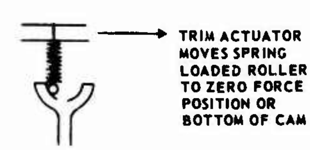


The complete control system, i.e., longitudinal, lateral, and directional, of the F-104B is a full hydraulically powered irreversible system.

Longitudinal control is provided by means of a controllable horizontal stabilizer mounted at the top of the vertical stabilizer. Artificial feel is provided by means of a bobweight and a variable feel cam arrangement, which is similar to a bungee or spring feel system. Shown below is a schematic of the cam and spring arrangement which indicates its principle of operation.



ARTIFICIAL FEEL SYSTEM FOR LONGITUDINAL CONTROL SYSTEM



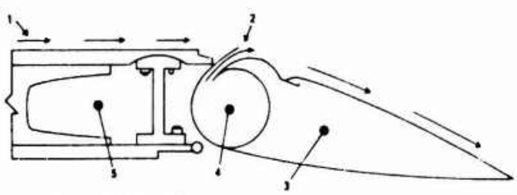
PRINCIPAL OF TRIM

This type of system is unique in that the shape of the curve of stick force versus stabilizer deflection depends on the slope of the cam. Longitudinal trim is provided by an electric actuator that returns the spring loaded roller on the cam to the zero force position. (See preceding sketch.)

Lateral control consists of conventional ailerons mounted on the outboard section of the wing. Artificial feel is provided by means of a bungee. To prevent a dangerously high rate of roll and possible inertial coupling problems at higher speeds the aileron deflections are limited to plus or minus 15 degrees with landing flaps up. This is 5 degrees less deflection than with landing flaps down. Trim is provided by means of an electric actuator that repositions the neutral position of the bungee.

Directional control is provided by a conventional rudder. Artificial feel is provided by means of a bungee. With landing flaps up maximum rudder deflection is decreased from plus or minus 20 degrees to plus or minus 6 degrees by mechanical stops at the rudder pedals. This is done to prevent structural failure of the vertical tail at high speeds. Rudder trim is provided through the yaw damper circuit to the rudder.

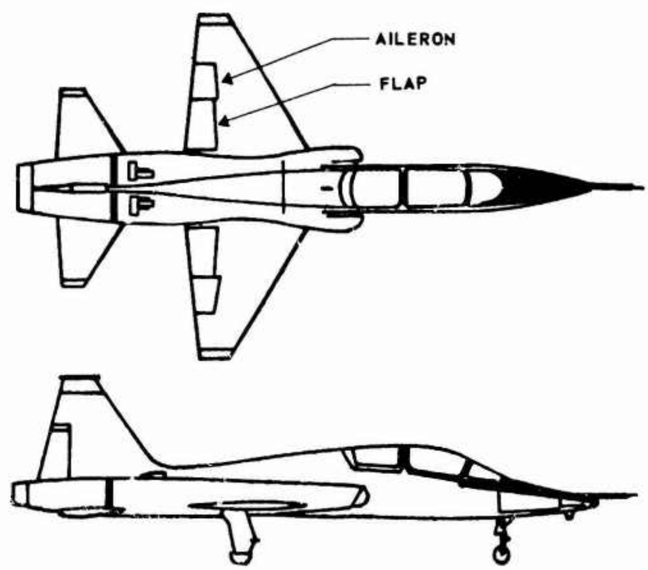
BOUNDARY LAYER CONTROL



1. Airflow
2. Boundary layer nozzle
3. Trailing edge flap
4. Boundary layer control duct
5. Aft wing section

In the above figure the air is bled from the compressor and ducted to the boundary layer control manifold which is located above the trailing edge flap hinge. The boundary layer control manifold has a series of nozzles which direct high pressure air over the upper surface of the flap when they are placed in the landing position. The high velocity created by this jet of air re-energizes the boundary layer, causing it to adhere to the curved fairing and bend around and pass over the upper surface of the flap. This induces the adjacent layer of air to adhere and bend through the flap deflection angle, thus preventing airflow separation and resulting in a reduced landing speed. The system operation is automatic.

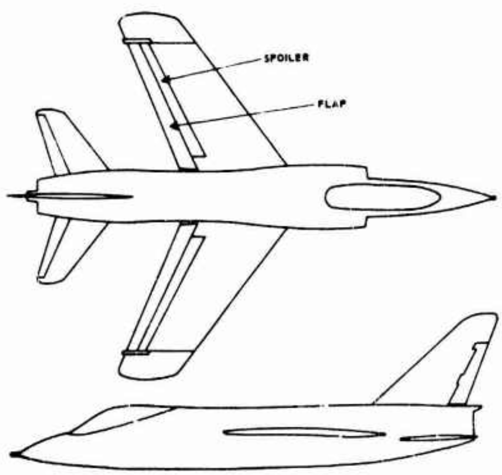
NORTHROP T-38



The T-38 has a fully powered irreversible control system with conventional ailerons and rudder, and an all-moveable horizontal tail. This system is quite similar to the F-104B except for the type of artificial feel in the horizontal stabilizer system. All control surfaces are hydraulically operated. The horizontal tail system's artificial

feel consists of a bungee and a positive bobweight. Artificial feel for the ailerons and rudder is achieved by means of bungees. Maximum rudder deflection is reduced also with the landing gear up to prevent structural damage to the vertical tail. Trim is provided by electric actuators that reposition the neutral (no load) position of the bungees. The flaps are mechanically interconnected to the horizontal tail to automatically change its angle to the trim position when the flaps are actuated - trailing edge down when the flaps are extended.

GRUMMAN F-11F (TIGER CAT)



The longitudinal control system of the F-11F is a hydraulically-operated irreversible system consisting of an all-movable horizontal stabilizer and geared elevator much like that of the F-86F. One unique feature of the system is that when the wing flap control is in the "up" position, the elevator is automatically locked in line with the stabilizer and the two units move as a single unit. When the wing flaps are in the down position the elevator is geared to stabilizer motion. When the control stick is pushed forward the leading edge of the stabilizer moves up and the trailing edge of the elevator moves down. When the control stick is pulled aft, the stabilizer leading edge moves down and the elevator trailing edge moves up. This, in effect, changes the camber of the tail as well as the angle of attack which improves pitch control in the lower speed range.

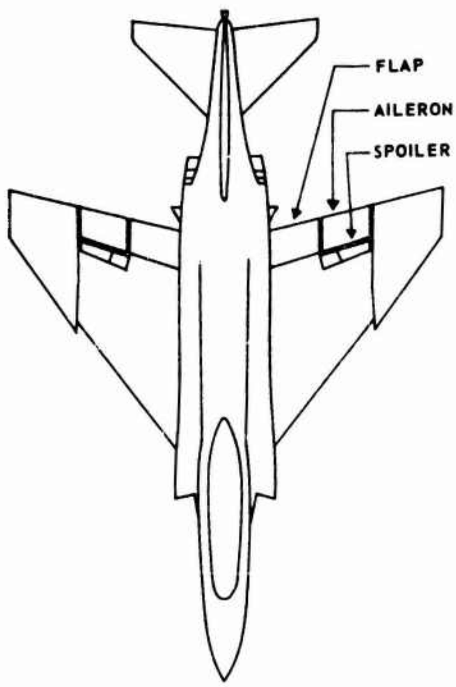
Artificial feel is supplied to the pilot by means of a cam and spring-loaded follower (modified bungee) that is quite similar to that of the F-104B. (See schematic of control system.) In addition, a positive bobweight is used in the system for feel. Longitudinal trim is provided by an electric actuator that varies the neutral position of the artificial feel system which in turn varies the neutral position of the stick.

The lateral control system is a hydraulically-operated irreversible system consisting of a movable flaperon (spoiler) on the top of each wing. This type of system is used on the F-11F so that full span wing flaps can be used to provide lower carrier approach speeds. The flaperons are hinged forward and move through an arc of 55 degrees when the control stick is deflected fully. When control stick is deflected right, the right flaperon rises with the left flaperon remaining flush and vice-versa. The artificial feel system consists of a cam and spring-loaded follower (modified bungee) that is almost identical to the longitudinal system arrangement. Lateral trim is provided by the same method as longitudinal trim, i.e., varying the neutral position of the artificial feel system.

The directional control system is irreversible and consists of a hydraulically-operated conventional rudder. To prevent possible structural failure of the vertical tail the rudder is limited to 5 degrees either side of neutral by automatic rudder pedal position stops with flaps in up position. With flaps in the down position rudder travel is increased to 22

degrees either side of neutral for increased directional control at lower speeds. This is quite similar to the system of the F-104B. The artificial feel system consists of a cam and spring-loaded follower (modified bungee) quite similar to that of the longitudinal and lateral system. Movement of the cam against the follower induces a force in the control system that opposes pedal movement. This force is dependent on pedal position only. Trim is provided by an electric actuator that operates through the yaw damper to trim the rudder in much the same way as the F-104B rudder trim.

MCDONNELL F-4C



The longitudinal control system of the F-4C consists of a hydraulically-operated horizontal stabilizer referred to by McDonnell as a "stabilator." Artificial feel is provided the system by the addition of a bobweight and a Q-spring which makes stick force a function of dynamic pressure. Trim is accomplished by adjusting the neutral position of the artificial feel system.

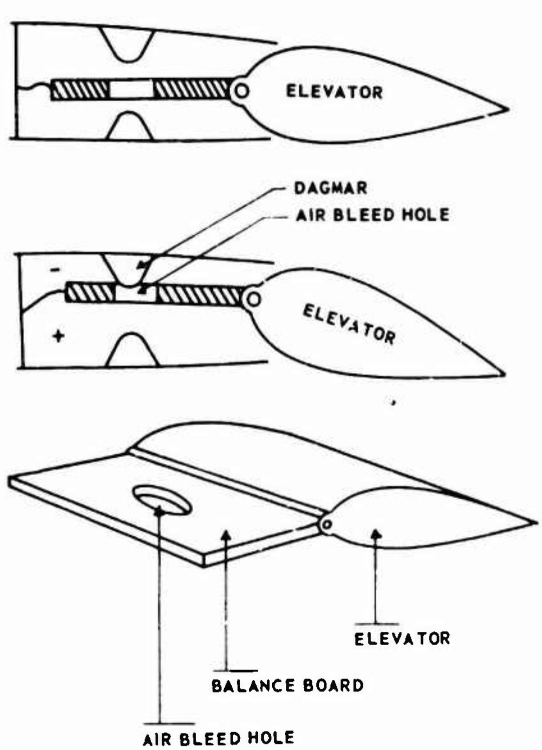
The lateral control system consists of hydraulically-operated ailerons and spoilers. The system is quite unique in that the ailerons deflect downward only. When the control stick is deflected right for a roll to the right, the right aileron remains flush, the right spoiler deflects upward, the left aileron deflects downward, and the left spoiler remains flush. The system operates opposite for a roll to the left. Spoilers are used in the F-4C to partially alleviate wing twisting during high-speed roll maneuvers, and the aileron spoiler combination is used to increase roll rate and roll effectiveness at low speeds. Maximum roll rate is decreased above 220 knots indicated airspeed by a pressure switch in the system that decreases the maximum available hydraulic pressure which in turn reduces the deflection of the aileron and spoilers in proportion with dynamic pressure. The reason for this is to avoid dangerously high rates of roll which might cause inertial coupling. Artificial feel is provided the system by means of a bungee. Trim is made available by adjusting the neutral position of the bungee.

An aileron-rudder interconnect is incorporated in the lateral and directional control system to automatically coordinate turns made at speeds below 225 knots. This is accomplished by providing rudder displacement as a function of aileron displacement.

CONVAIR 880

The longitudinal control system of the 880 is a reversible system that consists of conventional elevators mounted on the rear spar of the horizontal stabilizer. Movement of the elevators is accomplished indirectly by movement of a control or servo tab located on

the trailing edge of each elevator. An unusual feature in the system is that the left and right elevators are independently hinged and may be moved independently of each other on the ground. However, the elevators will always act together in flight since the control tabs are interconnected. Centering springs located at the elevator hinge line and attached to the control tabs are used to overcome system friction, and return the control tabs to neutral when the pilot releases stick pressure. An internal seal type of aerodynamic balancing called a balance board is used on the elevators to reduce stick forces. The balance boards are quite unique in their design in that they provide maximum assistance to the pilot at large deflection of the control surfaces where he needs assistance and very little at small deflection of the control surfaces where assistance is not required. The principle of operation of the balance boards is described as follows. (Refer to figure shown below.)



If the control surface is deflected in either direction, a differential pressure will exist between each side of the balance board, with the increased pressure being on the deflected side. This differential pressure will exert an additional force that will aid control surface movement. The differential pressure will increase as the control surface deflection is increased due to the dome-shaped "dagmar" entering the airbleed hole and effectively sealing off each side of the balance board. At maximum deflection of the elevator, the airbleed hole is closed completely and the differential pressure is at its maximum. This gives the maximum aid in moving the control surface at precisely the time it is required. Once the control surface is again centered differential pressures will no longer exist.

Longitudinal trim is normally provided by an electrical actuator that changes the angle of incidence of the horizontal stabilizer. In the event of electrical failure, trim can be accomplished manually by trim cables.

The lateral control system is a partially reversible system that consists of conventional ailerons augmented by two sets of wing spoilers on each wing. The ailerons, located approximately half way out the wing span, between the inboard and outboard flaps (see picture), are indirectly operated through control tabs on the trailing edge of each aileron. The ailerons are aerodynamically balanced by balance boards similar to those of the longitudinal system. The wing spoilers, located forward of each flap are hydraulically operated and incorporate a blowdown safety design that prevents damaging the spoilers at high speeds. The lateral control system operates as follows (right roll example): When the control wheel is rotated to the right, the right aileron and spoilers deflect up, the left

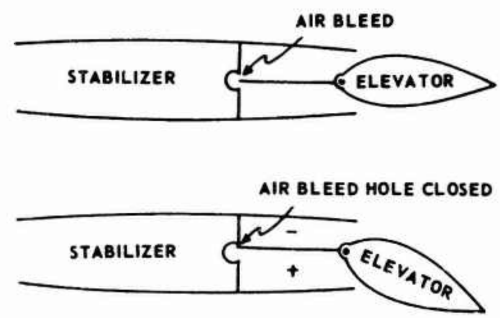
aileron deflects down, and the left spoilers remain flush. The spoilers are also used as speed brakes by deflection of both sets of wing spoilers together. If the spoilers are being used as speed brakes, and the control wheel is rotated right, the right spoiler will remain in the up position and the left spoiler will deflect downward toward the flush position. Lateral trim is provided by means of a manually-operated trim tab located on the trailing edge of each aileron.

The directional control system is reversible and consists of a conventional rudder that is operated indirectly by a control tab on the trailing edge of the rudder. A center spring mechanism is used in conjunction with the control tab to return it to its neutral position when pressure is removed from the rudder pedals. The rudder is aerodynamically balanced by the use of five balance boards identical to those used in the elevator. One unique feature of the control system is that although it is a reversible system, a feel system is designed into the rudder control system. This makes the system force limited at high airspeeds, thus preventing overstressing of the vertical tail through excessive side loads. Rudder trim is provided by means of a manually-operated trim tab located on the trailing edge of the rudder.

BOEING KC-135

The longitudinal control system is a reversible system and control is provided by means of elevators mounted on an adjustable horizontal stabilizer. The elevators are independently hinged in much the same way as those of the Convair 880. The elevators are deflected together by manually-operated interconnected control tabs (servo tabs) located on the trailing edge of the elevators. Once full control tab is attained, additional elevator deflection can be obtained by further movement of the control column, though this is not practical during flight due to high stick forces. Each elevator is aerodynamically balanced by means of five balance panels (internal seals) that operate quite similarly to the balance boards of the Convair 880.

Shown below is a sketch of the balance panel.



Trim is accomplished by varying the angle of incidence of the horizontal stabilizer by an electrical or manual actuator. In addition, a trim tab is located on each elevator and is actuated by movement of the horizontal stabilizer. The purpose of these tabs is to position the elevators in line with the stabilizer and reduce the upward and downward movement of the elevator which is a result of aerodynamic loads on the elevator caused by positioning of the stabilizer.

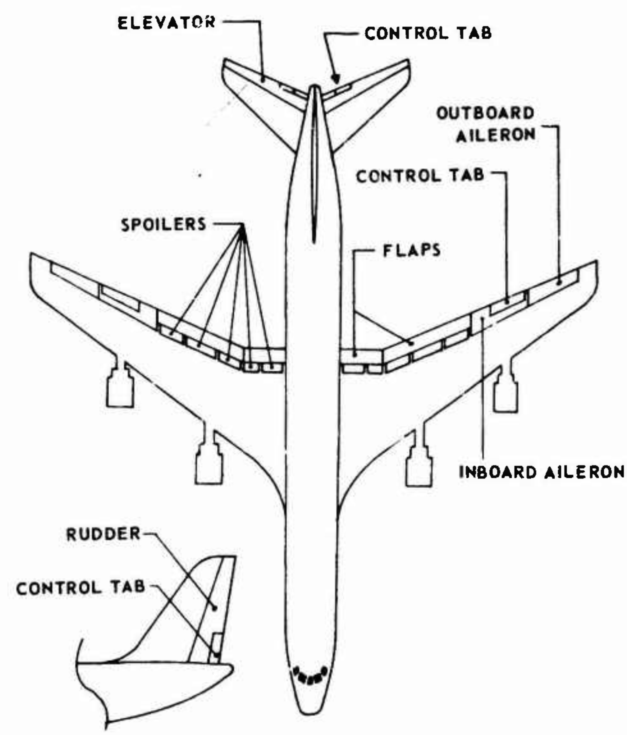
The lateral control system is a partially reversible system and consists of inboard and outboard ailerons that are used in conjunction with two sets of spoilers on the top of each wing. The inboard ailerons, which are considerably smaller than the outboard ailerons are used in conjunction with the spoilers for lateral control throughout the speed range of the aircraft. When flaps are lowered for low speed flight, the outboard ailerons are locked out of the system and remain faired in the neutral position. This prevents wing twist at high speeds. The outboard ailerons

are aerodynamically balanced by use of internal seals and a spring tab on each aileron. Movement of the inboard ailerons is by action through the spring tab. The spring tab is also used as a trim tab for lateral trim. The lateral control system operates as follows (right roll example): When the control wheel is rotated to the right, the right ailerons and right spoilers deflect upward. The left ailerons deflect downward, and the left spoilers remain flush. The spoilers on both wings are also used as speedbrakes by being deflected together. If the spoilers are being used as speed brakes, and the control wheel is rotated right, the right spoiler will remain up, and the left spoiler will deflect down.

The directional control system of the KC-135 consists of a conventional rudder that incorporates four rudder tabs, i.e., a trim tab, a spring tab, a control tab, and an antibalance tab. The antibalance tab is located on the trailing edge of the control tab. The purpose of tha antibalance tab is to increase the effectiveness of the control tab. In addition five balance panels (internal seal) are used for aerodynamic balancing of the rudder. The rudder control surface is mechanically operated from the pilot's rudder pedal through the control tab and spring tab. Rudder trim is provided by a mechanical manually-operated trim tab.

The control system of the Boeing 707 is almost identical to the KC-135 except for the directional control system. The 707's rudder incorporates aerodynamic balancing only in the form of balance panels. The rudder has one control tab used to deflect the rudder. Whenever the rudder is deflected in excess of 15 degrees, the control tab hits its stop, a rudder hydraulic control unit takes over the rudder system and it then becomes an irreversible system. Artificial feel is supplied to the pilot by a Q-spring assembly which provides an artificial feel proportional to the dynamic pressure and rudder deflection.

DC-8



The longitudinal control system of the DC-8 is a partially reversible system that consists of conventional elevators that are manually operated indirectly by control tabs. Aerodynamic balancing is incorporated in the elevator by means of a balance tab on each elevator and an overhang balance. A centering spring is provided in the system to provide more positive centering of the controls and also for additional control forces. Trim is provided by means of an electric actuator that varies the angle of incidence of the horizontal stabilizer.

The lateral control system consists of inboard and outboard ailerons interconnected by a torsion rod. Wing spoilers on the top of

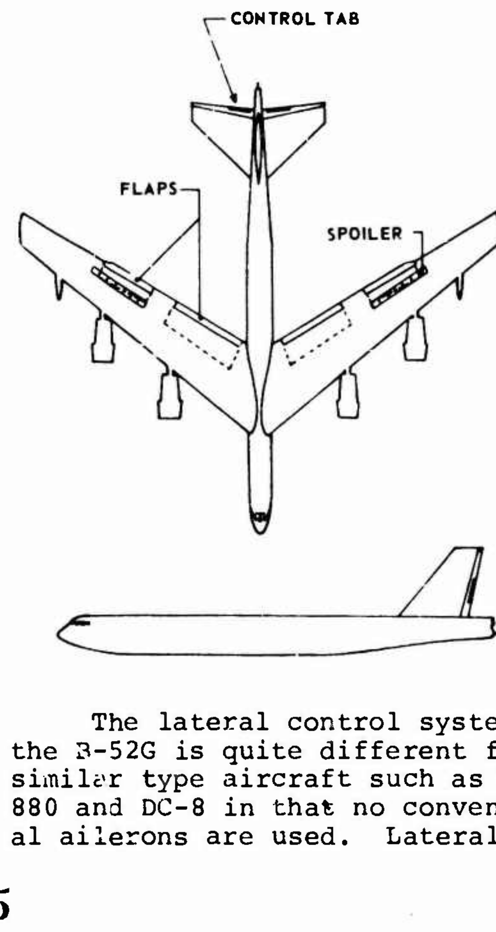
each wing are used in conjunction with the ailerons, whenever the landing gear is extended, for better lateral control during landing. Both inboard and outboard ailerons use aerodynamic balancing. Hydraulic power normally operates the inboard ailerons which in turn operate the outboard ailerons through the interconnecting torsion rods. At low speeds when the inboard ailerons are deflected, the outboard ailerons deflect an equal amount; however, as speed is increased, the outboard aileron deflection becomes less due to the twisting of the interconnecting torsion rods. This, in effect, reduces wing twist as speed is increased. Artificial feel is supplied to the pilot by means of a bungee spring. In the event that hydraulic power is not available, the inboard ailerons can be manually operated indirectly by control tabs located on the trailing edge of each aileron. These control tabs are normally locked in the neutral position whenever the ailerons are operated by hydraulic power, but automatically unlock when hydraulic power is not available. Lateral trim is accomplished manually by adjusting the neutral (no load) position of the bungee spring.

Directional control is provided by means of a hydraulically-powered conventional rudder. The rudder uses aerodynamic balancing in the form of an overhang balance. Artificial feel is supplied to the system by a bungee spring. When hydraulic power is not available, the rudder can be controlled indirectly by a mechanically-operated control tab. Like the aileron control tab, the rudder control tab is normally locked in the neutral position when hydraulic power is available, but automatically unlocks if hydraulic power is not available. Rudder trim is provided by adjusting the neutral (no load) position of the bungee spring.

B-52G

The longitudinal control system of the B-52G is quite similar to that of the 707. The system is reversible and consists of conventional independently hinged elevators positioned by manually-operated interconnected control tabs. The elevator is mass balanced by weights in the elevator forward of its hinge line. Aerodynamic balancing is incorporated in the elevator by use of balance panels (internal seals) which operate similarly to those used in the 707. Additional feel is supplied to the elevator by means of a Q-spring. In addition, a centering spring is used to assist in centering the elevator at low indicated airspeed. Longitudinal trim is provided by an electric actuator that varies the angle of incidence of the horizontal stabilizer.

BOEING B-52



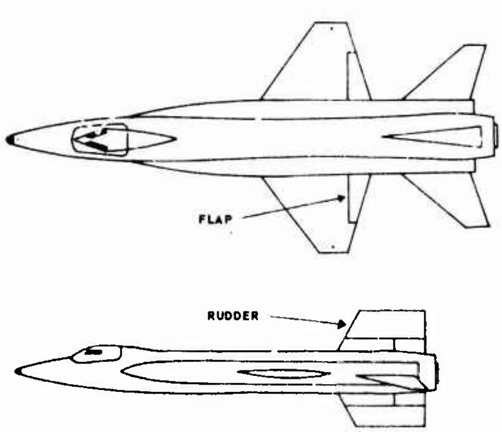
The lateral control system of the B-52G is quite different from similar type aircraft such as the 880 and DC-8 in that no conventional ailerons are used. Lateral con-

trol is provided by means of two sets of spoilers located on the top of each wing (see picture). The spoilers are hydraulically operated and artificial feel is supplied to the pilot by means of bungee springs. The spoilers can be deflected fully up to an indicated airspeed of 250 knots. Above this speed, hydraulic pressure is insufficient to obtain full deflection. The reason for this is to reduce wing twist at high indicated airspeeds. The spoilers are also used as speed brakes in the same manner as the 707.

Lateral trim is provided by an electric actuator that repositions the neutral (no load) position of the bungee springs.

The directional control system of the B-52G is reversible and consists of a conventional rudder operated by a control tab on the trailing edge of the rudder. The rudder is balanced aerodynamically by the same method as the elevator, i.e., balance panels. Additional feel is supplied to the pilot by a Q-spring which is almost identical to that of the elevator system. Directional trim is provided by manually adjusting the control tab through the Q-spring feel system.

X-15



The X-15 has two control systems, an aerodynamic flight-control system and a reaction control system. The reaction control system, often called a ballistic control system, is used to control the aircraft's attitude and altitude where the aerodynamic surfaces are relatively ineffective.

1. Aerodynamic Flight Control System

Longitudinal (pitch) and lateral (roll) control in this system is provided by a hydraulically-actuated horizontal stabilizer. The stabilizer consists of two all-movable, one-piece surfaces that can be moved simultaneously or differentially. Longitudinal (pitch) control is obtained by simultaneous movement of the left and right stabilizers. Lateral (roll) control is obtained by differential movement of the horizontal stabilizer surfaces. Combined pitch and roll control is obtained by compound movement of the horizontal stabilizer surfaces. Artificial feel is supplied to the system by bungees. Longitudinal trim is obtained by shifting the neutral (no load) position of the bungee. Lateral trim is adjustable only on the ground.

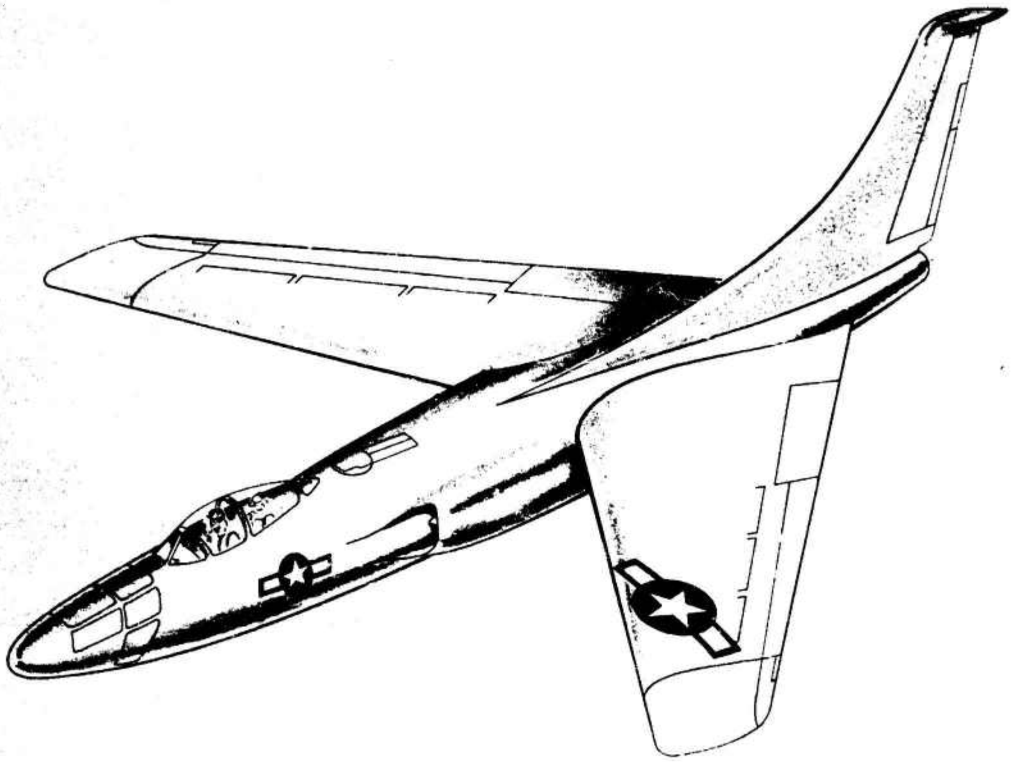
Directional control is obtained by deflection of the upper and lower stabilizers that are interconnected and hydraulically actuated. Artificial feel is provided by a bungee. Prior to landing, the lower stabilizer is dropped since it extends below the landing skids. Directional trim is adjustable only on the ground.

2. Reaction Control System

Reaction control is provided by small rockets located in the nose section and wing that use a monopropellant (hydrogen peroxide) which is converted by catalytic action to superheated steam and oxygen. The

reaction of the escaping gases causes the aircraft to move about the selected or combined axes. The control used by the pilot consists of a control stick handle located on the left console of the cockpit. Upward and downward movement of the control handle operates the rockets located on the top and bottom of the nose section, respectively, and gives the aircraft pitch control. Sideward movement of the control handle operates the rockets located on the sides of the nose section and provides the aircraft with directional control. Rotating the control handle operates the rockets located in the wing section and provides lateral control.

Artificial feel is provided the pilot for all three axes of operation by spring bungees connected to the system. The angular acceleration and velocity of the aircraft vary with the amount and duration of the ballistic control handle application. The velocity tends to sustain itself after the stick is returned to the neutral position. A subsequent stick movement opposite to the initial one is required to cancel the original induced velocity.



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